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CORRELATION BASED METHOD FOR COMPARING AND RECONSTRUCTING QUASI-IDENTICAL TWO-DIMENSIONAL STRUCTURES

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Abstract

We show a method for comparing and reconstructing two similar amplitude-only structures, which are composed by the same number of identical apertures. The structures are two-dimensional and differ only in the location of one of the apertures. The method is based on a subtraction algorithm, which involves the autocorrelations and cross-correlation functions of the compared structures. Experimental results illustrate the feasibility of the method.

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The ability to specify accurately the differences between similar structures becomes very important in pattern recognition. Visual methods for measuring distances between corresponding structural elements by using high-resolution optical equipment [1,2] are rather time cumbersome and subjective.

Vander Lugt holographic filters [3] provide a useful comparison of similar amplitude-only structures, which is based on correlation functions [4]. Such filters are holograms of the Fourier spectrum of the compared structures. If the Vander Lugt filter corresponding to a specific structure is illuminated by the Fourier spectrum of another one, the cross-correlation of the structures will be obtained.

If these structures are identical, their cross-correlation equals the autocorrelation of one of them, which must be symmetrical. But if they are not, the cross-correlation will be non-symmetrical. Thus, this symmetry distortion will reveal that the compared structures are different but it cannot specify such differences.

The concept of *class of pairs* [5,6] gives more insight to the correlation analysis applied to the study of amplitude-only structures. Depending on their separation, the pairs of structural elements can be classified into groups or classes. For regular structures, the smaller the separation of the pairs the greater the population of the corresponding class, i.e. the number of pairs of structural elements with such a separation.

The classes of pairs associated to a specific amplitude-only structure of identical elements can be determined directly in its correlation function. Usually, this correlation will consist of peaks, whose morphology is given by the autocorrelation of a structural element alone. The position of the peak maximum determines the separation vector of a class of pairs and its height the population of this class.

With a basis on these two parameters (i.e. peak positions and heights) it is possible to determine the distribution of the elements into the structure. After applying a deconvolution algorithm on the peak morphology, a complete reconstruction of the structure can be realised. On the other hand, given a specific structure we can determine its differences to another one by applying a subtraction algorithm, which involves both their autocorrelations and their cross-correlations. So, the classes of pairs on which they differ will be revealed accurately. The application of this last procedure in case of two-dimensional structures is the subject of the present short note.

Let us consider two similar structures to be compared, each one with N identical square apertures, distributed on an opaque screen as in Figs.1a,b,c. The transmittance of any aperture will be $A\, rect(x/a)\, rect(y/a) \otimes \delta(x-x_n, y-y_n)$, with $0 \leq A \leq 1$ the aperture transmission, a the side length of the aperture, and x_n, y_n the coordinates of the centre of the n-th aperture. The function $rect(\cdot)$ takes the value 1 inside the aperture and null otherwise, $\otimes$ denotes the convolution operator, and $\delta(\cdot)$ is the Dirac's delta function [4].

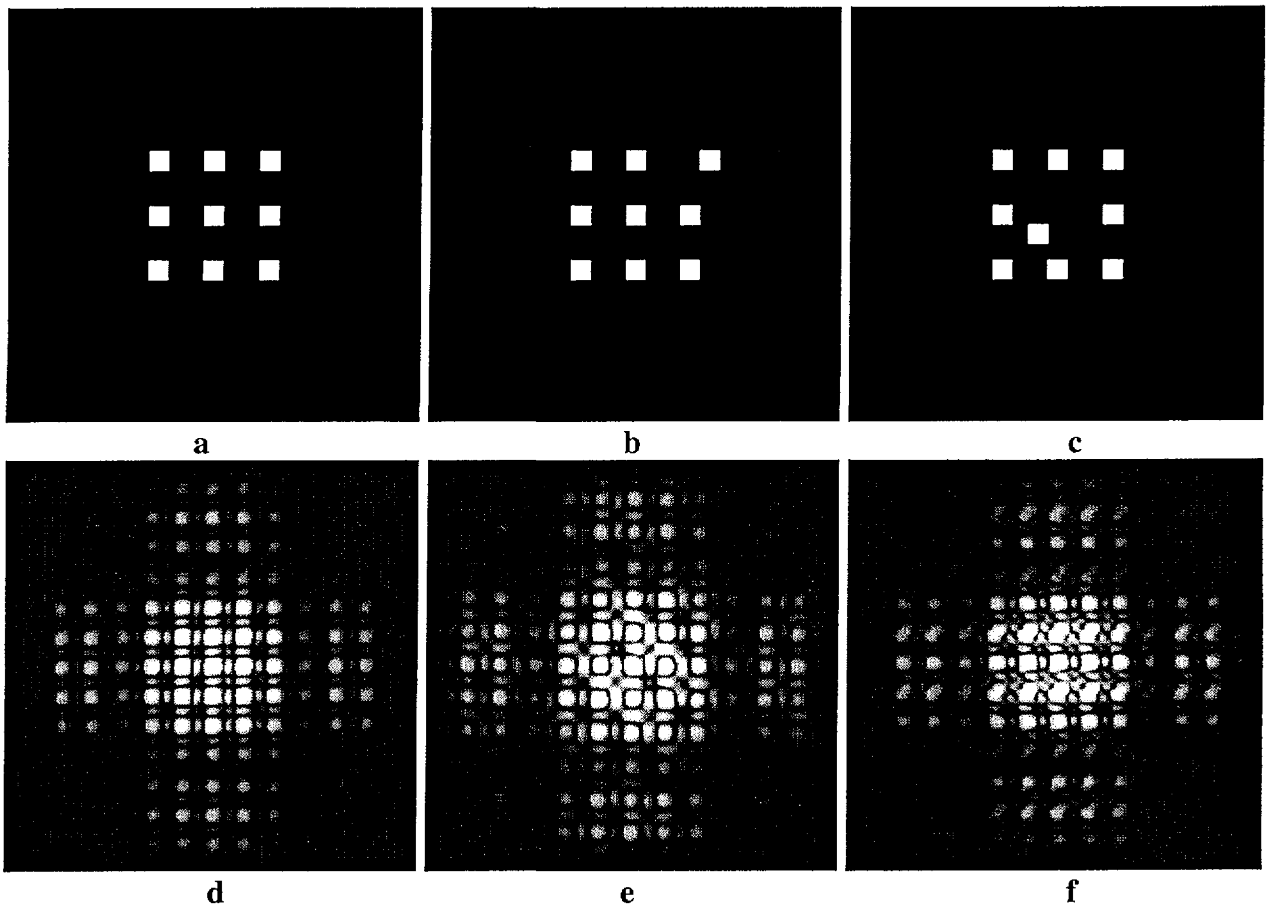


Fig. 1: a-c) similar structures of 9 elements (identical square apertures of side *a* and transmission *A*). In comparison with a), one aperture in b) and c) was shifted on the diagonal a distance equal to the aperture diagonal. d-f) Intensity distributions of the corresponding Fraunhofer interferograms

Then, the transmittance of a structure will be given by

$$t(x,y) = A\ rect(x/a)\ rect(y/a) \otimes \sum_{n=1}^{N} \delta(x-x_n; y-y_n) = t_0(x,y) \otimes \sum_{n=1}^{N} \delta(x-x_n; y-y_n). \quad (1)$$

By illuminating the structure with a spatially coherent optical field of wavelength λ, the intensity distribution of the Fraunhofer pattern can be obtained at the rear focal plane of a Fourier lens of focal length f (Figs. 1d,e,f), that is:

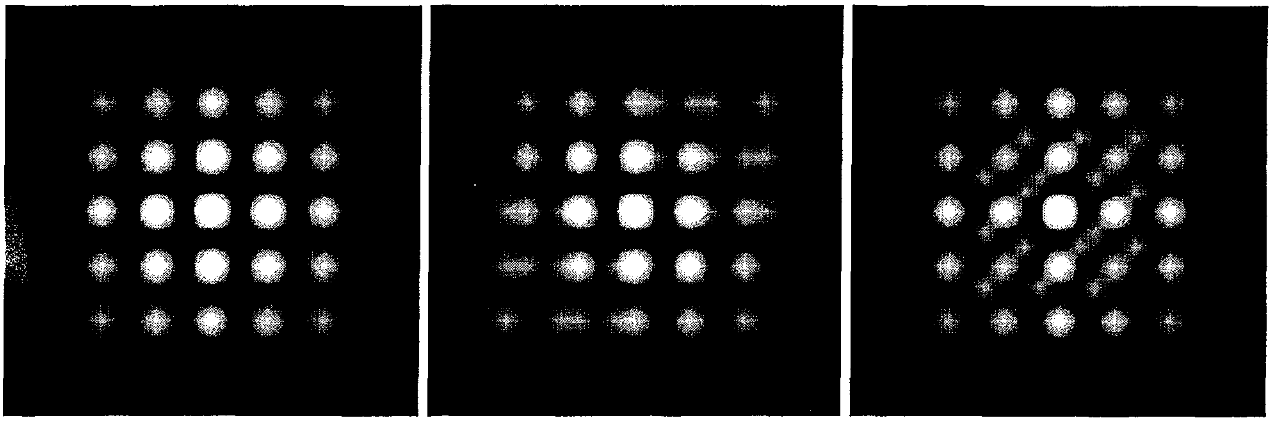
$$I(u,v) = I_0(u,v)\left[N + \sum_{m=1}^{N-1} \sum_{n=m+1}^{N} 2\cos\{2\pi[u(x_m - x_n) + v(y_m - y_n)]\}\right]. \quad (2)$$

$I_0(u,v) = (1/\lambda f)^2 \left(A\, a^2\, \mathrm{sinc}(\pi u a)\, \mathrm{sinc}(\pi v a)\right)^2$ denotes the intensity distribution of the diffraction pattern provided by any aperture, and $u = \xi_x / \lambda f$, $v = \xi_y / \lambda f$ are the spatial frequencies associated to the geometrical coordinates ξ_x, ξ_y of the Fraunhofer interferogram.

According to the Winner-Khintchine or autocorrelation theorem [7], the Fourier spectrum of eq.(2) yields the autocorrelation of the transmittance eq.(1), i.e.

$$\Gamma_{tt}(x,y) = \Gamma_0(x,y) \otimes \sum_{m=1}^{N}\sum_{n=1}^{N} \delta(x-(x_m - x_n)\,,\, y-(y_m - y_n))\,, \tag{3}$$

where $\Gamma_0(x,y)$ exhibits a pyramidal shape because it is the Fourier spectrum of $I_0(u,v)$. In practical situations, the autocorrelation eq.(3) can be obtained experimentally by using a Vander Lugt filter [4] or numerically by calculating the Fourier spectrum of the intensity distribution of the Fraunhofer interferogram, after its capture by a conventional CCD array (Fig. 2).



a **b** **c**

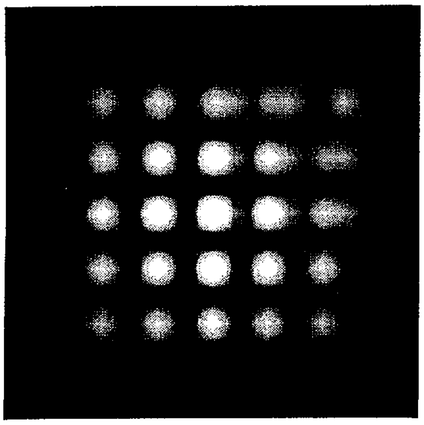
Fig. 2: Autocorrelation modules of the corresponding structures in Fig. 1 a-c

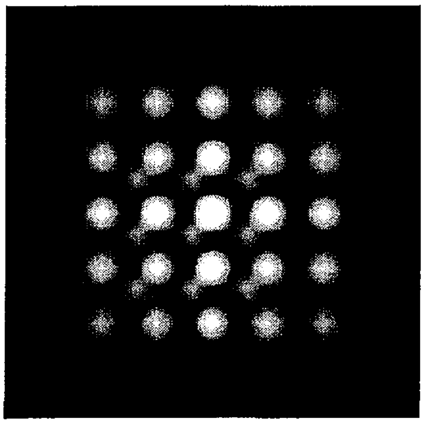
Now, let us consider two quasi-identical or similar structures of transmittances $f(x,y)$ and $g(x,y)$, which only differ in the location of one aperture, i.e. the k-th aperture, so that the coordinates of their apertures satisfy the condition $(x_n^f, y_n^f) = (x_n^g, y_n^g)$ for $n \neq k$. The superscripts are corresponding to each structure. As a consequence, $x_k^g - x_n^f = x_k^g - x_n^g$ and $y_k^g - y_n^f = y_k^g - y_n^g$ for $n \neq k$. So, the cross-correlation between such structures is given by (Fig.3):

$$\Gamma_{fg}(x,y) = \Gamma_0(x,y) \otimes \sum_{m=1}^{N}\sum_{n=1}^{N} \delta\{x-(x_m^g - x_n^f), y-(y_m^g - y_n^f)\}, \tag{4}$$

Once we have the autocorrelations of the structures to be compared and their cross-correlation, we calculate numerically the subtractions $\Gamma_{fg}(x,y) - \Gamma_{ff}(x,y)$ and $\Gamma_{fg}(x,y) - \Gamma_{gg}(x,y)$. The first one provides information about $f(x,y)$ and the second one about $g(x,y)$, as we will show in the following.

Taking into account the above condition between the aperture coordinates, these subtractions yield





a **b**

Fig. 3: a) cross-correlation between the structures in Figs 1a and 1b
b) cross-correlation between the structures in Figs 1a and 1c

$$\Gamma_{fg}(x,y)-\Gamma_{ff}(x,y)= \Gamma_0(x,y)\otimes\left(\sum_{m=1}^{N}\left[\delta\{x-(x_m^g-x_k^f),y-(y_m^g-y_k^f)\}-\delta\{x-(x_m^f-x_k^f),y-(y_m^f-y_k^f)\}\right]\right) \tag{5a}$$

and

$$\Gamma_{fg}(x,y)-\Gamma_{gg}(x,y)= \Gamma_0(x,y)\otimes\left(\sum_{m=1}^{N}\left[\delta\{x-(x_m^g-x_k^f),y-(y_m^g-y_k^f)\}-\delta\{x-(x_m^g-x_k^g),y-(y_m^g-y_k^g)\}\right]\right) \tag{5b}$$

respectively. The coordinates of the k-th apertures of both structures are related by $x_k^g = x_k^f + \varepsilon_k$ and $y_k^g = y_k^f + \eta_k$, with $\varepsilon_k \neq 0$ and $\eta_k \neq 0$ the components of the difference vector between the structures. Thus, eqs.(5a,b) become (Figs. 4, 5)

$$\Gamma_{fg}(x,y)-\Gamma_{ff}(x,y)=\Gamma_0(x,y)\otimes\left(\left[\delta\,(x-\varepsilon_k\,,y-\eta_k)-\delta\,(x,y)\right]\otimes\sum_{n=1}^{N}\delta\,(x+\Delta x_{nk}^f\,,y+\Delta y_{nk}^f)\right) \tag{6a}$$

and

$$\Gamma_{fg}(x,y)-\Gamma_{gg}(x,y)=\Gamma_0(x,y)\otimes\left(\left[\delta\,(x-\varepsilon_k\,,y-\eta_k)-\delta\,(x,y)\right]\otimes\sum_{m=1}^{N}\delta\,(x-\Delta x_{mk}^g\,,y-\Delta y_{mk}^g)\right), \tag{6b}$$

with $\Delta x_{mk}^{(f,g)} = x_m^{(f,g)} - x_n^{(f,g)}$ and $\Delta y_{mk}^{(f,g)} = y_m^{(f,g)} - y_n^{(f,g)}$ respectively.

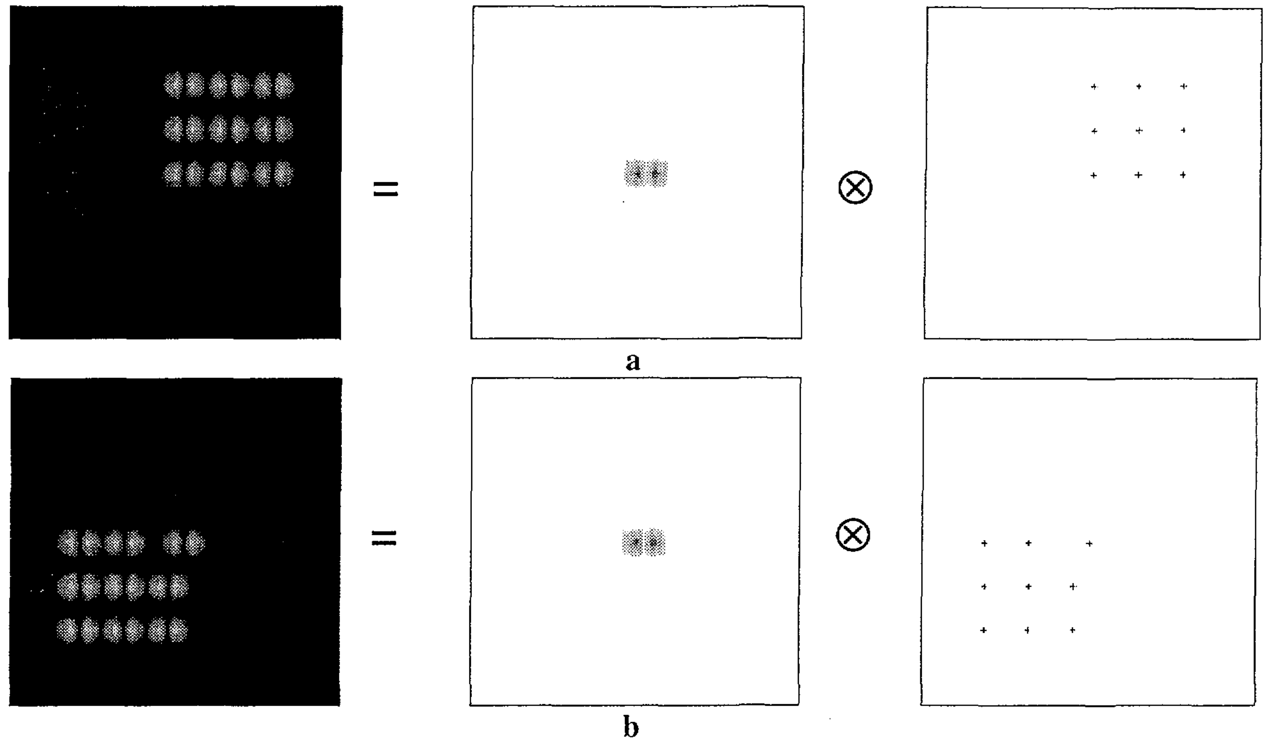


Fig. 4: Subtraction algorithm for comparing the structures in Figs. 1a ($f(x,y)$) and 1b ($g(x,y)$)

a) $\Gamma_{fg}(x,y)-\Gamma_{ff}(x,y)$ b) $\Gamma_{fg}(x,y)-\Gamma_{gg}(x,y)$

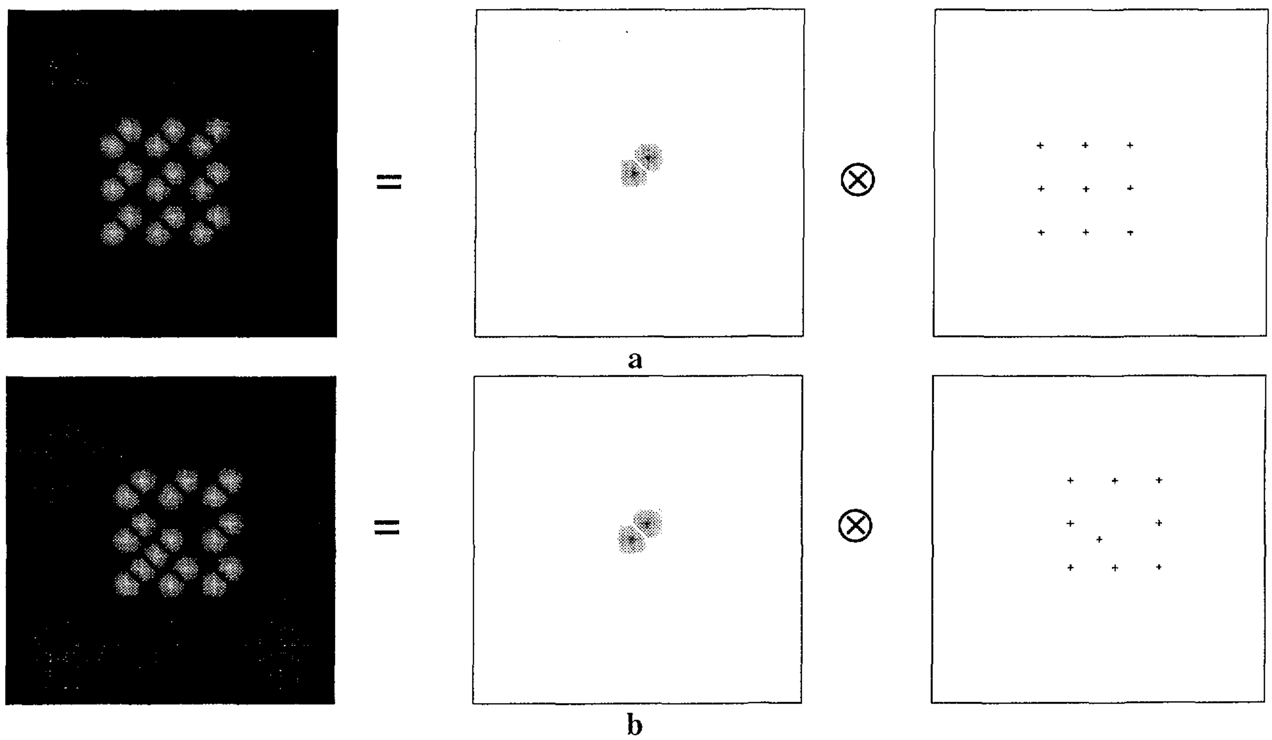


Fig. 5: Subtraction algorithm for comparing the structures in Figs. 1a ($f(x,y)$) and 1c ($g(x,y)$)

a) $\Gamma_{fg}(x,y)-\Gamma_{ff}(x,y)$ b) $\Gamma_{fg}(x,y)-\Gamma_{gg}(x,y)$

As illustrated in Figs. 4 and 5, the difference $\Gamma_{fg}(x,y)-\Gamma_{ff}(x,y)$ in eq.(6a) yields a set of correlation peaks shaped by $\Gamma_0(x,y)$. Its distribution results from the convolution between $\sum_{n=1}^{N}\delta\,(x+\Delta x_{nk}^{f}\,,y+\Delta y_{nk}^{f})$ (Figs. 4a and 5a, right pictures), which represents the aperture distribution of the structure *f(x,y)*, and $\delta(x-\varepsilon_k, y-\eta_k)-\delta(x,y)$ (Figs. 4a and 5a, middle pictures), which provides the information about the difference vector in the positions of the *k*-th aperture of both the compared structures.

The shape of the correlation peaks of the difference $\Gamma_{fg}(x,y)-\Gamma_{gg}(x,y)$ in eq.(6b) is also given by $\Gamma_0(x,y)$, but now their distribution results from the convolution between $\sum_{m=1}^{N}\delta\,(x-\Delta x_{mk}^{g}\,,y-\Delta y_{mk}^{g})$ (Figs. 4b and 5b, right pictures), which represents the aperture distribution of the structure *g(x,y)*, and $\delta(x-\varepsilon_k, y-\eta_k)-\delta(x,y)$ (Figs. 4b and 5b, middle pictures), too.

So, this method allows us not only to identify the difference between two quasi-identical structures to be compared, but also to reconstruct them.

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