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Abstract

Experimental evidence is shown that confirms the *Entendue* invariance in speckle fields. Because of this condition, the coherence patch of the speckle field can be significantly greater than the mean size of the speckles, as is shown by double exposure speckle interferometry.

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1. INTRODUCTION

The random interference pattern that appears after illuminating a diffuser with a laser beam is known as *speckle*[1,2]. Its statistical nature, properties and applications have been extensively studied[1,2]. Some of these applications show that there are experimental situations in which the coherence patch size significantly differs from the speckle size. In double exposition speckle interferometry, for example, we can obtain interference fringes when the shifts of corresponding speckles are longer than their mean diameter, provided that the coherence patch covers completely both speckles.

It is a consequence of the *Entendue* invariance[3-5] through the propagation of the speckle fields, as we show in this paper. This property should be taken into account to assure the accuracy of the techniques of the speckle metrology[2,6].

2. THEORY

Fig. 1 shows a conventional experimental set-up for recording subjective speckle patterns.[2,6]

The diffuser *D* is imaged onto the plane *IP* by the aberration free lens *L*, of focal length *f*. Thus, when a laser beam illuminates it, a subjective speckle pattern[2] is produced at *IP*, which is the image of the speckle field that emerges from the diffuser. Specifically, small areas *S'* of the speckle pattern that emerges from the diffuser are optically conjugated of areas *S* of the order of the speckle size at the plane *IP*.

The scatters inside *S'* are smaller than the resolution limit of the lens *L* and introduce a random phase distribution on the illumination. As a consequence, they can be considered as uncorrelated point sources[2].

The mean size of the speckles will depend on the size of the aperture stop *A*, located at the rear focal plane of *L*.[1,7] The detector (i.e. a photographic plate or a CCD array) for recording the speckle pattern will be located at the observation plane *OP*, which is in general defocused by an amount $z = Z_O - Z_S$.

Now, let us suppose that a gaussian laser beam illuminates the diffuser *D*. The optical field that emerges from each small area *S'* of the diffuser will be spatially incoherent and its cross-spectral density will be given by

$$W_{S'}(\mathbf{r}_{S'}, \mathbf{r}'_{S'}) = \frac{I_0}{\pi R^2} \exp\left(- \frac{|\mathbf{r}_{S'}|^2}{R^2} \right) \delta(\mathbf{r}_{S'} - \mathbf{r}'_{S'}) , \qquad (1)$$

where I_0 is the maximum intensity of the laser beam, R denotes the radius of the small area, $\mathbf{r}_{S'}$ is the position vector that localises the small area on the diffuser plane and $\delta(\mathbf{r}_{S'} - \mathbf{r}'_{S'})$ is the Dirac's delta function[8].

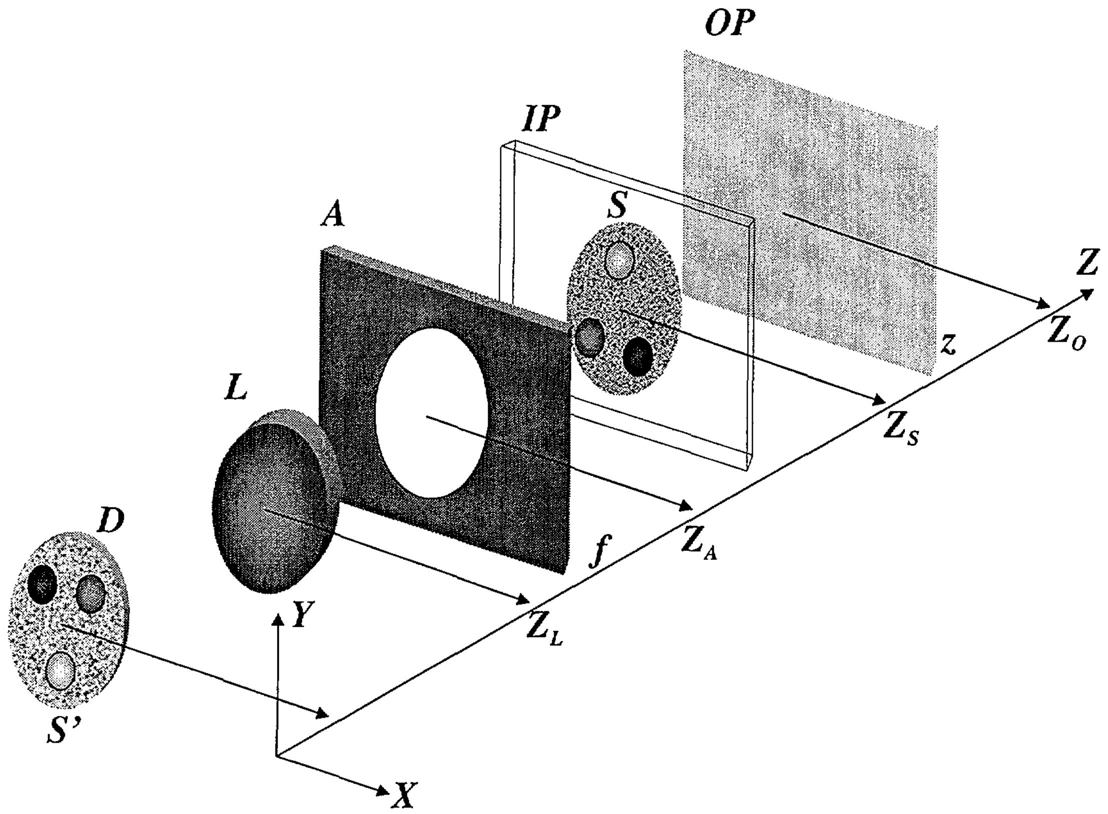

Fig. 1: Experimental set-up for recording the double exposition specklegrams: *S'* small area inside the diffuser *D*, *L* lens of focal length *f*, *A* aperture located at the rear focal plane of *L*, *S* subjective speckle optically conjugated to *S'*, *OP* observation plane, $z = Z_O - Z_S$ defocusing.

Because L is a diffraction limited (aberration free) lens, the intensity transfer function of the optical system will be[1,2]

$$T(\mathbf{k}) = \lambda^2 \exp\left(-\frac{|\mathbf{k}|^2}{K_A^2} \right), \tag{2}$$

where $\mathbf{k} = \mathbf{r}_A / \lambda f$ is the spatial frequency vector, with $\mathbf{r}_A$ the position vector on the aperture plane and λ the wavelength, and K_A is the cut-off spatial frequency determined by the aperture stop. By applying the usual procedure for partial coherent imaging[9] we obtain straightforwardly the cross-spectral density of the optical field on the small areas at the detector plane, which are corresponding to the areas *S'* on the diffuser, i.e.

$$W_{OP}(\mathbf{r}_{OP}, \mathbf{r}'_{OP}) = \frac{I_0\, \lambda\, K_A^2}{A^2} \exp\left(-\frac{|\mathbf{r}_{OP}|^2 + |\mathbf{r}'_{OP}|^2}{2\, A^2} \right) \exp\left(-\frac{|\mathbf{r}_{OP} - \mathbf{r}'_{OP}|^2}{B^2} \right), \tag{3}$$

where

$$A^2 = R^2 + \frac{1}{4}\left(\frac{1}{\pi\, K_A}\right)^2 + (\lambda\, z\, K_A)^2 \quad , \qquad B^2 = \left(\frac{1}{\pi\, K_A}\right)^2 \left(1 + \frac{1}{4\pi^2 K_A^2 R^2}\right) + \left(\frac{\lambda\, z}{\pi\, R}\right)^2 . \quad (4)$$

So, the intensity distribution across those areas will be given by

$$I_{OP}(\mathbf{r}_{OP}) = W_{OP}(\mathbf{r}_{OP}, \mathbf{r}_{OP}) = \frac{I_0\, \lambda\, K_A^2}{A^2} \exp\left(-\frac{|\mathbf{r}_{OP}|^2}{A^2}\right). \quad (5)$$

It is apparent that both the parameter A^2 and the lateral magnification of the lens determine the intensity distribution radius. Note that A^2 differs from R^2 in two terms, which depend on the cut-off spatial frequency and the defocusing. So, the sizes of the corresponding areas on both the diffuser and the detector will be similar and of the order of the speckle size if z=0 and an optical system with high numerical aperture and lateral magnification equals to 1 is used.

On the other hand, the coherence patch at the detector plane will also be gaussian, but now the parameter B^2 and the lateral magnification of the lens determine its radius. Indeed, the normalised cross-spectral density will give the degree of spatial coherence of the optical field at this plane, that is

$$\mu_{12}(\mathbf{r}_{OP} - \mathbf{r}'_{OP}) = \frac{W_{OP}(\mathbf{r}_{OP}, \mathbf{r}'_{OP})}{\sqrt{I_{OP}(\mathbf{r}_{OP})}\,\sqrt{I_{OP}(\mathbf{r}'_{OP})}} = \exp\left(-\frac{|\mathbf{r}_{OP} - \mathbf{r}'_{OP}|^2}{B^2}\right). \quad (6)$$

B^2 depends on the size of the illuminated area on the diffuser, the cut-off spatial frequency and the defocusing. Note that the coherence patch at the plane *IP* will be inversely proportional to the aperture stop size. So, the optical field at the image provided by an ideal imaging system with lateral magnification equals to 1 will have the same intensity distribution and the same spatial coherence properties as the optical field that emerges from the diffuser.

But, it is not the case for the experimental situations. Usually, we use well-corrected optical systems with relative small numerical apertures (or variable ones), we change the size of the illuminated area on the diffuser by using field stops or other devices, and the location of the best focus plane Z_S is determined with limited accuracy. So, the speckle pattern recorded by the detector will be a blurred version of the speckle pattern that emerges from the diffuser (even by adequate acquisition resolution). Furthermore, the spatial coherence properties also change. Specifically the coherence patch grows through the propagation from the diffuser to the detector planes.

However, the values of A^2 and B^2 for each experimental situation must satisfy the invariance of the *Entendue*[3-5]. In other words, the ratio between the coherence patch and the areas corresponding to *S'* should be the same in all the planes connected by Fourier transforms along the set-up[10], i.e.

$$E = \frac{\pi\, B^2}{\pi\, A^2} = \frac{1}{\pi^2\, K_A^2\, R^2} \quad (const.). \tag{7}$$

So, it is possible to adjust properly both the field and the aperture stops in the set-up in order to obtain speckle fields with the following characteristics:

- $A=B$, i.e. the coherence patch fits the mean size of the speckles. The adjustment condition for this case is $\pi\, K_A^2 = \frac{1}{\pi\, R^2}$.
- $A>B$, i.e. the coherence patch is smaller than the mean size of the speckles. Now, $\pi\, K_A^2 > \frac{1}{\pi\, R^2}$, so that the speckle field will be essentially spatially incoherent. Such speckle fields can be observed at the best focus plane if R is adequately greater than K_A. Double exposure speckle interferometry with these speckle fields will produce fringes with very low visibility because the corresponding speckles will be uncorrelated. Thus, they are not useful for conventional metrology purposes.
- $A<B$, i.e. the coherence patch is greater than the mean size of the speckles. Here, $\pi\, K_A^2 < \frac{1}{\pi\, R^2}$, so that the speckle field will be spatially partially coherent. Such speckle fields can be observed at the best focus plane if K_A is adequately greater than R. Double exposure speckle interferometry with these speckle fields will produce fringes with relative high visibility, provided the coherence patch covers the pairs of corresponding speckles. Then, these speckle pairs will be correlated.

On the other hand, from eq.(6) it is apparent that the spatial coherence properties of the speckle fields produced by the experimental set-up in Fig. 1 satisfy the mathematical requirements of the Schell-model beams[3,4,11]. Particularly, the degree of spatial coherence is a real valued function and the coherence patch is spatially invariant on the observation plane, due the circular symmetry of the gaussian profile of the degree of spatial coherence.

3. EXPERIMENTAL RESULTS

Double exposure specklegrams[2] were photographically recorded by using the experimental set-up in Fig. 1 under the following conditions:

- The two speckle patterns of each specklegram were recorded under the same spatial coherence conditions. So, the coherence patch is well defined for each double exposure plate.
- The relative shift between the two-recorded speckle patterns was greater than the mean diameter of their speckles. Specifically, relative displacements of 3, 4, 5 and 6 times the mean diameter of the speckles were introduced. Defocusing was also well controlled.

It is well known that the double exposure specklegrams constitute an ensemble of Young's pairs if the corresponding speckle pairs are correlated, i.e. if the relative shift between the two speckle patterns is smaller than the size of their coherence patch[2,6].

Such specklegrams produce Young's fringe patterns with relative high visibility, whose period provides a measurement of the shift between the recorded speckle patterns. On the other hand, the size of the coherence patch of the speckle field can be estimated by determining the visibility of the Young's fringes. Thereafter, it is possible to compare the coherence patch with the mean size of the speckles and to verify the *Entendue* invariance.

Using the set-up in Fig. 2 captured the Young's fringe patterns generated by each specklegram.

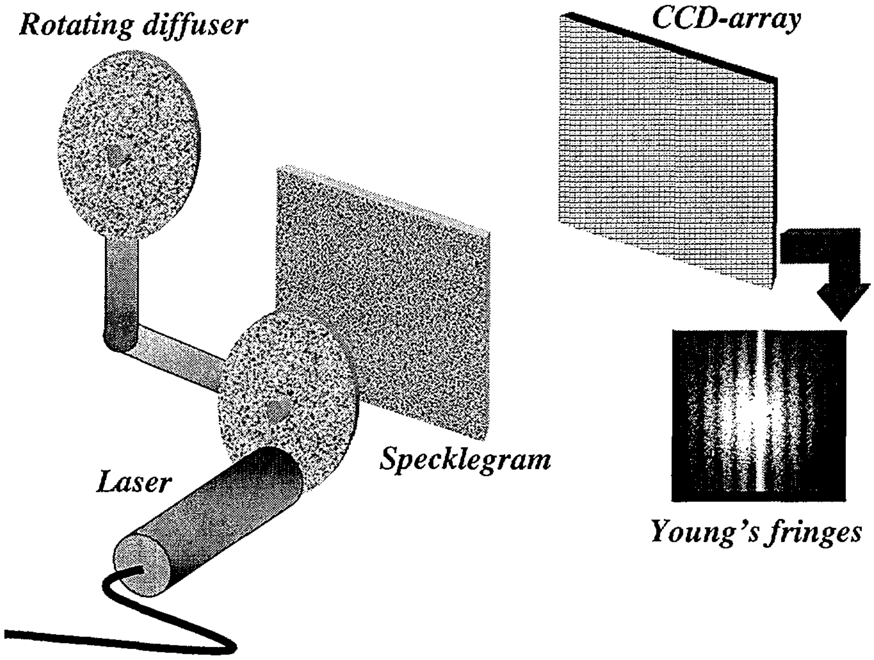

Fig. 2: Experimental set-up for capturing Young's fringes from a double exposure specklegram. The rotating diffuser is mounting in an arm that allows putting it in or out the set-up.

Each specklegram was illuminated with a He-Ne laser in two different modes: non-expanded beam and spread beam through a rotating diffuser. A CCD sensor captured the Young's fringe patterns produced by the both illumination modes that are shown in Tables 1 and 2.

Note the increase of the number of fringes with the increase of the relative displacement of the recorded speckle fields as expected. So, for these cases the coherence patch must be greater than the mean diameter of the speckles, in such a way that the pairs of corresponding speckles are included completely inside the coherence patch. For this reason, they remain correlated although their relative shift is significantly greater than their sizes.

The increase of the fringe visibility in the spread beam mode of illumination is apparent too. It is a consequence of the reduction of noise introduced by the speckles on the fringes, because of the use of the rotating diffuser.

The *Entendue* invariance is also illustrated in Table 2, by using specklegrams with different relative displacements, recorded at the best focus plane and at two other planes in its neighbourhood. It confirms the compromise between the area corresponding to S' and the coherence patch established in eq.(7).

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TABLE 1: Young's fringe patterns from different double exposure specklegrams. Relative displacement = Shift factor X Mean diameter of the speckles

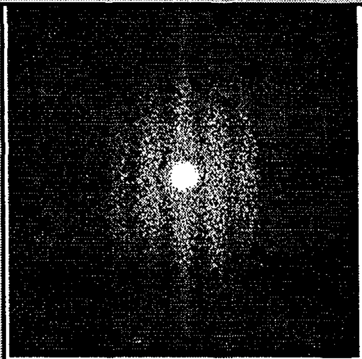
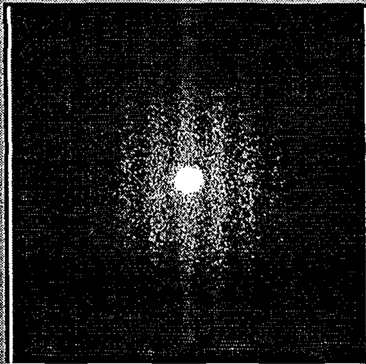
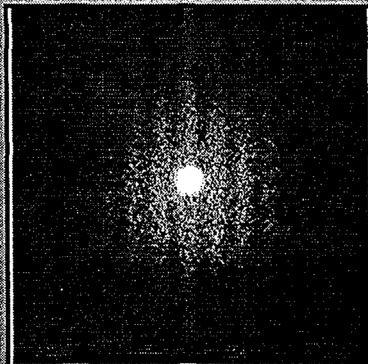
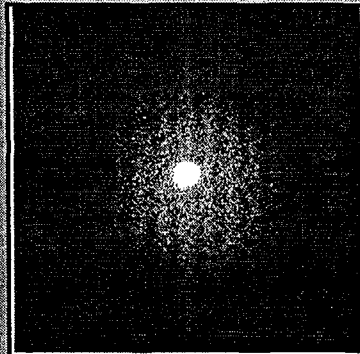
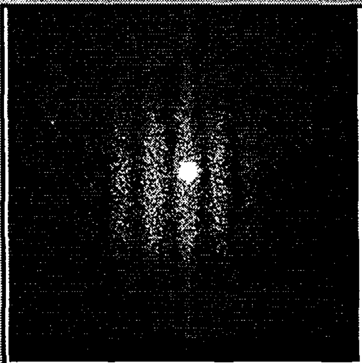
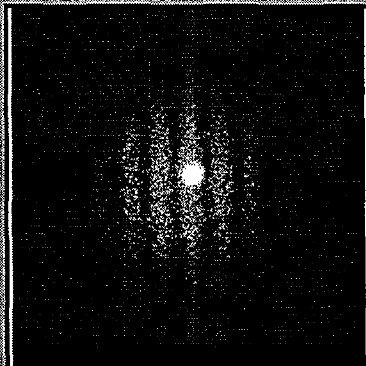
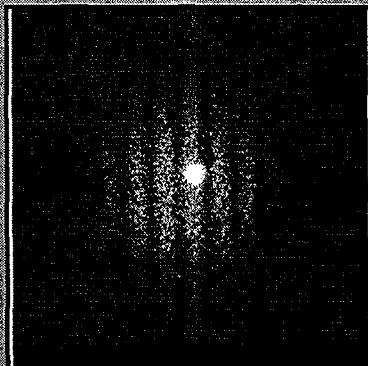
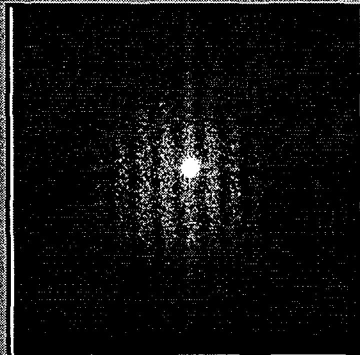
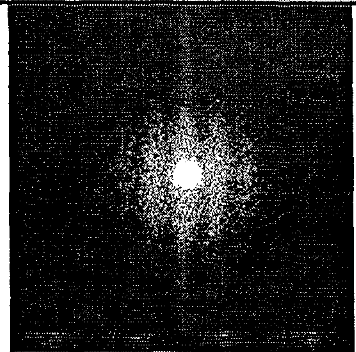
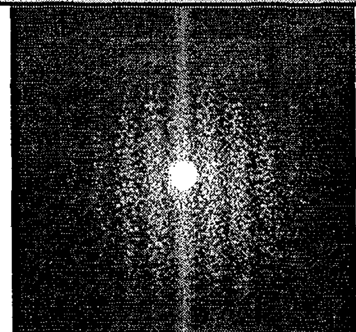
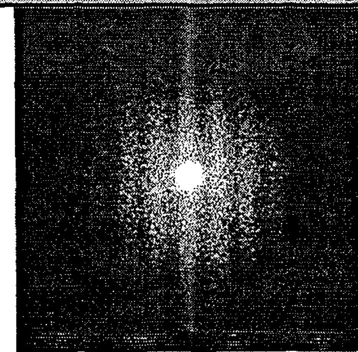
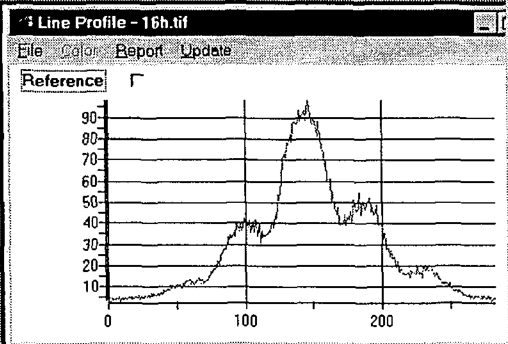
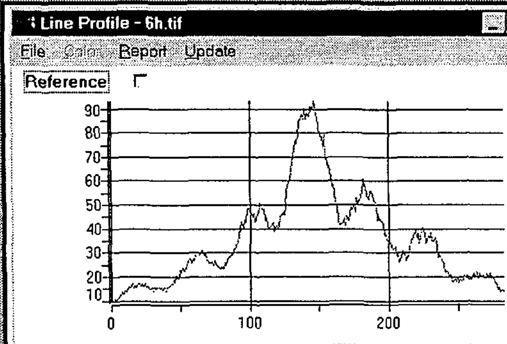
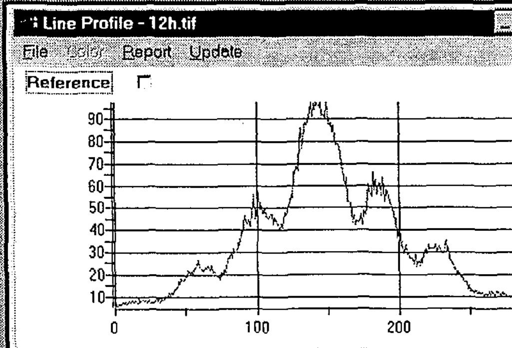
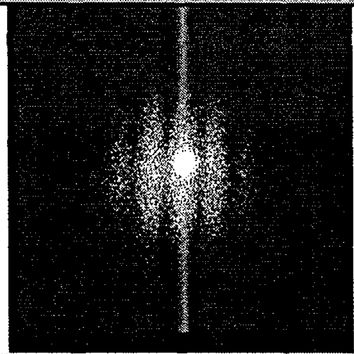
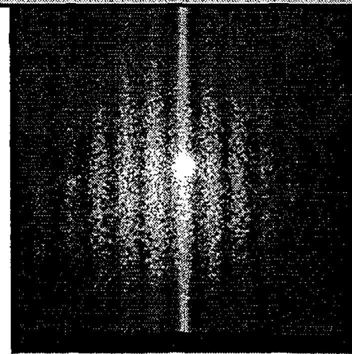
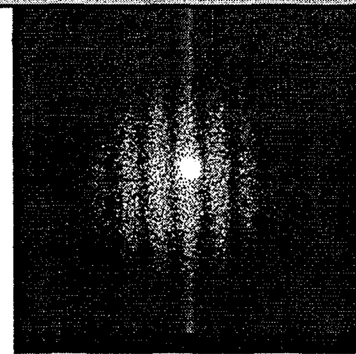
Shift factor	3	4	5	6
Illumination with non-expanded beam (without rotating diffuser)				
Illumination with spread beam (with rotating diffuser)				

TABLE 2: Young's fringe patterns acquired at different observation planes (i.e. different values of z). The speckle separation and the *Entendue* value are corresponding to a specific observation plane

Measurements with non-expanded beam (without rotating diffuser)			
Speckle separation (μm)	**70**	**90**	**100**
Entendue value in arbitrary units	**23**	**22**	**24**
Young's fringe patterns			
Profiles of the above patterns			

Measurements with spread beam (with rotating diffuser)			
Speckle separation (µm)	**70**	**90**	**100**
Entendue value in arbitrary units	**37**	**38**	**36**
Young's fringe patterns			
Profiles of the above patterns