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United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency

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MIRAMARE – TRIESTE

March 2000

Abstract

A minimal lepton number violation (LNV) is proposed which could naturally appear in SUSY theories, if Yukawa and LNV couplings had a common origin. According to this idea properly implemented into MSSM with an additional abelian flavor symmetry the prototype LNV appears due to a mixing of leptons with superheavy Higgs doublet mediating Yukawa couplings. As a result, all significant physical manifestations of LNV reduce to those of the effective trilinear couplings $LL\overline{E}$ and $LQ\overline{D}$ aligned, by magnitude and orientation in a flavor space, with the down fermion (charged lepton and down quark) effective Yukawa couplings, while the effective bilinear terms appear generically suppressed relative to an ordinary μ-term of MSSM. Detailed phenomenology of the model related to the flavor-changing processes both in quark and lepton sectors, radiatively induced neutrino masses and decays of the LSP is presented. Remarkably, the model can straightforwardly be extended to a Grand Unified framework and an explicit example with SU(7) GUT is thoroughly discussed.

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1 Introduction

Until recently all the confirmed experimental data indicated that lepton and baryon number are conserved in agreement with Standard Model (SM) of quarks and leptons. These global conservation laws are in essence an important by–product of a generic $SU(3)_C \otimes SU(2)_W \otimes U(1)_Y$ gauge invariance of SM which leads for an ordinary SM particle spectrum together with a general baryon number conservation to the special conservation laws for the every known lepton flavor. Therefore, in contrast to the quark case where the (baryon number conserving) mixing among different generations occurs, being given by the Cabbibo–Kobayashi–Maskawa (CKM) matrix elements, leptons cannot mix as an orthodox SM contains the left–handed massless neutrinos only. However, the recent SuperKamiokande data on atmospheric neutrinos [1] seem to finally change the situation clearly indicating that neutrino oscillations ($\nu_\mu \to \nu_\tau$ or sterile ν_s) actually take place, thus opening a way for new physics beyond the SM.

On the other hand, strict baryon and lepton number conservation could not have a firm theoretical foundation unless associated with some local gauge invariance and, as a result, with new long–distance interactions which are experimentally excluded [2]. Leaving aside for the moment baryon number violation (BNV), there could be, in principle, several reasons for lepton number violation (LNV) related with possible extensions of the SM, such as a presence of higher–dimensional neutrino mass operators induced by gravity [3], an existence of superheavy right–handed neutrinos inducing small neutrino masses through the known see–saw mechanism [4], new weak triplet Higgs bosons giving tiny masses to neutrinos directly [5] etc. However the minimal supersymmetric extension of the SM, being in all other aspects also well–motivated, where not only fermions but also their scalar superpartners automatically become carriers of lepton (and baryon) numbers, seems to be the most natural and appealing framework for LNV.

In general, the basic renormalisable dimension–four BNV and LNV couplings expected in the low–energy MSSM superpotential W, unless forbidden by some side symmetry such as R–parity [6], are given by

$$\Delta W = \mu_i L_i H_u + \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k. \tag{1}$$

where i, j, k are generation indices (i, j, $k = 1, 2, 3$) and an associated summation is implied (color and weak isospin indices are suppressed); $L_i(Q_j)$ denote the lepton (quark) $SU(2)$–doublet superfields and $\overline{E}_i(\overline{U}_i, \overline{D}_i)$ are $SU(2)$–singlet lepton (up quark, down quark) superfields; μ_i are mass parameters which mix lepton superfields with the "up" Higgs superfield H_u, while λ_{ijk} ($\lambda_{ijk} = -\lambda_{jik}$), λ'_{ijk} and λ''_{ijk} ($\lambda''_{ijk} = -\lambda''_{ikj}$) are dimensionless couplings. The first three terms in ΔW (1) violate lepton number, while the last one violates baryon number. Thus there are 48 new and *a priori* unconstrained parameters (beyond those of the R–parity conserving MSSM) with arbitrary flavor structure in general. Needless to say, this fact presents serious difficulties for a study of the R–parity violating (RPV) phenomena at a theoretical as well as phenomenological level (for the recent reviews see [7] and references therein).

When estimating possible contributions of RPV interactions to the low energy processes one typically assumes that only one of the RPV couplings or one combination of their products is predominant, while the rest are negligibly small. This assumption made for quarks and leptons taken in the physical mass basis looks in some cases unnatural with respect to the

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starting flavor structure of the couplings involved (1) where quarks and leptons appear in gauge ("unrotated") basis. Nevertheless, what we have learned from such studies is that the RPV couplings are typically smaller than the ordinary gauge couplings. although some of them taken alone could quite be of order $O(1)$ [7] for sparticle masses of $O(100)$ GeV. Whereas the lowest individual bounds follow for BNV couplings $\lambda''_{112} \leq 10^{-7}$ and $\lambda''_{113} \leq 10^{-5}$ from double nucleon decay and $n - \bar{n}$ oscillation, respectively, the strictest combined bounds appear due to the simultaneous presence of LNV (λ'_{ijk}) and BNV (λ''_{ijk}) interactions in (1) that inevitably leads to the unacceptably fast proton decay, unless $\lambda'_{11k}\lambda''_{11k} \leq 10^{-22}$ (for $k = 2, 3$) and $\lambda'\lambda'' \leq 10^{-10}$ (for any combination of flavors) are taken[7]. So, one could expect, unless somewhat enormous flavor anisotropy for RPV couplings is assumed that, while SUSY-inspired baryon number violation might be highly (or even fully) suppressed, lepton number violation could occur and at a level which seems large enough for possible observation of many of its spectacular manifestations.

Meanwhile, R–parity is not the only symmetry known to ensure proton stability. In principle, it is not difficult to arrange the general RPV couplings (1) in such a way to have both lepton number violation and baryon number conservation, i.e. $\mu_i \neq 0$, $\lambda \neq 0, \lambda' \neq 0$, $\lambda'' = 0$. For example, it can be achieved by imposing some discrete symmetry on the quark and lepton fields. The simplest choice might be the reflection Z_2 symmetry under which quark and leptons transform as

$$Q, \overline{U}, \overline{D}, L, \overline{E} \rightarrow -Q, -\overline{U}, -\overline{D}, L, \overline{E}. \tag{2}$$

However, the Z_3 symmetry

$$Q, \overline{U}, \overline{D}, L, \overline{E} \rightarrow Q, \omega^2\overline{U}, \omega\overline{D}, \omega^2 L, \omega^2\overline{E} \quad (\omega = e^{i2\pi/3}), \tag{3}$$

known as the baryon parity[8], seems to be of particular interest. The reason is that the baryon parity happens to be the superstring–inherited gauge discrete symmetry which is stable under gravitational corrections. This symmetry, as was shown[8], forbids not only the dimension–four BNV interactions in (1), but still dangerous dimension–five operators as well. At the same time, acting more selectively than R–parity, Z_3 baryon parity allows all LNV interactions in (1). In this connection it seems reasonable to suppress fully all BNV couplings ($\lambda'' = 0$) and be focused further on LNV only. From here on we assume that it is the case when considering MSSM.

However, the situation becomes further complicated when one tries to embed MSSM with LNV couplings (1) into the more fundamental framework of a Grand Unified Theory (GUT) which typically keeps quarks and leptons in common multiplets. By contrast, discrete symmetries protecting the proton stability, such as Z_3 baryon parity mentioned, treat quarks and leptons differently and hence they come into conflict with the known minimal GUTs. Nevertheless, a number of the properly extended GUTs have been constructed [9] where the low-energy lepton number violation versus baryon number conservation appears once the starting symmetry breaks down to MSSM.

An excessively wide variety of the possible LNV couplings (36 of the λ and λ' terms) brings up another critical point: what is a basic prototype LNV form which could naturally appear in MSSM? The bilinear terms $\mu_i L_i H_u$ in the superpotential ΔW (1) might be such a minimal possibility, if all trilinear couplings were generically absent (i.e. $\lambda = \lambda' = 0$). These trilinear

couplings could be prohibited by some additional symmetry – typically by a gauge (ordinary or anomalous) $U(1)$ symmetry concerning the quark and lepton flavors [10] in the properly extended MSSM. The bilinear terms in themselves can be rotated away, thus recovering the effective trilinear λ and λ' terms from the ordinary Yukawa couplings for leptons and down quarks, respectively. Such a type of minimal theory with a generic alignment between LNV terms and Yukawa couplings was considered in a number of papers [9-11] and many of its interesting features had been established. The most appealing one is that the magnitide and flavor structure of LNV couplings are essentially given now by those of Yukawa couplings so as to naturally overcome all the currently available experimental constraints (among them the most stringent ones which might follow from $K^0 - \overline{K}^0$ mixing, $K^0 \to e_i \bar{e}_j$ decays etc.).

However, this model has, at least, two serious drawbacks. The first one concerns the natural scale of new, arbitrary in general, mass parameters μ_i relative to the basic MSSM mass given by an ordinary μ-term, $\mu H_u \overline{H}_d$. All the μ_i certainly must be arranged to be at most of order of μ not to disturb significantly an electroweak symmetry breaking in MSSM. Furthermore, when extending to the GUT framework the bilinear terms lead in general, together with the lepton mixing with a weak Higgs doublet, to the quark mixing with a color Higgs triplet, thus inducing baryon number violation as well. Again, the only possible solution to the problem might use the electroweak scale order masses μ_i in the GUT symmetry-invariant bilinear couplings. This would require that new fine-tuning conditions, besides a notorious gauge hierarchy one, should be satisfied in a very ad hoc way.

The second problem is that when the SUSY soft breaking terms are included the bilinear couplings cannot be rotated away and lead to an enormously complicated scalar sector with sneutrinos condensed, and neutrinos and neutralinos mixed by a proper (seven–by–seven in a general $\widetilde{\gamma} - \widetilde{Z} - \widetilde{H}_u^0 - \widetilde{H}_d^0 - \nu_e - \nu_\mu - \nu_\tau$ basis) mass matrix. As a result, there appear the tree-level neutrino masses which are generally too large unless some precise alignment between the bilinear LNV and corresponding SUSY soft–breaking terms is provided. There was made some progress in recent years towards his problem in the framework of supergravity theories [12].

Nevertheless, the Higgs-lepton mixing model (or HLM model hereafter) of LNV seems to be, against all the odds, the most appealing one as it uniquely links with flavour physics peculiarities both in quark and lepton sectors. We take it as the starting point towards a new framework related with a possible common origin of the LNV and Yukawa couplings. While our model is also based on a generic Higgs-lepton mixing, we propose, in contrast to an ordinary picture (1), that this mixing appears not with a standard MSSM Higgs doublet H_u but with some superheavy weak doublets $\Phi + \overline{\Phi}$ mediating the down fermion (down quark and charged lepton) effective Yukawa couplings. This mechanism is readily extended to the GUT framework as well, since it allows any large $\Phi - L$ mixing masses. As a result, we drive at another picture where only effective trilinear LNV couplings appear being aligned with the Yukawa ones, whereas the ordinary bilinear $H_u - L$ mixings (1) are proved to be strongly suppressed. So, the model considered can be qualified as a purely trilinear model or, equally (if expressed in terms of primary couplings, see below), as the heavy Higgs-lepton mixing (HHLM) model of LNV.

Depending on the $U(1)_A$ charges assigned to matter and Higgs superfields involved one can

come to an ordinary HLM model or the HHLM model considered here. Some of the predictions of both models, particularly those concerning quark flavor conservation in the LNV inspired processes are very similar. However, there are principal differences as well. The point is that an influence of the SUSY soft breaking sector, being predominant for an ordinary HLM, is quite negligible for the HHLM model. Therefore, the LNV-Yukawa alignment, while appeared in both of models, leads just in the latter case to the distinctive relations between various LNV processes arising from the slepton and squark exchanges, which are basically conditioned by quark and lepton mass hierarchy (see Section III). By contrast, in an ordinary HLM case these processes appear to be essentially determined by W and Z boson exchanges and, as a result, are largely flavor-independent. Whilst at the moment one cannot phenomenologically distinguish them, further extension to the GUT framework seems, as we argue below, to favor the HHLM model.

We construct an explicit example of the R-parity violating $SU(7)$ GUT[13]. The preference given to $SU(7)$ model over other grand unified frameworks is essentially determined by the missing VEV solution to the doublet-triplet splitting problem which naturally appears in $SU(N)$ GUTs starting from the $SU(7)$[14]. In this model the effective LNV couplings with the Yukawa-aligned structure immediately evolve, while the baryon number non-conserving RPV couplings are safely projected out by the proper missing VEV vacuum configuration that breaks the starting $SU(7)$ symmetry down to the one of MSSM. This implies that a missing VEV solution to gauge hierarchy problem can generically protect baryon number conservation in the RPV $SU(7)$ SUSY GUT.

The rest of the paper is organized as follows. In Section II we present the proposed HHLM model of minimal LNV in MSSM whose a detailed phenomenological study will be given in Section III. In Section IV the $SU(7)$ framework for lepton number violation versus baryon number conservation is presented. Finally, in Section V our conclusions are summarized.

2 Minimal lepton number violation in MSSM

We argue here that the proposed HHLM model with a heavy Higgs-lepton mixing is in fact the minimal generic form of LNV whose basic predictions are related with masses and mixings of quarks and leptons.

Following to the observed up-down hierarchy in quark mass spectrum, where the top mass is clearly leading, we propose that, while the up quark Yukawa couplings have a usual trilinear form

$$W_U = Y^u_{jk} Q_j \overline{U}_k H_u \tag{4}$$

the down fermion (down quark and charged lepton) Yukawa couplings could have one more dimension being generically mediated by superheavy Higgs doublets $\Phi + \overline{\Phi}$ which simultaneously mix with a standard "down" Higgs $\overline{H}_d$ and leptons L_i:

$$W_D = G^d_{jk} Q_j \overline{D}_k \overline{\Phi} + G^l_{jk} L_j \overline{E}_k \overline{\Phi} + f \Phi \overline{H}_d S + M_\Phi \overline{\Phi} \Phi, \tag{5}$$

$$W_{LNV} = h_i L_i \Phi T \tag{6}$$

(Y^u_{jk}, $G^{l,d}_{jk}$, f and h_i are dimensionless coupling constants properly introduced). The couplings (4-6) are in substance the most general ones which can appear in MSSM complemented by

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some abelian flavor symmetry as an anomalous $U(1)_A$ symmetry in the case considered which is supposed to properly arrange the rest of hierarchy in quark and lepton mass spectra [15]. We have introduced the singlet scalar superfields S and T, the basic carriers of the $U(1)_A$ charge. They are presumed to develop the high scale (up to string scale order) VEVs through the Fayet-Iliopoulos D-term related with $U(1)_A$ symmetry [16]. In terms of Q_A^S and Q_A^T all the other charges appear to be properly expressed so that the direct Higgs-lepton mixing terms $\mu_i L_i H_u$ (1) are strictly prohibited until $U(1)_A$ symmetry breaks[1]. Such a mixing is allowed only with superheavy Higgs doublet system $\Phi + \overline{\Phi}$ having Planck scale order mass, $M_\Phi = O(M_P)$.

Once $U(1)_A$ symmetry breaks ($< S > \neq 0$, $< T > \neq 0$) we come to the common mass matrix of all Higgs and lepton superfields involved:

	$\mathbf{L}_i$	$\overline{\mathbf{H}}_d$	$\overline{\Phi}$	
$\mathbf{L}_i$	0	0	0	
$\mathbf{H}_u$	0	μ	0	(7)
Φ	$h_i < T >$	$f < S >$	M_Φ	

whose diagonalization up to the second order mixing terms leads to the effective down fermion Yukawa and LNV couplings of type (keeping the same notation for the "rotated" superfields).

$$W_D^{eff} = G_{jk}^d f \frac{< S >}{M_\Phi} Q_j \overline{D}_k \overline{H}_d + G_{jk}^l f \frac{< S >}{M_\Phi} L_j \overline{E}_k \overline{H}_d \tag{8}$$

$$W_{LNV}^{eff} = h_i \frac{< T >}{M_\Phi} [G_{jk}^l L_i L_j \overline{E}_k + G_{jk}^d L_i Q_j \overline{D}_k + \frac{< S >}{M_\Phi} \mu L_i H_u] \tag{9}$$

Thereby, the effective coupling constants included in Eqs. (8, 9) are read off as

$$Y_{jk}^l = f G_{jk}^l \frac{< S >}{M_\Phi}, \quad Y_{jk}^d = f G_{jk}^d \frac{< S >}{M_\Phi} \tag{10}$$

and

$$\lambda_{ijk} = h_i G_{jk}^l \frac{< T >}{M_\Phi}, \quad \lambda'_{ijk} = h_i G_{jk}^d \frac{< T >}{M_\Phi} \tag{11}$$

from which the basic LNV-Yukawa coupling relations immediately follows

$$\lambda_{ijk} = \epsilon_i Y_{jk}^l, \quad \lambda'_{ijk} = \epsilon_i Y_{jk}^d \tag{12}$$

with the proportionality parameters ϵ_i determined as

$$\epsilon_i = \frac{h_i}{f} \frac{< T >}{< S >}. \tag{13}$$

Remarkably, as one can see from Eqs. (9) and (13), the effective $L - H_u$ mixing masses μ_i appear generically related to the basic MSSM mass μ

$$\mu_i = f \epsilon_i (\frac{< S >}{M_\Phi})^2 \mu, \tag{14}$$

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[1]We introduced two singlet scalar superfields S and T in order to have an ordinary μ–term $\mu H_u \overline{H}_d$ in superpotential, while the bilinear LNV terms $\mu_i L_i H_u$ are still prohibited (a minimal case with one scalar superfield S would admit both of terms in superpotential). In $SU(7)$ GUT framework considered in Section IV they appear as the non-trivial $SU(7)$ multiplets.

while being properly suppressed. When taking $\frac{<T>}{M_\Phi} \sim \frac{<S>}{M_\Phi} \sim 10^{-2}$ to provide the observed up-down mass hierarchy (or, equally, the observed mass scale for the b quark and τ lepton) in Yukawa coupling constants (10) one comes to a natural bound for μ_i parameters, $\mu_i \lesssim 0.01 \; GeV$ for $\mu = O(100 \; GeV)$. This is in fact too small to have any sizeable influence on the scalar sector and tree–level neutrino masses, as in a bilinear HLM case mentioned above. Therefore, as to the significant physical manifestations of LNV, one has only those related with the effective trilinear couplings (9) being aligned (by magnitude and orientation in flavor space) with down fermion Yukawa couplings (9)[2].

And the last is that the model also allows a straightforward extension to a GUT, particularly, to the $SU(7)$ GUT with a natural solution to the hierarchy problem due to the basic vacuum configuration appeared with no VEVs along all the color directions. There the singlet scalar superfields S and T are replaced, respectively, by one of the fundamental Higgs multiplets which break $SU(7)$ to $SU(5)$ and a basic adjoint multiplet of $SU(7)$ which just projects out all the BNV couplings and leaves the LNV ones only. Thus, the extra scalar superfields specially introduced in MSSM framework are turned out to naturally exist in the $SU(7)$ GUT. We consider this in more detail in Section IV.

3 Phenomenology of HHLM model

During the last few years the SUSY inspired baryon and lepton number non-conservation has called a considerable attention. As a result, an extensive study of the bounds from various low–energy processes on RPV couplings (1) was carried out, as well as many specific manifestations of RPV interactions at present and future colliders were investigated (see [7, 17] and references therein). In this Section we consider some of the immediate consequences of lepton number violation which could emerge in the minimal HHLM (Heavy Higgs-Lepton Mixing) model proposed. We pursue this study on the purely phenomenological level as given by the basic Yukawa-aligned trilinear LNV interactions (see Eq. (12)) with no significant bilinear terms generated in the soft SUSY breaking sector either on tree level (see Eq. (14)) or through the radiative corrections.

3.1 LNV couplings in physical basis

Towards this end one must go in the general alignment conditions (12) to the physical (mass) basis where the down fermion Yukawa matrices Y^l and Y^d are diagonal[3]. As a result, the basic relations between the effective λ and λ' couplings and masses of down fermions follow, which are

$$\lambda_{ijk} = \frac{\epsilon_i}{V c_\beta} \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix}_{jk} \tag{15}$$

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[2]There may appear, for the effective Yukawa and LNV operators mediated by the superheavy Higgs doublet pair $\Phi + \bar{\Phi}$, the competitive gravitational corrections smearing out the alignment conditions (12). However, these corrections are naturally suppressed if the bilinear $\Phi\bar{\Phi}$ has a non-zero $U(1)_A$ charge, thus getting mass once the $U(1)_A$ symmetry breaks. Indeed, if so, all the high-dimension operators which appear through the $\Phi + \bar{\Phi}$ exchanges are not neutral under $U(1)_A$ and, therefore, cannot be induced by gravity.

[3]We have assumed that the fermion and sfermion mass matrices can simultaneously be brought to the diagonal form as is usually taken in ordinary MSSM.

and

$$\lambda'_{ijk} = \frac{\epsilon_i}{V c_\beta} \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}_{jk} ,$$ (16)

respectively (here $V \approx 174\ GeV$ is the electroweak VEV, while $\tan\beta$ is an usual ratio of the VEVs of "up" and "down" Higgses H_u and $\overline{H}_d$). There appear clear in physical basis some of the generic features of the HHLM model which essentially determine its phenomenological implications considered further in this Section.

The first and foremost is that, while both of basic LNV couplings $L_i L_j \overline{E}_k$ and $L_i Q_j \overline{D}_k$ generally violate lepton number $\left|\Delta N^L\right| = 1$ and conserve baryon (quark) number $\Delta N^Q = 0$, some additional selection rules related with flavor (generation) species of leptons and quarks come into play in the HHLM model:

- A partial lepton flavor conservation according which only one of lepton flavor numbers is violated at a time, while the other two are still retained

$$\left|\Delta N^{L_i}\right| = \delta_{im} \quad (i, m = 1, 2, 3),$$ (17)

- An exact quark flavor conservation

$$\Delta N^{Q_i} = 0.$$ (18)

Next is a large reduction of the possible LNV couplings being conditioned by the selection rules (17) and (18). One can quickly confirm that only λ and λ' couplings with the last two indices equal are left in the physical basis. This gives $6 + 9 = 15$ physical λ and λ' couplings in total (instead of $9 + 27 = 36$ as in a general case) depending, in effect, on three unknown parameters ϵ_i only.

And the last (but certainly not least as it will be seen below) is a natural smallness of LNV coupling constants as they are seen from Eqs. (15, 16) being largely determined by the known masses of leptons and quarks

$$\lambda_{211} = \frac{\epsilon_3}{\epsilon_2} \lambda_{311} \simeq 2.9 \cdot 10^{-6} \frac{\epsilon_2}{c_\beta}$$

$$\lambda_{122} = \frac{\epsilon_3}{\epsilon_1} \lambda_{322} \simeq 6.4 \cdot 10^{-4} \frac{\epsilon_1}{c_\beta}$$

$$\lambda_{133} = \frac{\epsilon_2}{\epsilon_1} \lambda_{233} \simeq 1.1 \cdot 10^{-2} \frac{\epsilon_1}{c_\beta}$$ (19)

$$\lambda'_{i11} = (2.9 \div 8.6) \cdot 10^{-5} \frac{\epsilon_i}{c_\beta}$$

$$\lambda'_{i22} = (0.6 \div 1.7) \cdot 10^{-3} \frac{\epsilon_i}{c_\beta}$$

$$\lambda'_{i33} = (2.4 \div 2.6) \cdot 10^{-2} \frac{\epsilon_i}{c_\beta}$$ (20)

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where the numerical values shown follow from the masses of leptons and quarks (including the corresponding uncertainties in the down quark masses $m_d = 5 \div 15\ MeV$, $m_s = 100 \div 300\ MeV$ and $m_b = 4.1 \div 4.5\ GeV$ [18]). The unknown parameters ϵ_i in (19) and (20) can be all of the same order or follow to some hierarchy dictated by flavor symmetry of the underlying theory, if it is the case [10, 19].

3.2 Constraints from low–energy processes

Consider first the possible effective LNV interactions which follow from still unconstrained primary couplings (1) when all the intermediate sleptons and squarks are integrated out. The typical four–fermion operators appeared are listed in TABLE I. Generally, these operators mediating the possible flavor-changing transitions both in quark and lepton sectors could contribute to known processes leading to the deviations from the observed rates of $K^0 - \overline{K}^0$ and $B^0 - \overline{B}^0$ oscillations, charged current universality, $e - \mu - \tau$ universality, atomic parity violation and others. Also they could induce the rare decays of mesons and leptons many of which are highly suppressed or even forbidden in the SM. Using all the related data and observations presently existed one extracts rather severe bounds on the LNV couplings λ and λ' and/or on their products [7, 17].

Now, going from general case to the HHLM model with a generic LNV-Yukawa alignment presented in physical basis by the selection rules (17, 18) one can immediately confirm that there are no significant LNV contributions to any of the flavor-changing neutral current (FCNC) processes. The usually dangerous tree level LNV processes, such as leptonic and semi-leptonic decays of pseudoscalar mesons ($K^0 \to e_i \bar{e}_j$, $K^+ \to \pi^+ e_i \bar{e}_j$, $K^+ \to \pi^+ \nu_i \bar{\nu}_j$, $B^0 \to e_i \bar{e}_j$, $B \to X_q \nu_i \bar{\nu}_j$) as well as the possible LNV contributions to $K^0 - \overline{K}^0$ and $B^0 - \overline{B}^0$ oscillations, are naturally forbidden in our scenario. Moreover, since the typical strength of the LNV couplings in the HHLM model (see Eqs. (19, 20)) is smaller than a strength of electroweak interactions, even the loop contributions to the above processes (to those with $i = j$) are largely dominated by the usual SM interactions.

At the same time in cases when the LNV induced processes are allowed in HHLM model they appear to readily satisfy the existing bounds due a generic smallness of the LNV couplings appeared, thus leading to the quite acceptable limitations on the ϵ-parameters and $\tan\beta$ involved (19, 20). For example, one of the most stringent bounds that can be extracted from the atomic transition conversion process[7] $\mu^- + {}^{45}Ti \to e^- + {}^{45}Ti$ gives a bound $\frac{\epsilon_1 \epsilon_2}{c_\beta^2} < 1$ for the mediating sfermion masses of $m_{\tilde{f}} = 300\ GeV$.

Another important constraint comes from the current experimental limits on neutrinoless double beta decay[7]. This process is triggered by the six–fermion effective operators of the form $eu\bar{d}eu\bar{d}$ which appear in the low-energy theory after one integrates out sleptons and squarks. The analysis of the disintegration process of ${}^{76}Ge$ (with half–life time $T_{\frac{1}{2}} > 1.1 \cdot 10^{25}$ years) leads to the bound $\frac{\epsilon_1}{c_\beta} \lesssim 1.1$.

For the ϵ-parameters fixed, one can further get the bound on $\tan\beta$ preferably according to the largest LNV couplings λ_{i33} and λ'_{i33} appeared (19, 20). The most stringent bound comes from the charged current universality constraints in τ–decays which imply $\tan\beta \lesssim 6$, if $\epsilon_{2,3} \sim 1$ is taken.

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3.3 Three-body leptons decays

The rare decays of leptons, along with the processes mentioned above, are usually treated as the most promising ones in searching for lepton flavor violation phenomenon [20]. Here we consider three–body leptons decays of μ and τ triggered by the last of four–fermion operators listed in TABLE I. All these processes can be presented by a generic transition of type $e_j \to e_k + e_l + \bar{e}_m$ which proceeds by the exchange of a sneutrino $\tilde{\nu}_i$ in the t as well as u channel. The effective Lagrangian can be expressed as [21]

$$\mathcal{L}(e_j \to e_k + e_l + \bar{e}_m) = \mathcal{F}_{jklm}\bar{e}_{kR}e_{jL}\bar{e}_{lL}e_{mR} + \mathcal{F}_{mlkj}\bar{e}_{kL}e_{jR}\bar{e}_{lR}e_{mL}$$
$$+\mathcal{F}_{jlkm}\bar{e}_{lR}e_{jL}\bar{e}_{kL}e_{mR} + \mathcal{F}_{mklj}\bar{e}_{lL}e_{jR}\bar{e}_{kR}e_{mL} \tag{21}$$

where

$$\mathcal{F}_{jklm} = \sum_i \left(\frac{1}{m_{\tilde{\nu}_i}^2}\right)\lambda_{ijk}\lambda_{ilm}^\star. \tag{22}$$

The first two terms in (21) correspond to t channel exchange diagrams, while the last two terms correspond to u channel exchange diagrams.

Substituting the couplings from (19) into (22), we have calculated the branching ratios of various decays of μ and τ. They are exposed in TABLE II. Unless considerable experimental progress is made the most of branchings from TABLE II are clearly too small for the corresponding decays to be detected in a near future. The only exclusion might exist for the dominant τ decay mode $\tau \to 3\mu$, although even for the favorable parameter area with $\epsilon_{2,3} \sim 1$, $\tan\beta = 6$ and sneutrino masses of 100 GeV, its branching is turned out to be $Br(\tau \to 3\mu) \approx 10^{-8}$. Meanwhile, although presently one hardly expects a sensitivity better than 10^{-7} for any rare decay mode of τ [20], there is planned at LHC to reach the branching level 10^{-9}[22], particularly, for the mode mentioned.

In this connection the model predicts some of the quite specific signals that could be tested at LHC or other future facilities such as a muon collider and τ–factories. The first is that according to the flavor selection rule (18) the 3-lepton τ decays with the identical di-leptons in final state, like as $\tau^\mp \to e^\mp e^\mp \mu^\pm$ and $\tau^\mp \to \mu^\mp \mu^\mp e^\pm$, are strictly prohibited. The second is the characteristic relations appeared among the partial decay widths which can be expressed through the lepton masses and ϵ_i parameters in a following way

$$\frac{\Gamma(\mu \to 3e)}{\Gamma(\tau \to 3e)} \simeq \left(\frac{\epsilon_2 m_\mu}{\epsilon_3 m_\tau}\right)^2 \cdot \rho$$

$$\frac{\Gamma(\tau \to 3e)}{\Gamma(\tau \to 3\mu)} = \left(\frac{\epsilon_1 m_e}{\epsilon_2 m_\mu}\right)^2 \tag{23}$$

$$\frac{\Gamma(\tau \to 3e)}{\Gamma(\tau \to e\mu\bar{\mu})} = \left(\frac{m_e}{m_\mu}\right)^2$$

$$\frac{\Gamma(\tau \to \mu e\bar{e})}{\Gamma(\tau \to 3\mu)} = \left(\frac{m_e}{m_\mu}\right)^2$$

together with an elegant "model-independent" branching combination

$$\frac{\Gamma(\tau \to 3e)\Gamma(\tau \to 3\mu)}{\Gamma(\tau \to e\mu\bar{\mu})\Gamma(\tau \to \mu e\bar{e})} = 1 \tag{24}$$

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containing neither lepton masses nor the ϵ_i parameters (in the first relation in (23) the approximately equal masses for the second and third generation sneutrinos were taken for simplicity; ρ stands for the phase volume factor, $\rho \simeq (m_\mu/m_\tau)^5$).

One can readily confirm that as the distinctive suppression of some of τ decay modes mentioned, so the above strict relations between its modes allowed appear as a direct consequence of the Yukawa-aligned structure of the LNV λ couplings clearly manifested itself when taking in a physical basis (15).

3.4 LSP decays

The main predictions of the HHLM model proposed certainly belong to the lightest neutralino (LSP) decays[4], which could, in principle, be tested even at the currently working facilities if the sparticle masses were in a proper area.

Recall that the LNV decays of the LSP drastically changes a standard missing–energy signature being in the ordinary R–parity conserving MSSM. Instead, the LSP gives rise to the high-energy particles all of which but neutrinos can be detected directly. These decays are in effect the two-step processes being properly mediated by sleptons and/or squarks. At the first step the LSP freely goes to lepton-slepton (quark-squark) pair and then slepton (squark) decays through the proper LNV coupling into final lepton (or quark-lepton) pair. While the first step is practically flavor-independent (since the LSP goes to all fermion-sfermion pair but possibly the top-stop system mainly due to its electroweak eigenstate components photino $\tilde{\gamma}$ and zino $\tilde{Z}$), the second one, according to the basic coupling equations (15) and (16), crucially depends on the lepton and /or quark species.

As a result, only some particular (distinctively configured in a flavor space by the selection rules (17) and (18)) LSP decay modes appear. For the leptonic and semi-leptonic LSP decays proceeding in the final LNV stage through the λ and λ' couplings, respectively, they are

$$\chi_1^0 \rightarrow e_i + \bar{e}_k + \nu_k, \tag{25}$$

$$\chi_1^0 \rightarrow \nu_i + \bar{e}_k + e_k \tag{26}$$

and

$$\chi_1^0 \rightarrow e_i + \bar{d}_k + u_k, \tag{27}$$

$$\chi_1^0 \rightarrow \nu_i + \bar{d}_k + d_k \tag{28}$$

where indices i and k shown correspond to the lepton and quark generation species appeared. In this connection the most experimentally interesting cases are the decays (25) and (27) whose branchings are essentially determined by flavors of the charged leptons (and down quarks) involved:

$$\frac{\Gamma(\chi_1^0 \rightarrow e_i \bar{e}_k \nu_k)}{\Gamma(\chi_1^0 \rightarrow e_j \bar{e}_m \nu_m)} \simeq \left(\frac{\epsilon_i m_{e_k}}{\epsilon_j m_{e_m}}\right)^2 \tag{29}$$

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[4]We consider the lightest neutralino as the LSP.

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$$\frac{\Gamma(\chi_1^0 \to e_i \bar{d}_k u_k)}{\Gamma(\chi_1^0 \to e_j \bar{d}_m u_m)} \simeq \left(\frac{\epsilon_i m_{d_k}}{\epsilon_j m_{d_m}}\right)^2$$

We have assumed for simplicity, while deriving these relations, that all generation sleptons and squarks mediating decay processes (25-28) have approximately the same masses; also the CKM mixing was neglected for quarks.

From these general relations a variety of particular ones follow when taking some special orientation of flavor indices. Among them are cases when $i = j$ ($k \neq m$) and $k = m(i \neq j)$, respectively,

$$\frac{\Gamma(\chi_1^0 \to e_i \bar{e}_k \nu_k)}{\Gamma(\chi_1^0 \to e_i \bar{e}_m \nu_m)} \simeq \left(\frac{m_{e_k}}{m_{e_m}}\right)^2, \tag{30}$$

$$\frac{\Gamma(\chi_1^0 \to e_i \bar{e}_k \nu_k)}{\Gamma(\chi_1^0 \to e_j \bar{e}_k \nu_k)} \simeq \left(\frac{\epsilon_i}{\epsilon_j}\right)^2 \tag{31}$$

for leptonic decay modes and

$$\frac{\Gamma(\chi_1^0 \to e_i \bar{d}_k u_k)}{\Gamma(\chi_1^0 \to e_i \bar{d}_m u_m)} \simeq \left(\frac{m_{d_k}}{m_{d_m}}\right)^2, \tag{32}$$

$$\frac{\Gamma(\chi_1^0 \to e_i \bar{d}_k u_k)}{\Gamma(\chi_1^0 \to e_j \bar{d}_k u_k)} \simeq \left(\frac{\epsilon_i}{\epsilon_j}\right)^2 \tag{33}$$

for the semi-leptonic ones, as well as one more relation connecting both of sets

$$\frac{\Gamma(\chi_1^0 \to e_i \bar{e}_k \nu_k)}{\Gamma(\chi_1^0 \to e_j \bar{e}_k \nu_k)} \simeq \frac{\Gamma(\chi_1^0 \to e_i \bar{d}_k u_k)}{\Gamma(\chi_1^0 \to e_j \bar{d}_k u_k)} \tag{34}$$

which just like as the τ lepton branching relation (24) contains no any parameter at all.

The basic signature of the LSP decays, as it can quickly be read off the relations (29), (30) and (32), is an overdominance of the modes with the heaviest lepton and quark families, clearly manifested when comparing the cases with the same charged lepton taken. Among them the semileptonic modes $\chi_1^0 \to e_i \bar{b} t$ are, of course, dominant, if LSP heavier than top quark. Otherwise, the leptonic modes $\chi_1^0 \to e_i \bar{\tau} \nu_\tau$ dominate. Should the class of decays (26) and 28), no tagged by charged leptons, is considered the modes $\chi_1^0 \to \nu_i \bar{b} b$ and $\chi_1^0 \to \nu_i \bar{\tau} \tau$ appear to be leading. On the other hand, which class of decays from the above two dominates strongly depends on the nature of the LSP by itself [23]. If the LSP mainly consist of the $\tilde{\gamma}$ component, branching fractions to the modes (25, 27) are large, while the dominant $\tilde{Z}$ component gives preference to decay channels(26, 28).

Remarkably, even with the small couplings (19) and (20) appeared in HHLM model the LSP could decay inside a typical detector. In fact, three–body decays of the LSP (25-28) drive at the widths [24]

$$\Gamma_{ikk} = \frac{\alpha c_f}{128\pi^2} (\epsilon_i \frac{m_{f_k}}{V c_\beta})^2 \frac{M_{\chi_1^0}^5}{\tilde{m}_f^4} \tag{35}$$

where the corresponding effective LNV coupling constants were taken from basic equations (15) and (16) for lepton ($f_k = e_k$, $c_e = 1$) and quark ($f_k = d_k$, $c_d = 3$) cases, respectively ($\tilde{m}_f$ stands

for masses of the intermediate sfermions involved, while c_f is a color factor)). Assuming then that the LSP decays inside the detector $(c\gamma_L\tau(\chi_1^0) \lesssim 1$ m, γ_L is the Lorentz boost factor) one obtains the lower bounds on the generic LNV parameters ϵ_i

$$\epsilon_i \gtrsim 10^{-6} \left(2\gamma_L \frac{3}{c_f}\right)^{1/2} \left(\frac{174 GeV}{m_{f_k}} c_\beta\right) \left(\frac{\tilde{m}_f}{200 GeV}\right)^2 \left(\frac{100 GeV}{M_{\chi_1^0}}\right)^{5/2} \tag{36}$$

Therefore, even for the possible smallest ϵ_i value, $\epsilon_i \sim 0.01$, as is likely to be conditioned by neutrino masses (see next Subsection), one can expect to observe the LSP decays (including those into light leptons and quarks) inside the detector, if sfermions are not enormously heavy.

All the distinctive features of the LSP decays listed above taken together constitute a main basis for a global testing of HHLM model. Some of relations shown, such as (30), 32) and, especially, (34) suggest the direct testing of the model, the rest allows to extract actual values of the ϵ-parameters to compare them with those extracted from τ lepton decays (23) or quite the reverse.

3.5   Neutrino masses and oscillations

The SUSY inspired lepton number violation opens in substance the shortest way to neutrino masses and, indeed, many interesting attempts were made towards this problem (see [7, 11, 25] and references therein).

In the HHLM model with the highly suppressed direct Higgs-lepton mixing terms (14) one can neglect the tree-level neutrino masses and be focused only on their radiative masses caused by the trilinear LNV couplings (9). Generally, they contribute to each entry of the neutrino Majorana mass matrix through the diagrams with lepton–slepton and quark–squark loops. It is apparent with the hierarchies in the basic LNV couplings(15, 16) taken that the diagrams involving tau–stau and bottom–sbottom loops are highly dominant. Therefore, to the obviously good approximation the neutrino mass matrix comes finally to the remarkably transparent form

$$M_{ij}^\nu \simeq \frac{3}{8\pi^2} \frac{m_b^2}{V^2 c_\beta^2} \frac{m_b^2}{\tilde{m}_b^2} (A^b + \mu \tan\beta) \begin{pmatrix} \epsilon_1^2(1+\Lambda) & \epsilon_1\epsilon_2(1+\Lambda) & \epsilon_1\epsilon_3 \\ \epsilon_1\epsilon_2(1+\Lambda) & \epsilon_2^2(1+\Lambda) & \epsilon_2\epsilon_3 \\ \epsilon_1\epsilon_3 & \epsilon_2\epsilon_3 & \epsilon_3^2 \end{pmatrix} \tag{37}$$

where

$$\Lambda = \frac{m_\tau^4}{3m_b^4} \frac{\tilde{m}_b^2}{\tilde{m}_\tau^2} \frac{A^\tau + \mu\tan\beta}{A^b + \mu\tan\beta}, \tag{38}$$

$\tilde{m}_\tau$ and $\tilde{m}_b$ stand for stau and sbottom masses, while $A^{\tau,b}$ are the corresponding trilinear soft terms in the stau and sbottom left-right masses squared $m_{\tau,b}(A^{\tau,b} + \mu\tan\beta)$, respectively.

Ignoring for the moment the relatively small contributions stemming from the tau–stau loop $(\Lambda \ll 1)$ we come to one massive neutrino state with mass

$$m_3 \approx 4.5 \cdot 10^{-4} \frac{(A^b + \mu\tan\beta)}{\tilde{m}_b^2 c_\beta^2} (\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2) GeV^2 \tag{39}$$

and two massless states. To account for the SuperKamiokande data [1] for $\Delta m_{atm}^2 \approx 0.005$ eV^2 we require that

$$\frac{(A^b + \mu\tan\beta)}{\tilde{m}_b^2 c_\beta^2} (\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2) \approx 1.6 \cdot 10^{-7} GeV^{-1}. \tag{40}$$

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Furthermore, according to the same data[1] this massive state should be about the maximal mixture of ν_μ and ν_τ, while an admixture of ν_e should be small. This requires the following hierarchy[5]

$$\epsilon_1 \ll \epsilon_2 \sim \epsilon_3. \tag{41}$$

The Λ terms (tau–stau loop contributions) in M_{ij}^ν(37) result in some non-zero mass value $m_2 \approx \Lambda m_3$ for the next state, thus leaving finally only one neutrino state to be strictly massless, $m_1 = 0$. Therefore, for the neutrino mass-squared differences presently being of particular interest we have

$$\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2} = \frac{m_2^2 - m_1^2}{m_3^2 - m_2^2} \approx \frac{m_2^2}{m_3^2} \approx \Lambda^2. \tag{42}$$

Demanding that $\Lambda \approx 0.1$ (say, $\frac{\tilde{m}_b}{\tilde{m}_\tau} \approx 3$ and $A^\tau \approx A^b$ in Eq. (37)), one can account for the solar neutrino data in the small angle MSW solution [28] context. Thus, it looks like that both atmospheric and solar neutrino data are well accommodated within the minimal LNV model with the radiatively induced neutrino masses.

On the other hand, even a simple order of magnitude estimate shows (see Eq. (40)) that to get the observed neutrino mass scale one must require a decrease of the basic LNV parameters ϵ_i down to the order of $O(0.01)$ (unless the unnatural cancellation in neutrino mass scale (39) resulting in $A^{b(\tau)} + \mu \tan\beta = O(1)$ MeV) somehow occurs). If so, then LNV interactions practically have no direct implications for low-energy physics (such as those discussed in Subsections III.B, C) other than the neutrino phenomenology, while they will still significantly alter SUSY signal related with the LSP decays. Remarkably, even in this case, since the parameters μ_i are turned out to be properly diminished (see Eq. (14)) the LNV-Yukawa alignment (15, 16) continues to work successfully, and with it all related predictions for the LSP decays (Subsection III.D).

Finally, it is worthy of note that the simple structure of the neutrino mass matrix (37) could somewhat be altered within the extended GUTs due to the additional mixings of active neutrinos with those of sterile, generally presented in higher GUT multiplets (for some attempts to accommodate neutrino data within GUTs, see [29]).

3.6 HHLM versus HLM

Now, let us consider briefly an ordinary HLM (Higgs-Lepton Mixing) model, another minimal framework for LNV that could be given solely by just the generic bilinear terms $\mu_i L_i H_u$ in the superpotential ΔW (1). Rotating them away one recovers the Yukawa-aligned trilinear LNV couplings which, as those in the HHLM model (9), naturally overcome all the currently available experimental constraints following from low-energy physics (Subsection III.B).

However, the fundamental part of this scenario is related with SUSY soft-breaking sector owing to which the condensation of sneutrinos and, as a result, new sets of the physical (mixed) states both in gauge and Higgs sector arise [11]. Because of this there appear some principal differences with the HHLM model, which might manifest themselves at an observational level.

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[5]Actually, the CHOOZ data on ν_e disappearance [26] can be accommodated with $\epsilon_1^d/\epsilon_{2,3}^d \lesssim 0.1$ (for a coherent analysis of the neutrino mixings dictated by atmospheric and solar neutrino data see [27])

The main point is that, while the effective trilinear LNV couplings are generated in the HLM model.

$$\lambda_{ijk} = \xi_i Y_{jk}^l, \quad \lambda'_{ijk} = \xi_i Y_{jk}^d \quad (\xi_i = \frac{<\tilde{\nu}_i>}{V c_\beta} - \frac{\mu_i}{\mu})$$ (43)

they, being properly weakened by the Yukawa couplings, appear too small even for the dominant couplings[30]

$$\lambda_{i33} \approx 7 \cdot 10^{-9} \frac{\eta}{c_\beta^2}, \quad \lambda'_{i33} \approx 2 \cdot 10^{-8} \frac{\eta}{c_\beta^2}$$ (44)

(where $\eta = (M_1 M_2 / M_Z M_{\tilde{\gamma}} - M_Z s_{2\beta} / \mu)^{1/2}$ with $M_{1,2}$ standing for the $U(1)_Y$ and $SU(2)_W$ soft-breaking gaugino mass terms and $M_{\tilde{\gamma}} = c_W^2 M_1 + s_W^2 M_2$) for not marginally high values of $\tan\beta$. Hence, their contributions to the LNV processes relative to those from the direct LNV admixtures in the physical neutralino and chargino states are quite negligible. As a result, LNV processes in the HLM model, being dominantly mediated by W and Z bosons, are in essence family-independent (exclusive of the dependence on the generic mixing parameters ξ_i by themselves) in sharp contrast to HHLM model where they essentially conditioned by quark and lepton mass hierarchy.

For example, the rare leptonic decays of τ considered in Subsection III.C appear also in the HLM model. However, they practically do not depend now on the final lepton masses. The proper relations between their branchings follow from those of the HHLM model when taking in Eqs. (23) all lepton masses equal (while $\epsilon_i/\epsilon_j \to \xi_i/\xi_j$):

$$\Gamma(\tau \to 3e) = \Gamma(\tau \to e\mu\bar{\mu}) = \left(\frac{\xi_1}{\xi_2}\right)^2 \Gamma(\tau \to \mu e\bar{e}) = \left(\frac{\xi_1}{\xi_2}\right)^2 \Gamma(\tau \to 3\mu)$$ (45)

The same can be stated about the LSP decays as well (III.D). Their branching relations are also largely quark and lepton mass-independent in the HLM model. Therefore, again, instead of the hierarchical relations (29) one has the "democratical" ones

$$\frac{\Gamma(\chi_1^0 \to e_i \bar{e}_k \nu_k)}{\Gamma(\chi_1^0 \to e_j \bar{e}_m \nu_m)} \simeq \frac{\Gamma(\chi_1^0 \to e_i \bar{d}_p u_p)}{\Gamma(\chi_1^0 \to e_j \bar{d}_q u_q)} \simeq \left(\frac{\xi_i}{\xi_j}\right)^2$$ (46)

(i, j, k, m, p, q are any generation indices, no summing is implied) for decays (25, 27) mediated now by W bosons and similar relations for decays (26, 28) mediated by Z boson. The Z boson exchange gives one more peculiarity to the HLM model opening a way to two new decay modes in the leptonic and semi-leptonic sector, respectively. They are $\chi_1^0 \to \nu_i \bar{\nu}_k \nu_k$ and $\chi_1^0 \to \nu_i \bar{u}_k u_k$ coming solely from the neutrino (ν_i) admixtures in the LSP.

As to the partial decay rates of the τ lepton and LSP in the HLM model, they, according to the presently deduced constraints on the ξ_i parameters coming from the existing bounds for lepton flavor-changing decays of Z–boson and neutrino masses(see, e.g., [31, 32]), are turned out to be approximately in the same area as those in the HHLM model.

In closing one can summarize that, despite some generic similarity, the HHLM and HLM models present two directly opposed observational possibilities, each supplied with a quite clear signature manifesting itself respectively in the flavor hierarchy or flavor democracy of the final states produced.

4 Grand unification

We argue in this Section that the HHLM model can naturally be embedded in the grand unified framework, particularly in the $SU(7)$ model[13]. The preference given to the $SU(7)$ model over other grand unified schemes is essentially determined by the missing VEV solution to the gauge hierarchy problem which naturally appears in some $SU(N)$ GUTs starting from the $SU(7)$[14]. This is shown to lead to a similar hierarchy of baryon vs lepton number violation.

We discuss first briefly how one can come to the $SU(7)$ GUT. Towards this end let us consider a general $SU(N)$ SUSY GUT with the simplest anomaly–free set combination of the fundamental and 2-index antisymmetric representations

$$3 \cdot \left[(N-4)\overline{\Psi}^A + \Psi_{[AB]} \right] \tag{47}$$

($A, B = 1, ..., N$ are the $SU(N)$ indices) for the three quark–lepton generations like as $3 \cdot [\overline{5} + 10]$ in the prototype $SU(5)$ model. As to Higgs sector of the model there are an adjoint Higgs multiplet Σ_B^A responsible for the starting breaking of $SU(N)$ and conjugated pair of multiplets H and $\overline{H}$ (being specified later) where the ordinary electroweak doublets reside. Besides, there should be $N-5$ scalar superfields φ^r and $\overline{\varphi}^r$ ($r = 1, ..., N-5$) which break $SU(N)$ to $SU(5)$ by their own. It is also expected that certain of the matter and / or Higgs superfields in the model can carry charges of some protecting side symmetry like as an anomalous $U(1)_A$, as in the case considered.

Now we suppose that all the generalized Yukawa couplings as the R-parity conserving (ordinary up and down Yukawas), so R–parity violating ones allowed by $SU(N) \otimes U(1)_A$ symmetry are given by the similar set of the dimension-5 operators of the form (i, j, k are the generation indices, the SU(7) indices are omitted)

$$\mathcal{O}_{ij}^{up} \propto \frac{1}{M_P}(\Psi_i \Psi_j)(H\varphi) \tag{48}$$

$$\mathcal{O}_{ij}^{down} \propto \frac{1}{M_P}(\overline{\Psi}_i \Psi_j)(\overline{H}\varphi) \tag{49}$$

$$\mathcal{O}_{ijk}^{rpv} \propto \frac{1}{M_P}(\overline{\Psi}_i \Psi_j)(\overline{\Psi}_k \Sigma) \tag{50}$$

which can be viewed as an effective interactions generated through the exchange of some heavy states with Plank scale order masses (they might be treated as states inherited from the massive string modes)[6]. When being generated by an exchange of the same superheavy multiplet the operators (49) and (50) appear with the dimensionless coupling constants to be properly aligned.

However, one must ensure first the suppression of the BNV interactions since they are generated from the coupling (50) as well. The key idea here is that the adjoint field Σ involved in the RPV coupling (50) could develop the missing VEV pattern with zero color components:

$$< \Sigma > = diag[0, 0, 0, a_4, a_5, ..., a_N]V_{GUT} \tag{51}$$

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[6]The up-down hierarchy in the quark mass spectrum is assumed to be properly given in this case by the VEV ratio of different extra scalars φ involved in $\mathcal{O}_{ij}^{up}$ and $\mathcal{O}_{ij}^{down}$, respectively.

where $\sum_{k=4}^{N} a_k = 0$. It is easy to verify that with such a basic vacuum configuration in the model the baryon number violating part of RPV interactions are projected out from the low energy effective superpotential.

As it was shown in the recent papers[14] the missing VEV configurations like (51) naturally appear in some extended $SU(N)$ GUTs from $SU(7)$ to solve the doublet-triplet splitting problem. In the minimal $SU(7)$ case[13] which still remains an ordinary local symmetry of MSSM at low energies the solution (51) has a form

$$< \Sigma >= diag[0,0,0,1,1,-1,-1]V_{GUT}. \tag{52}$$

This breaks the $SU(7)$ symmetry to

$$SU(7) \rightarrow SU(3)_C \otimes SU(2)_W \otimes SU(2)_E \otimes U(I)_1 \otimes U(I)_2 \tag{53}$$

while the extra symmetry $SU(2)_E \otimes U(I)_1 \otimes U(I)_2$ breaking to the standard hypercharge $U(I)_Y$ appears due to the additional fundamental scalars $\varphi^{1,2}$ and $\overline{\varphi}^{1,2}$ (septets and anti-septets of $SU(7)$) mentioned above. They are supposed to develop their VEVs along the "extra" directions

$$\varphi_A^1 = \delta_{A6} V_1, \varphi_A^2 = \delta_{A7} V_2 \tag{54}$$

only through the proper Fayet-Iliopoulos $D-$term related with anomalous $U(1)_A$ symmetry [16] This is specially introduced in the $SU(7)$ model as a protecting symmetry which keeps extra symmetry-breaking scalars (54) untied from the basic adjoint scalar Σ not to influence the missing VEV solution appeared (52).

One can readily check that a solution (52) gives a minimum to a general adjoint superpotential containing any even powers of Σ (conditioned by the reflection symmetry $\Sigma \rightarrow -\Sigma$ imposed):

$$W_A = \frac{1}{2}m\Sigma^2 + \frac{\lambda_1}{4M_P}\Sigma^4 + \frac{\lambda_2}{4M_P}\Sigma^2\Sigma^2 + ... \tag{55}$$

with $V_{GUT} \sim \frac{1}{\lambda}(mM_P)^{1/2}$ which, for the properly chosen adjoint mass m and coupling constants $\lambda_{1,2,...}$, can easily comes up to the string scale M_{str}. The superpotential W_A can also be viewed as an ordinary renormalizable two-adjoint superpotential with the second heavy adjoint scalar to be further integrated out.

Now, let us see how this missing VEV mechanism works to solve the gauge hierarchy problem or, equivalently, the doublet-triplet splitting problem in the $SU(7)$ SUSY GUT. There is, in fact, the only reflection-invariant coupling of the basic adjoint Σ with a pair of the ordinary Higgs-boson containing supermultiplets H and $\overline{H}$

$$W_H = f\overline{H}\Sigma H \quad (\Sigma \rightarrow -\Sigma, \overline{H}H \rightarrow -\overline{H}H) \tag{56}$$

having the zero VEVs, $H = \overline{H} = 0$, during the first stage of the symmetry breaking. Thereupon W_H turns to the mass term of H and $\overline{H}$ depending on the missing VEV pattern (52). This vacuum, while giving generally heavy masses (of order of M_{GUT}) to them, leaves their weak components strictly massless. To be certain we must specify the multiplet structure of H and $\overline{H}$ in the case of the color-component missing VEV solution (52) appeared in the $SU(7)$. One can see that H and $\overline{H}$ multiplet must be the 2-index antisymmetric 21-plets of $SU(7)$ which after

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starting symmetry breaking (53) contain just a pair of the massless weak doublets of MSSM (for more detail see[13]). Thus, there certainly is a natural doublet-triplet splitting although we are coming to the strictly vanishing μ-term at the moment. However, at the next stage when SUSY breaks, radiative corrections shift the missing VEV to some nonzero value of order M_{SUSY}, thus inducing the ordinary μ-term of MSSM[7], on the one hand, and BNV couplings with the hierarchically small constants $\lambda''_{ijk} = O(M_{SUSY}/M_{GUT})$, on the other.

Now, substituting the VEVs for scalars Σ (52) and φ (54) in the basic operators (48-50), one obtains at low energies the effective renormalizable Yukawa and LNV interactions, while the BNV interactions are proved to be properly suppressed. Besides, if one further introduces the superheavy intermediate $SU(7)$ septets $\Phi + \overline{\Phi}$, whose exchange generates the effective operators $\mathcal{O}_{ij}^{down}(49)$ and $\mathcal{O}_{ijk}^{rpv}$ (50) simultaneously, the alignment between their dimensionless coupling of type (12) follows immediately. This can easily be read off the effective Yukawa and LNV couplings(8, 9) properly specified to the $SU(7)$ case ($\overline{H}_d \to \overline{H}$, $S \to \varphi$, $T \to \Sigma$).

The $SU(7)$ model is thoroughly considered in our forthcoming paper[13].

5   Conclusions

The recent neutrino data[1] strongly suggest that neutrinos are massive. While some other modifications of the SM could lead to neutrino masses, SUSY extension of the SM with a generic lepton number violation seems to be the most plausible and attractive framework.

In the present paper we have proposed some prototype model for a minimal LNV which could appear in SUSY theories, if all the generalized Yukawa coupling, both R-parity conserving and R-parity violating, had a common origin.While our model is based on a generic Higgs-lepton mixing, we propose, in contrast to the known picture, that this mixing appears not with a standard MSSM Higgs doublet but with some superheavy weak doublets mediating the down fermion (down quark and charged lepton) Yukawa couplings. As a result, all significant physical manifestations of LNV are no other than those of the effective trilinear couplings $LL\overline{E}$ and $LQ\overline{D}$ aligned, by magnitude and orientation in flavor space, with the down fermion effective Yukawa couplings.

One of the immediate consequences of the HHLM model is a natural suppression of the flavor-changing processes both in quark and lepton sectors due to the additional flavor selection rules (17, 18) appeared. According to them a large reduction of a number of the possible LNV coupling constants, from 36 of λ_{ijk} and λ'_{ijk} to 3 of ϵ_i (13), takes place.

The model predicts a number of the potentially interesting signals (Section III.B, C, D, E) which can be tested in future experiments. Experimental study of the LSP decays is certainly of main interest for the model. These decays, which even for the small LNV coupling constants could occur inside the typical detectors, suggest the global testing of the HHLM model.

Simultaneously, we have shown that the present model leads to a self-consistent picture of the radiatively induced neutrino masses and mixings, thus successfully accommodating all the presently available neutrino data, particularly, in the small angle MSW solution[28] context. However, the observed neutrino mass scale[1] requires by itself to suppress the magnitude of the

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[7]At this stage the effective bilinear LNV terms are also generated but they are still suppressed relative to the ordinary μ-term just as in the case of MSSM discussed in Section II (see Eq.(14)).

LNV couplings demanding ϵ -parameters down to the order of 10^{-2}. If so, one can hardly expect any experimentally interesting LNV signal in low–energy physics (Sections III.B, C) beyond the neutrino phenomenology. Nevertheless, even in this case the most significant HHLM predictions, which are related with the decay modes of the LSP (Section III.D), remain in force.

Finally, we have presented the $SU(7)$ GUT framework for the minimal lepton number violation. Remarkably enough, the SUSY inspired baryon number violation is proved to be projected out from the low-energy superpotential by the missing VEV vacuum configuration giving a solution to doublet-triplet problem. So, a natural gauge hierarchy seems to lead to a similar hierarchy of the baryon vs lepton number violation, at least, in the SUSY $SU(7)$ GUT.

Acknowledgments

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Tables

TABLE I. Four-fermion operators resulting from LNV interactions.

Effective operators	Couplings involved	Particles exchanged	Example processes
$d_j\bar{d}_k d_l\bar{d}_m$	$\lambda'_{ijk}\lambda'^{\star}_{iml}$	$\tilde{\nu}_i$	$K^0 - \overline{K}^0,\ B^0 - \overline{B}^0$
$u_j\bar{d}_k d_l\bar{u}_m$	$V^{\dagger}_{pj}V_{mq}\lambda'_{ipk}\lambda'^{\star}_{iql}$	$\tilde{e}_i$	$B \to K\pi$
$u_j\bar{e}_k e_l\bar{u}_m$	$V^{\dagger}_{pj}V_{mq}\lambda'_{kpi}\lambda'^{\star}_{lqi}$	$\tilde{d}^c_i$	$\pi^0 \to \bar{\mu}e,\ D^+ \to \pi^+\bar{\mu}e$
$d_j\bar{e}_k e_m\bar{d}_l$	$V_{ip}\lambda'^{\star}_{kpj}\lambda'_{mil},\ \lambda^{\star}_{ikm}\lambda'_{ilj}$	$\tilde{u}_i,\ \tilde{\nu}_i$	$K^0 \to e_l\bar{e}_k,\ K^+ \to \pi^+ e_l\bar{e}_k$
$d_l\nu_j\nu_m\bar{d}_k$	$\lambda'^{\star}_{jil}\lambda'_{mik},\ \lambda'^{\star}_{jki}\lambda'_{mli}$	$\tilde{d}_i,\ \tilde{d}^c_i$	$K^0 \to \pi^0\nu_j\bar{\nu}_m,\ B^0 \to K^0\nu_j\bar{\nu}_m$
$u_j e_k\nu_l\bar{d}_m$	$\lambda'_{ijk}\lambda'_{iml},\ \lambda_{ijk}\lambda'_{iml}$	$\tilde{e}_i,\ \tilde{d}_i$	$\pi^- \to \bar{\nu}_l\mu,\ B^0 \to K^+ e\bar{\nu}_l$
$e_l\nu_j\nu_m\bar{e}_k$	$\lambda_{ijk}\lambda^{\star}_{ilm},\ \lambda_{jki}\lambda^{\star}_{mli}$	$\tilde{e}_i,\ \tilde{e}^c_i$	$\mu \to e\nu_j\bar{\nu}_m,\ \tau \to \mu\nu_j\bar{\nu}_m$
$e_l e_j e_m\bar{e}_k$	$\lambda_{ijk}\lambda^{\star}_{ilm}$	$\tilde{\nu}_i$	$\mu \to ee\bar{e},\ \tau \to ee\bar{\mu}$

TABLE II. Three body decays of μ and τ, their expected branchings and the current upper limits [18].

Decay process	Branching	Upper limit (CL=90%)
$\mu \to 3e$	$7.2\cdot 10^{-18}\left(\frac{100GeV}{m_{\tilde{\nu}_2}c_\beta}\right)^4(\epsilon_1\epsilon_2)^2$	$< 1.0\cdot 10^{-12}$
$\tau \to 3e$	$3.5\cdot 10^{-16}\left(\frac{100GeV}{m_{\tilde{\nu}_3}c_\beta}\right)^4(\epsilon_1\epsilon_3)^2$	$< 2.9\cdot 10^{-6}$
$\tau \to 3\mu$	$1.5\cdot 10^{-11}\left(\frac{100GeV}{m_{\tilde{\nu}_3}c_\beta}\right)^4(\epsilon_2\epsilon_3)^2$	$< 1.9\cdot 10^{-6}$
$\tau \to e\bar{e}\mu$	$3.5\cdot 10^{-16}\left(\frac{100GeV}{m_{\tilde{\nu}_3}c_\beta}\right)^4(\epsilon_2\epsilon_3)^2$	$< 1.7\cdot 10^{-6}$
$\tau \to \mu\bar{\mu}e$	$1.5\cdot 10^{-11}\left(\frac{100GeV}{m_{\tilde{\nu}_3}c_\beta}\right)^4(\epsilon_1\epsilon_3)^2$	$< 1.8\cdot 10^{-6}$
$\tau \to ee\bar{\mu}$	forbidden	$< 1.5\cdot 10^{-6}$
$\tau \to \mu\mu\bar{e}$	forbidden	$< 1.5\cdot 10^{-6}$