FORMATION NUMBER OF LAMINAR VORTEX RINGS. NUMERICAL SIMULATIONS.

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Abstract

The formation time scale of axisymmetric vortex rings is studied numerically for relatively long discharge times. Experimental findings on the existence and universality of a formation time scale, referred to as the "formation number," are confirmed. The formation number is indicative of the time a vortex ring acquires its maximal circulation. For vortex rings generated by impulsive motion of a piston, the formation number was found experimentally to be approximately 4. Numerical extension of the experimental study to thick shear layers indicates that the scaled circulation of the pinched-off vortex is relatively insensitive of the details of the formation process, such as the velocity program, velocity profile or vortex generator geometry. In contrast, the formation number does depend on the velocity profile.

Introduction

A new feature of vortex ring formation was recently observed experimentally by Gharib, Rambod & Shariff (1998) (henceforth denoted as GRS) for vortex rings formed by a piston pushing a column of fluid out of a tube. As the duration of the piston stroke increased, GRS showed evidences on the existence of a limiting process that imposes an upper bound on the circulation a vortex ring can acquire for a set of flow parameters. It was shown both experimentally and analytically that this maximum in the circulation is attained at a narrow range of *formation time* $4.5 > t^* > 3.6$ for a number of cases differing in the velocity program and in the velocity or diameter of the piston. This universal number is referred to as the *formation number*. The formation time t^* is defined as $t^* = \overline{U}_p t/D$, where $\overline{U}_p$ is the average velocity discharged from the orifice of diameter D and t is the time. The duration of the discharge time is T and the corresponding formation time is T^*.

The existence of a universal formation number is intriguing as it may hint at the possibility of nature using this time scale for certain evolutionary incentives, such as optimum ejection of blood into the left ventricle or aquatic locomotion processes where ejection of vortices might have been utilized for the purposes of propulsion.

Numerical model

The formation time scale of axisymmetric vortex rings is studied in the present work numerically for relatively long discharge times. The axisymmetric, time-dependent and incompressible Navier-Stokes equations are solved by the FIDAP package for a large set of cases differing in the velocity program, velocity profile of the discharging flow, geometry configuration and Reynolds number.

Results

The formation and propagation of a vortex ring are shown in Fig. 1 by the vorticity field for two different discharge times (stroke ratios) of T^*=2 and 6. In the case of the short discharge time, all the vorticity generated at the orifice during the ejection is essentially entrained into the downstream propagating vortex ring, leaving behind a calm flow region. The situation is very different at the higher discharge time of T^*=6: the vortex ring disconnects from the bulk of flow, leaving behind a noticeable trailing jet-like region. Any excess vorticity (–circulation) that cannot be entrained into the vortex remains in the wake.

The evolution of the total circulation and the circulation of the vortex ring is of major interest in the present study. These quantities are plotted in Fig. 2 for T^*=6. Very good agreement is obtained between the numerical and the experimental results.

The numerical simulations confirm the existence of a maximum in the vortex circulation for $T^* > 4$. The numerical simulations considered also the effects of several factors on the circulation and on the formation number. It was found that the formation number does depend on the velocity profile Non-impulsive velocity programs slightly increase the formation number, while non-uniform velocity profiles may decrease it. In the case of a parabolic velocity profile of the discharged flow, for example, the formation number decreases by a factor as large as 4. These findings indicate that the major source of the small variations found experimentally in the magnitude of the formation number is the different evolution of the velocity profile of the discharged flow.

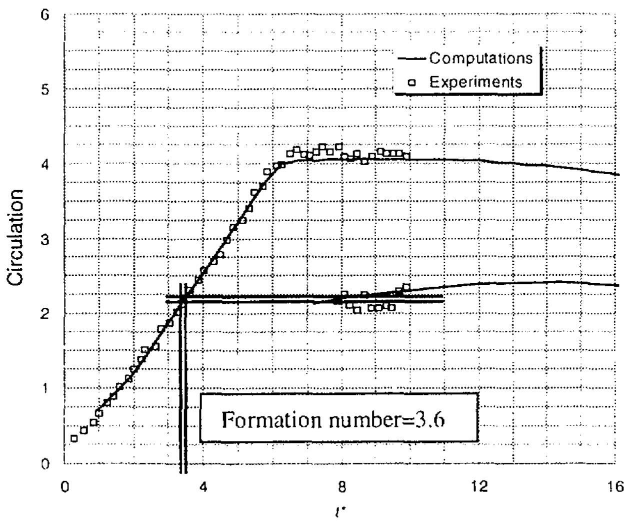

Figure 2: Total and vortex circulation and the determination of the formation number

Table I lists the maximal total circulation and the vortex circulation for T^*=6 and various velocity programs (impulse, trapezoidal or linear ramp), velocity profiles (uniform or parabolic), *Re* numbers and configuration geometry. In all the wide variety of cases considered in Table I, the vortex circulation (scaled by U_pD, where U_p is the maximal piston velocity and D is the oriffice diameter) is in the range of $2.61 > \Gamma_{vortex} > 1.85$. The maximal circulation for the same cases varies in a much wider range of $12.08 > \Gamma_{vortex} > 2.24$. Theoretical as well as experimental results of GRS also indicate that the variations in Γ_{vortex} are indeed relatively small. An analysis based on the variational principle of Kelvin-Benjamin (assuming a constant limiting value of the non-dimensional kinetic energy) predicts $1.90 > \Gamma_{vortex} > 1.42$ for cases with specified discharge velocity profile, while the experimental results of GRS are in the range of $2.7 > \Gamma_{vortex} > 2.2$.

These observations might hint at the possibility that the vortex circulation (scaled properly) is yet another universal quantity related to vortex rings. In other words, the vortex circulation is relatively insensitive to the formation conditions, once its asymptotic state (e.g. $T^* > 4$) has been reached. This conjecture should be further investigated in the future.

References

Gharib, M., Rambod, E. & Shariff, K. 1998 A universal time scale for vortex ring formation. to be published in *J. Fluid Mech.*

Velocity Profile	**Velocity Program**	$\Delta T/T$	*Re*	Γ_{max}	Γ_{vortex}	**Formation number**
Uniform	Impulse		2500	3.33	2.05	3.60
Nozzle	Impulse		2500	3.98	2.47	3.97
Orifice	Impulse		1250	4.43	2.61	3.81
Orifice	Impulse		2500	3.99	2.42	3.83
Orifice	Impulse		5000	3.74	2.30	3.80
Parabolic	Impulse		2500	12.08	1.85	0.90
Uniform	Trapezoidal	0.1	2500	3.16	2.06	3.54
Uniform	Trapezoidal	0.2	2500	2.99	2.06	3.78
Uniform	Trapezoidal	0.3	2500	2.78	2.05	3.95
Uniform	Trapezoidal	0.4	2500	2.51	1.90	3.97
Uniform	Linear		2500	2.24	1.85	5.22

Table I: Maximal circulation Γ_{max}, vortex circulation Γ_{vortex} and formation number dependence on the generation factors for T^*=6.

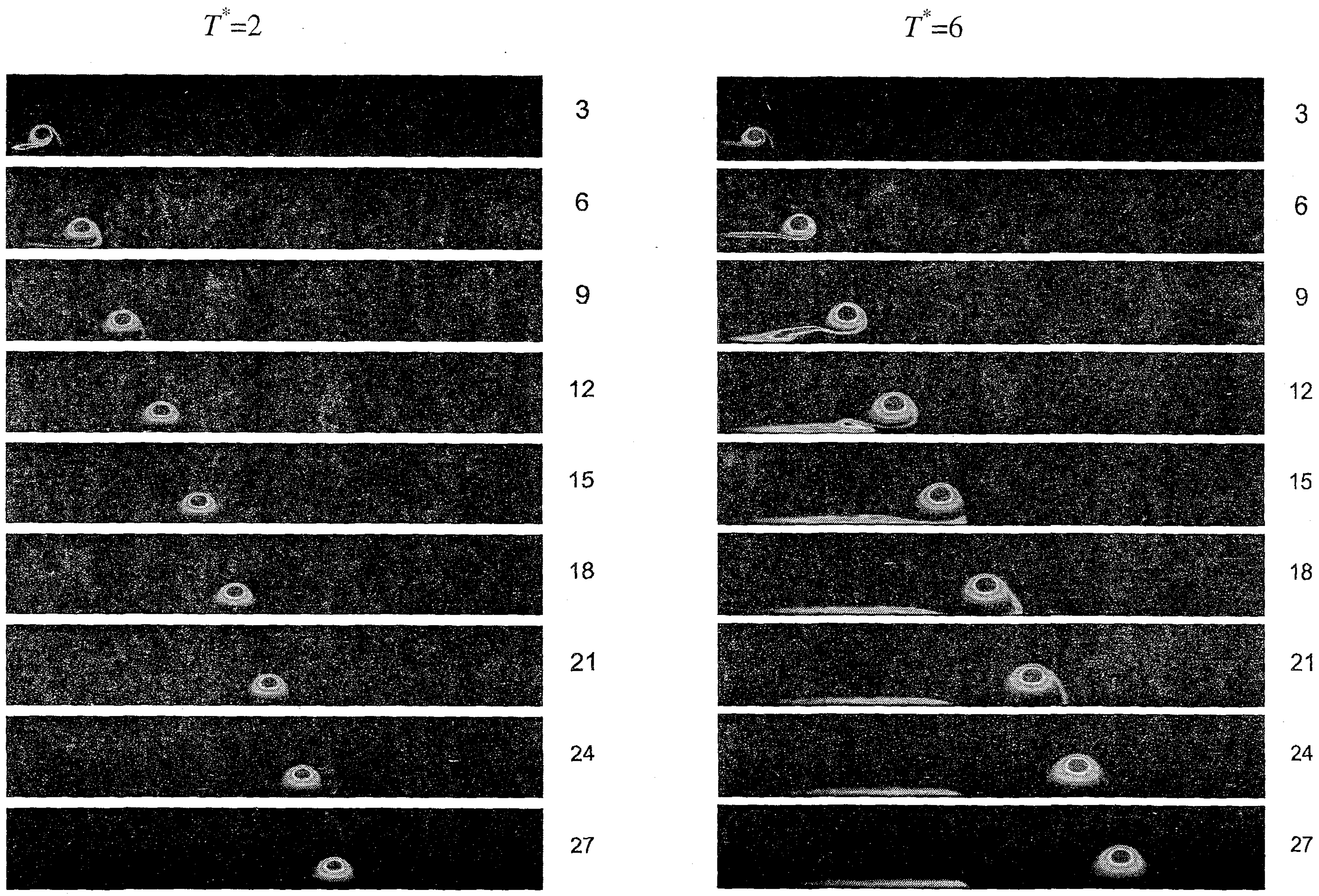

Figure 1: The formation and propagation of the vortex ring for short (T^*=2) and long ((T^*=2) non-dimensional discharge time.