A NUMERICAL SIMULATION METHOD FOR RADIATIVE TRANSPORT IN GENERAL PARTICIPATING MEDIA

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ABSTRACT

Many cases of radiation transport in nature and technology can be described as a continuum transport process through an interacting Participating Medium, governed by the Radiative Transfer Equation. Current numerical methods for solving the RTE, in particular Discrete-Ordinates based methods, are in a process of rapid development during recent years. However, they are still lacking in generality and sometimes suffer from performance limitations in three-dimensional problems involving strong coupling between ordinate directions. We present a new numerical solution procedure for the Discrete Ordinates approximation of the RTE, including treatment of anisotropic optical properties and generalized boundary conditions. The numerical schemes used here are well established in CFD, but have not been applied previously to the solution of the RTE. The new scheme is guaranteed to converge, subject to a numerical stability condition. We demonstrate the validity of the developed code on a series of verification cases.

INTRODUCTION

Radiative energy transport through a medium that interacts with the radiation is an important problem with many applications in science and technology. General treatment of radiation transport through Participating Media (PM) is usually modeled by the Radiative Transfer Equation (RTE). A popular approximation of the RTE in recent years is the Discrete Ordinate Method (DOM), analogous to finite-volume discretization in directional space [1], [2]. It has been developed and extended over the last decade to treat complex problems with irregular grids, anisotropic scattering, and various schemes for spatial discretization [3]-[7]. However, the common solution procedure using sequential sweeps in each ordinate direction may be ineffective in the presence of strong scattering, and may yield conflicts for complex geometry [8]. The procedure is not guaranteed to converge when complex geometry, scattering, opaque boundary conditions or anisotropic properties are present. Advanced procedures commonly used in CFD codes cannot be easily applied to this algorithm.

The *Discrete Ordinates with Time Stepping* (DOTS) method presented here uses the 'pseudo-time' iteration, which is widely used in CFD. This technique transforms the original boundary value problem to an initial value problem by adding derivatives of the dependent variables by a time-like parameter to left-hand-side of the equations [9]. The solution of the original boundary value problem is obtained in the limit of 'time' going to infinity. The time-iterative method is well developed and widely used for solving CFD problems. Its application to the radiation transport problem will therefore provide access to the wealth of theoretical knowledge and enhancement procedures that have accumulated through CFD experience during the last few decades.

MATHEMATICAL FORMULATION

The governing equation for radiation transfer in a general Participating Medium is the RTE:

$$\frac{\partial I(s,\omega)}{\partial s} = -\beta(s,\omega)I(s,\omega) + \frac{1}{4\pi}\int_{4\pi}\sigma(s,\omega')I(s,\omega')\Phi(s,\omega,\omega')d\omega' + \eta(s,\omega) \qquad (1)$$

On the boundaries, we distinguish between outgoing directions, where the radiation propagates outwards from the domain towards the boundary ($\underline{s}\cdot\underline{n}>0$); and incoming directions, where the radiation propagates from the boundary into the domain ($\underline{s}\cdot\underline{n}<0$). We present the boundary condition in a form analogous to the RTE, using the directional-hemispherical reflectivity $\rho(\omega)$ and a boundary phase function $\Phi'(\omega,\omega')$:

$$I'_{in}(\omega) = \frac{1}{\pi}\int_{s\cdot n>0}\rho(\omega')I'_{out}(\omega')\Phi'(\omega,\omega')I(\omega)d\omega' + q'(\omega) \qquad (2)$$

Note that the optical properties of both the medium and the boundary are in general anisotropic, as shown in equations (1) and (2). This statement of the RTE is adequate for a gray medium, or may be used as spectral representation of non-gray medium and boundaries.

The RTE (1) is discretized in the directional coordinates by dividing angular space into a finite number N of solid angle subdomains ω_m, referred to as ordinates, where $m=1,...N$. A function $f(\omega)$ within an ordinate m in the interior of the domain is represented by its average value f_m. We shall use the following definitions: $J_m=I_m\omega_m$, $Q_m=q_m\omega_m$, since these new quantities represent radiative energy fluxes in the ordinate direction, rather than intensities, and are therefore convenient in expressing radiative energy conservation. Applying this directional discretization to the RTE results in a coupled set of ordinary differential equations for all the fluxes J_m of ordinates ω_m, m=1,..,N, as expressed in the following matrix equation:

$$\frac{\partial}{\partial s}\mathbf{J} + \mathbf{B}\,\mathbf{J} - \mathbf{Q} = 0 \qquad (3)$$

where **J** is a vector with elements J_m, **Q** has elements Q_m, and the elements of the matrix **B** are:

$$\mathbf{B}_{mn} = \beta_m\delta_{mn} - \frac{\omega_m}{4\pi}\Phi_{mn}\sigma \qquad (4)$$

A similar discretization is applied to the boundary condition (2). We continue the discretization process by applying the finite volume method (FVM) to the DOM system of equations (3). The spatial domain is divided to computational cells. The cell is denoted by P, its six neighbors are denoted by W (west), E (east), S (south), N (north), L (low) and H (high). Appropriate approximations of convective terms are used [9], [10], with a central difference scheme, and addition of 'artificial dissipation'. We add a pseudo-time term $\partial J_m/\partial t$ for the iterative process, where t is an artificial time-like variable, and an artificial viscosity term $D(J_m)$. We chose a scheme where the convective and artificial dissipation terms (east contribution, say) are given by:

$$C_{m_E} = \frac{1}{2V_P}\hat{\mathbf{s}}_m \cdot \hat{\mathbf{n}}_e A_e$$
$$D_{m_E} = \frac{1}{2V_P} A_e \max\{ |\hat{\mathbf{s}}_m \cdot \hat{\mathbf{n}}_e|, C_{lin} \} \tag{5}$$

where C_{lin} is some small non-negative number. The scheme then becomes the first order upwind scheme. The discretized DOM equations are then:

$$\frac{\partial}{\partial t}J_m + \frac{\partial}{\partial s}J_m + \sum_n B_{mn}J_n - Q_m - D(J_m) = 0 \tag{6}$$

The pseudo-time explicit discretization is now introduced. We define the local time step for each ordinate and each cell as $\Delta t/W_{mP}$, where W_{mP} is a positive local weight function. Application of the simple Euler forward-difference scheme in time to (6) yields:

$$W_{m_P}\frac{J_{m_P}^{(+)} - J_{m_P}}{\Delta t} = -\left(C_{m_W}J_{m_W} + C_{m_E}J_{m_E} + \ldots + C_{m_P}J_{m_P}\right) - \sum_n B_{mn_P}J_{n_P} + Q_{m_P} + \left(D_{m_W}J_{m_W} + D_{m_E}J_{m_E} + \ldots - D_{m_P}J_{m_P}\right) \tag{7}$$

The RHS is the residual at cell P, ordinate m. The first term on the RHS is the convection, the next is due to absorption and scattering, the third is the source due to emission, and the last is the artificial viscosity. As the residual diminishes, the pseudo-transient term tends to zero, and convergence to the stationary solution is reached.

Based on stability analysis, the pseudo-time step and the weight function W_{mP} are constrained according to:

$$\Delta t < 1$$
$$W_{m_P} = \beta_{m_P} + C_{m_P} + D_{m_P} \tag{8}$$

Typically $\Delta t=1$ was used, without any numerical stability problems.

EXAMPLES

Two-Dimensional Annular Sector

Moder et al. [11] solved the problem of a 2D annular sector by a DOM implementation specifically tailored for cylindrical geometry, and compared to exact solutions, either analytical or by accurate ray-tracing, which makes it suitable for code validation. A 2D black annular sector of $0° \le \vartheta \le 60°$ span and inner and outer radii R_i and R_o encloses a cold (T=0) medium. One of the walls is hot ($T=T_h$) and the others are at T=0. Periodic boundary conditions prevail in the axial direction. The nondimensional radial heat fluxes into the walls vs. the angle ϑ are compared to the reference solutions (exact and numerical) in Figure 1. It is interesting to note that the present results are almost identical to the reference numerical results, including even the 'ray effect' errors identified and explained in this reference.

Three-Dimensional Annular Sector

This problem is similar to the former one, except that it has a finite axial extent and only the hot inner wall case is considered. It validates a 3D case with available exact solutions [11]. The problem is solved using several variations of the optical properties and dimensions and compared to the available results (Figure 2). Due to memory limitation our resolution for was lower than that used in [11]. The agreement is usually good in spite of the present low resolution, except for a stronger ray effect which is most pronounced in the transparent medium cases.

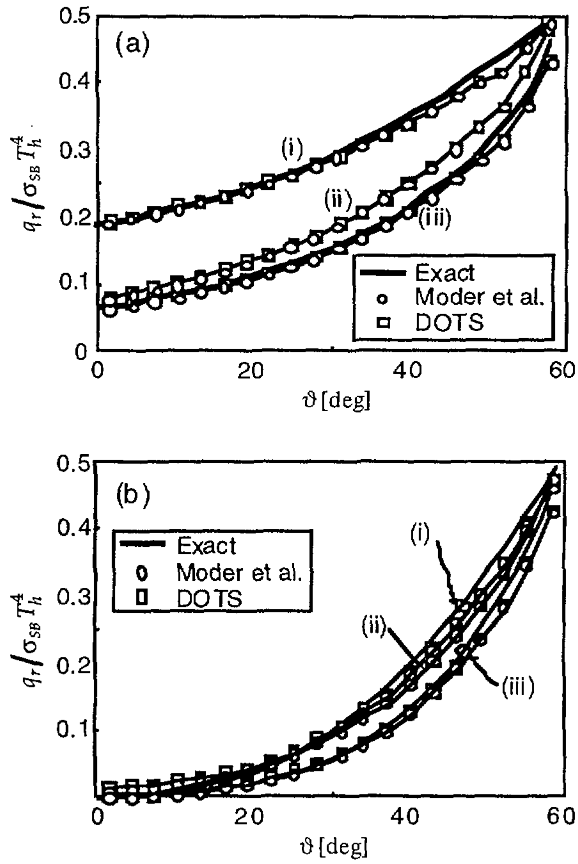

Figure 1. Two-dimensional annular sector (a) Outer wall radial heat transfer with a heated right wall, R_i=0.5 m, and R_o=1 m; (i) α=0, σ=0; (ii) α=0.5 m^{-1}, σ=0.5 m^{-1}; (iii) α=1 m^{-1}, σ=0. (b) Inner wall radial heat transfer with a heated right wall, R_i = 0.5 m R_o = 1 m; (i) a=0, σ=0; (ii) α=0.5 m^{-1}, σ=0.5 m^{-1}; (iii) α=1 m^{-1}, σ=0.

DISCUSSION

A general numerical method was developed for the solution of the radiative transfer equation applied to a general anisotropic participating medium. The DOTS implementation was verified by comparison to available exact and numerical solutions. Some features of the present code were not exercised and verified by these cases, due to limitations of the cases available in the literature.

Acceleration techniques such as multigrid, residual smoothing, and Runge-Kutta integration [9] can be applied to the DOTS iteration, in analogy to their CFD application. We expect that a combination of these well-proven acceleration methods will significantly improve convergence rates, and hence enable access to high-accuracy and high-complexity radiation problems that are now too computationally expensive. Reduction in turn-around time can be achieved by parallelization. The time-iterative method is purely explicit and therefore is easy to parallelize. When the radiation problem is very complex, or when it is coupled to other modes of heat transfer, parallelization becomes a serious issue since the overall problem can be very compute-intensive.

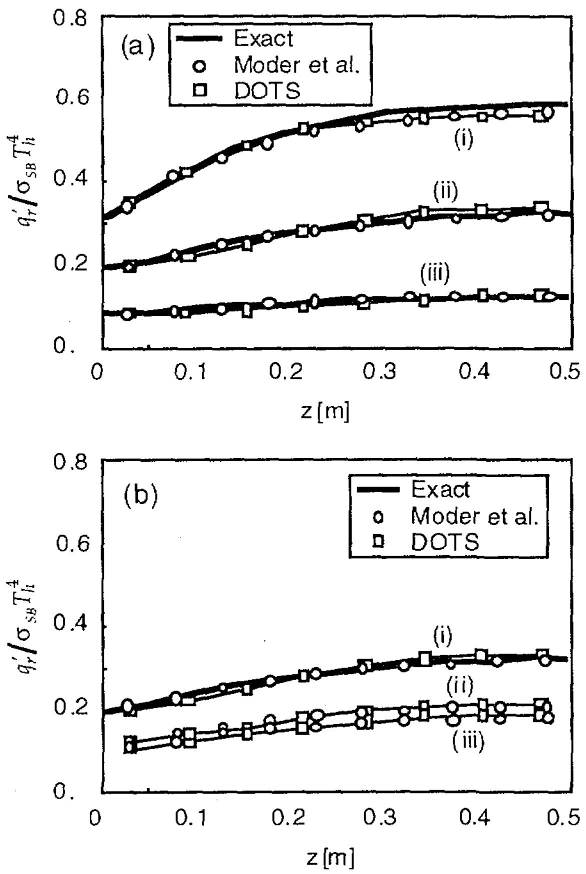

Figure 2: Three-dimensional annular sector with a heated inner wall. (a) Outer wall radial heat transfer along ϑ=30°, transparent medium: (i) R_i=0.5 m, R_o=0.75 m; (ii) R_i=0.5 m, R_o=1 m; (iii) R_i=0.5 m, R_o=1.5 m. (b) Outer wall radial heat transfer along ϑ=30°, participating medium. (i) α=0, σ=0; (ii) α=0.5 m^{-1}, σ=0.5 m^{-1}; (iii) α=1 m^{-1}, σ=0. (c) Outer wall radial heat transfer along z=0.5m, transparent medium: (i) R_i = 0.5 m, R_o = 0.75 m; (ii) R_i = 0.5 m, R_o = 1 m; (iii) R_i = 0.5 m, R_o = 1.5 m. (d) Outer wall radial heat transfer along z=0.5m, participating medium: (i) α=0, σ=0; (ii) α=0.5 m^{-1}, σ=0.5 m^{-1}; (iii) α=1 m^{-1}, σ=0.

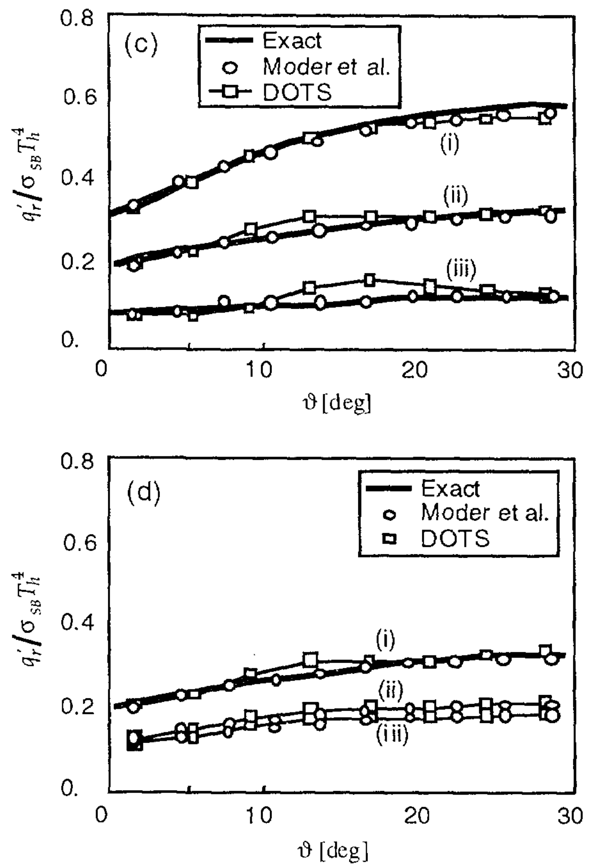

Figure 2 (Continued)

Usually the radiation transport problem is coupled with other modes of heat transfer, conduction and convection, and through convection to the flow problem. Standard simulation packages, implementing the methods and features described above for flow, convection and conduction problems, are available today. It is therefore quite natural to formulate the radiation problem also using the methodology that is common to the other parts of the problem. We are currently developing such an interface to link the DOTS code to the commercial CFD code PHOENICS.

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