FLOW RESISTANCE IN ROD ASSEMBLIES

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Abstract

The general form of relation between the resistance force and the velocity vector, resistance tensor structure and possible types of anisotropy in the flow thorough such structures as rod or tube assemblies are under discussion. Some questions of experimental determination of volumetric resistance force tensor are also under consideration.

There is a resistance force that acts from the solid component of the porous body on the fluid flow in it. The specific value of this force, i.e. divided to the unit volume of liquid phase, can be performed as:

$$\vec{f} = -\rho K\vec{u}. \tag{1}$$

The porous body model approximation in application to the fluid flow in such structures as pin or tube assemblies was developed in works [1, 2] in IPPE. In order to determine resistance force components in the flow thorough the tube assembly with the arbitrary angle between flow and assembly axis following correlations were recommended:

$$\vec{f}_z = -\rho\frac{\lambda_{fz}}{2d_h}uu_z, \quad \vec{f}_r = -\rho\frac{\lambda_{fr}}{2d_h}uu_r \tag{2}$$

Comparison of (1) and (2) shows, that the friction factor *K* has been a tensor with the components along the main axes:

$$k_{\eta\eta} = \frac{\lambda_{fz}}{2d_h}u, \quad k_{\xi\xi} = \frac{\lambda_{fr}}{2d_h}u. \tag{3}$$

The correlations (2) practically have no direct experimental confirmation and as it has been shown in [3] where are reasons to doubt in their adequacy, especially in determination of the resistance force component normal to the velocity vector, i.e. the buoyancy force.

The analysis fulfilled in [3] by means of the matrix polynomial theory, showed that the resistance force can be presented as vector correlation:

$$\vec{f} = -\rho\, a(u, \mathrm{Cos}^2\varphi)\vec{u} - \rho\, b(u, \mathrm{Cos}^2\varphi)(\vec{u}\vec{n})\vec{n} \tag{4}$$

or in the equivalent tensor form:

$$f_i = -\rho K_{ij}u_j, \tag{5}$$

where K_{ij} – the resistance tensor with the components:

$$\{k_{ij}\} = \begin{pmatrix} a + bn_xn_x & bn_xn_y & bn_xn_z \\ bn_xn_y & a + bn_yn_y & bn_yn_z \\ bn_xn_z & bn_yn_z & a + bn_zn_z \end{pmatrix} \tag{6}$$

and where: $\vec{u}$, u – velocity vector and its modulus,
$\vec{n}$ – unit vector along the assembly axis,
n_x, n_y, n_z – its projections on the co-ordinate axes,
φ – the angle between the velocity vector and the assembly axis.
Matching the z- axis with the direction along the pins ($n_x = n_y = 0$, $n_z = 1$) we get:

$$\{k_{ij}\} = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a+b \end{pmatrix}, \quad (7)$$

i.e. the main resistance tensor components are equal to

$$k_{xx} = a(u, \cos^2\varphi);\ k_{hh} = a(u, \cos^2\varphi) + b(u, \cos^2\varphi), \quad (8)$$

and they are even cosines functions of the angle between the velocity vector and the assembly axis.

Taking into account the relation (8) the resistance tensor can be written in the following form:

$$k_{ij} = k_{\xi\xi}\delta_{ij} + (k_{\eta\eta} - k_{\xi\xi})n_i n_j, \quad (9)$$

where δ_{ij} is the unit vector.

Analysing the correlations (4) and (5) some possible types of resistance anisotropy can be marked out:

1 type *:* $b = 0$, $a = a(u, \cos^2\varphi)$.

The resistance depends on the direction, but for any velocity vector the resistance force is opposite to it (4). The resistance tensor becomes the scalar value. The $a(u, \cos^2\varphi)$ correlation is defined from the experimental measurement results for flow resistance over inclined assembly

$$a(u, \cos^2\varphi) = \frac{\lambda_f(\varphi)}{2d_h} u. \quad (10)$$

2 type *:* $a = a(u)$, $b = b(u)$.

Factors a and b in (4) do not depends on the angle between the velocity vector and assembly axis and are determined by the friction factor for longitudinal and transversal flow over the tube assembly. This case agrees with the hypothesis (2). According to the correlation (4) the buoyancy force is equal to

$$f_b = \rho b u \cos\varphi \sin\varphi = \rho(k_{\eta\eta} - k_{\xi\xi})u \cos\varphi \sin\varphi.$$

3 type *:* The general case when $b \neq 0$ and one or both factors depends on the angle.

Experiment has to show what type of anisotropy takes place in fact.

The rough experimental scheme is shown on Fig. 1. The porous structure (tube or pin assembly) with the length l is inserted into the flat channel with the height h. The angle between the channel and assembly axes is a. The flow in the channel is longitudinal.

The steady-state flow velocity distribution in the porous media is described by the equations:

$$\frac{\partial}{\partial y} u_y u_z + \frac{\partial}{\partial z} u_z^2 = -\frac{1}{\rho}\frac{\partial P}{\partial z} + v_{y\delta}\left(\frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2}\right) - k_{zy}u_y - k_{zz}u_z, \quad (11)$$

$$\frac{\partial}{\partial y} u_y^2 + \frac{\partial}{\partial z} u_y u_z = -\frac{1}{\rho}\frac{\partial P}{\partial y} + v_{y\delta}\left(\frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2}\right) - k_{yy}u_y - k_{yz}u_z, \quad (12)$$

$$\frac{\partial u_z}{\partial z} + \frac{\partial u_y}{\partial y} = 0, \quad (13)$$

$$\left.\frac{\partial u_z}{\partial y}\right|_{y=0} = 0, \quad \left.\frac{\partial u_z}{\partial y}\right|_{y=h} = 0. \quad (14)$$

According to (9) the tensor components are equal to: ($n_x = 0$, $n_y = \sin\alpha$, $n_z = \cos\alpha$):

$$k_{yy} = k_{\eta\eta}\sin^2\alpha + k_{\xi\xi}\cos^2\alpha,$$

$$k_{zz} = k_{\eta\eta}\cos^2\alpha + k_{\xi\xi}\sin^2\alpha, \quad (15)$$

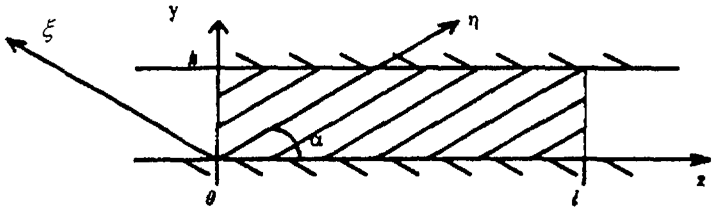

Fig. 1.

$k_{yz} = k_{zy} = (k_{\eta\eta} - k_{\xi\xi})\sin\alpha \cos\alpha$.

The effective viscosity can be approximated as:

$$\nu_{ef} = C_V\, u\, d_h , \qquad (16)$$

where C_V is constant value.

Far away from the entrance of the porous body the velocity distribution becomes equable over the cross section:

$$u_z = u_0 = \text{const},\ u_y = 0. \qquad (17)$$

According to (11) and (12) the pressure drop distribution is:

$$-\frac{1}{\rho}\frac{\partial P}{\partial z} = k_{zz} u_0 ,\quad -\frac{1}{\rho}\frac{\partial P}{\partial y} = k_{yz} u_0 , \qquad (18)$$

i.e. for such flow, when the angle between the velocity vector and assembly axis exists, not only longitudinal but also transversal pressure gradient occurs. The k_{zz} and k_{yz} factors can be calculated by means of the measured pressure gradients and in accordance with the correlation (15) the main tensor components for the angle set can be defined:

$$k_{\eta\eta} = k_{zz} + k_{zy}\,\mathrm{tg}\alpha ,\quad k_{\xi\xi} = k_{zz} - k_{zy}\,\mathrm{ctg}\alpha . \qquad (19)$$

The experimental measurement of the transversal pressure gradient as well as the longitudinal one is essential and important because it is impossible to define two unknown values ($k_{\eta\eta}$, $k_{\xi\xi}$) only by measuring one parameter - the resistance of the longitudinal flow.

Fig. 2 shows some results of numerical calculations of the problem (11 – 14) from the article [3]. The calculations result show that in a case of liquid flow in the channel with the anisotropic porous insert the non-uniformity of the longitudinal velocity through the channel height occurs while the pressure is approximately constant. Alignment of the non-uniformity of the velocity distribution (Fig. 2a) and increasement of the transversal pressure drop (Fig. 2b) take place on the "inlet" section of the porous insert. Near the porous insert exit the "outlet" section exists, where the inverted changes take place. The transversal pressure drop decreases and the uniform velocity distribution becomes non-uniform. The velocity distribution non-uniformity in the "outlet" section id opposite to those in the "inlet" section. In the central part of the insert the velocity and pressure drop distributions conform to the solution (17), (18).

The numerical calculation results and approximate analytical solution [4] show that the "inlet" and "outlet" sections of the flow lightly influence to the longitudinal pressure drop, and the longitudinal pressure gradient can be defined be measuring total pressure drop for the rod assembly. The transversal pressure drop ought to be measured outside of the "inlet" or "outlet" sections of the porous insert, as it springs out the Fig. 2b.

In order to estimate the "inlet" and "outlet" section length dependence on the angle and the regime parameters the approximate analytical solution of the problem (11 – 14) has been done. It has been based on the one-parameter lineal approximation for the longitudinal velocity and has used the integral form for the momentum equations (11, 12).

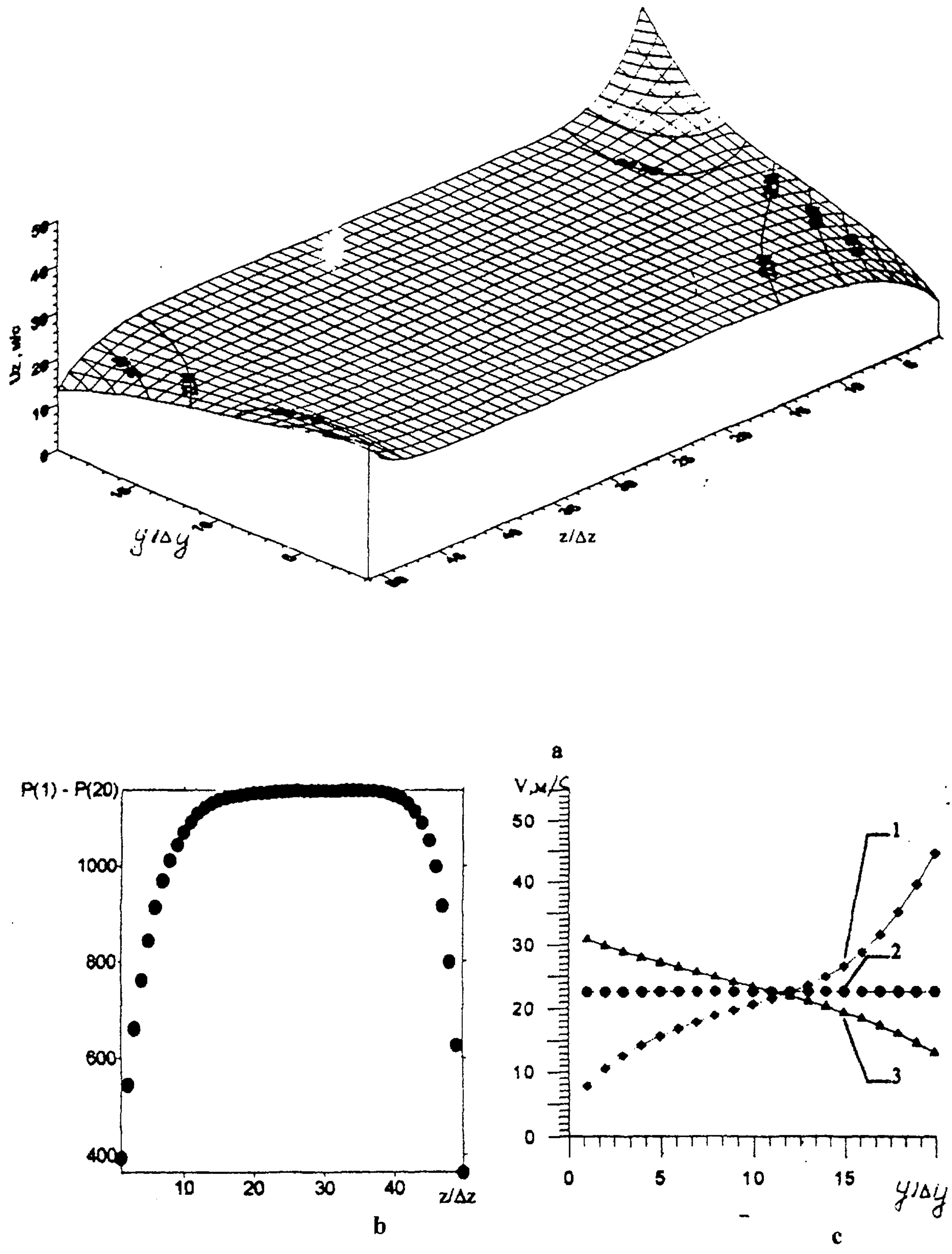

Fig. 2. The flow through the porous structure (α=*10°*, *h=3 m*, $k_{\eta\eta}$=*3.8*, $k_{\xi\xi}$ =*62*, u_0=*22.6 m/c/c*): a) longitudinal velocity component distribution in the channel; b) transversal pressure drop distribution along the channel: c) longitudinal velocity distribution through the height of the channel in the entrance (1), the exit (3) and in the centre (2) of the poorhouse insert.

According to the solution above the velocity irregularity decreases in accordance with the exponential law with the exponent equal to l_1 and l_2, accordingly (length of "inlet" and "outlet" sections are equal to 2-3 l_i,):

$$l_1 / h = \left(1 + \sqrt{1 + \tilde{k}_{zz}\left(8\tilde{v} + \tilde{k}_{yy} / 3\right)}\right) \Big/ 2\tilde{k}_{zz}\,, \tag{20}$$

$$l_2 / h = \left(4\tilde{v} + \tilde{k}_{yy} / 6\right)\left(1 + \sqrt{1 + \tilde{k}_{zz}\left(8\tilde{v} + \tilde{k}_{yy} / 3\right)}\right)^{-1} \tag{21}$$

where $\tilde{k}_{ij} = k_{ij} h / u_0$, $\tilde{v} = v_{ef} / u_0 h$.

Fig. 3 shows the l_1 and l_2 dependence on the angle. The approximation (3) has been used for the main tensor components calculations.

The general form of the resistance force dependence on the velocity vector is defined. The main tensor resistance components are established to be the functions of the angle between the assembly axis and velocity vector. Different types of the flow resistance are possible in dependence if the different types of the functions, including the case when the resistance force vector is strictly opposite to the velocity vector for any angle , i.e. the buoyancy is absent.

The measurements of the assembly resistance for various angles of the flow only are not adequate for experimental determination of the resistance tensor. It is necessary also to measure the transversal pressure gradient in the flow. This measurement ought to be fulfilled on the stabilised part of the flow, outside of the "inlet" or "outlet" sections of the porous insert.

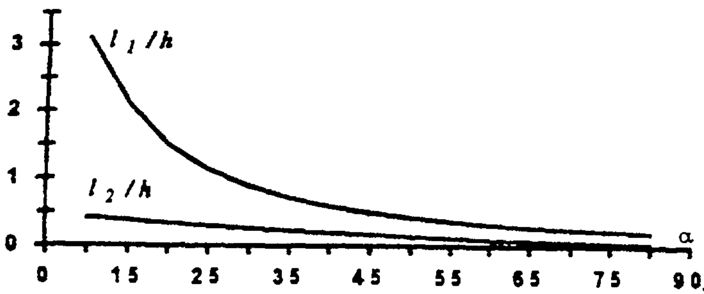

Fig. 3. Dependence of l_1/h and l_2/h on the angle of the flow *(d = 5 mm ,s/t = 1.25 , h = 60 mm, Re = 3e+4)*.

NOMENCLATURE

a, b – factors for $\vec{f}$ in (4);
d_h – hydraulic diameter, mm;
K, k_{ij} – resistance tensor and its components;
$\vec{n}$, n_i – unit vector and its components ($n_i = \cos(\vec{n}\vec{x}_i)$);
$\vec{f}$ – volumetric resistance force;
u, $\vec{u}$ – mean fluid velocity modulus, vector;
x, y, z, r, x_i – co-ordinates;
α – angle between assembly and channel axis (Fig. 1);
φ – angle between assembly axis and velocity vector;
λ_{fz}, λ_{fr} – friction factors for longitudinal and transversal flow through the assembly;
ρ – liquid density, kg/m;
ξ, η – main anisotropy axes directions.

REFERENCES

[1] SUBBOTIN V.I. et al., Reactor thermophysics problem solutions by means of the computers. M.: Atomizdat, 1979.

[2] MITENKOV P.M., GOLOVKO V.F., USHAKOV P.A., JURJEV JU.S. Design of Heat exchanging apparatus for APP. M.: Energoatomizdat, 1998.

[3] KORSUN A.S., VIKULOVA S.V. To the definition of the resistance of the anisotropic porous body. "Proc. of the Second Russian National Conference on Heattransfer", M.: 26-30 Oct. 1998, (in print.)

[4] KORSUN A.S., KOROTKOVA I.S. Flow velocity distribution in the channel with anisotropic porous insert. "Proc. of the Second Russian National Conference on Heattransfer", M.: 26-30 Oct. 1998, (in print.)