

# ON DEVELOPMENT OF ANALYTICAL CLOSURE RELATIONSHIPS FOR LOCAL WALL FRICTION, HEAT AND MASS TRANSFER COEFFICIENTS FOR SUB-CHANNEL CODES

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XA0055064

## Abstract

The purpose has been to describe an approach suggested for constructing generalized closure relationships for local and subchannel wall friction, heat and mass transfer coefficients, with not only axial and transversal parameters taken into account, but azimuthal substance transfer effects as well. These constitutive relations that are primary for description of one- and two-phase one-dimensional flow models can be derived from the initial 3-D drift flux formulation. The approach is based on the Reynolds flux, boundary layer and generalized coefficient of substance transfer. One more task has been to illustrate the validity of the "conformity principle" for the limiting cases. The method proposed is based on the similarity theory, boundary layer model, and a phenomenological description of the regularities of the substance transfer (momentum, heat, and mass), as well as on an adequate simulation of the forms of flow structure by a generalized approach to build (an integrated in form and semi-empirical in maintenance structure) analytical relationships for wall friction, heat and mass transfer coefficients.

## 1. INTRODUCTION

A detailed understanding of the momentum, heat and mass transfer intensities is of great importance for the analysis of complex processes in the transient and abnormal nuclear reactor clustered rod bundles behavior. Also, it is an urgent problem for validation and verification of the closure relationships for such lumped parameters "best estimate" codes as RELAP, TRAC, COBRA, and so on. Averaged by area, the field equations obtained become one-dimensional. Therefore, the information on changes of variables in the direction normal to the main (axial) flow is actually lost. Thus, the transfer of momentum, energy and mass between the wall and fluid should be specified by closure relationships in forms of theoretical, semi-empirical, or empirical correlations.

In the nominal and steady state conditions, the package of friction, heat and mass transfer coefficients have been known well enough. However, as a rule, for complicated conditions (when there are several physical effects in an interaction), the knowledge of closure relationships leaves much to be desired. For example, in the low mass flux tube flow, the friction factor (by homogeneous model) the calculated results appear to be 500-1000% as small as experimental data (Nakorajkov et al, 1980) with saddle-shape void fraction profiles. Frequently rather than not, there appear limitations of experiments and/or lack of measured parameters necessary, that is why it is not possible to estimate individual effects of different phenomena. In particular, in LOCA experiments there were no simultaneous measurements of shear stresses and local void fraction. That is why the anomalous increasing effects of friction and heat transfer (Kornienko, 1997a) were not taken account in the "best estimate" codes. On the other hand, for clarifying such effects and developing the mathematical models for their closure relationships, it is necessary to create simple and appropriate method for description of such transversal profile effects.

It should be noted that a similar situation takes place in the evaluation of the gravity term of the momentum equation at mixed convection (Iannello et al, 1988). In order to properly evaluate this term, the spatially-averaged fluid density must be known. Typically, however, the bulk-averaged (mixing-cup) fluid properties are measured in experiments and

calculated by lumped parameter analysis codes. However, this is the case not only with the density term but also with ones inertial and spatial mass flux gradient ones. Moreover, this is also the case for energy and mass conservation equations. For low mass flux the bulk-averaged and spatially-averaged densities may significantly differ, and thus there is an inherent error introduced by using the bulk density to compute the gravity pressure gradient and other components. Alternatively (Iannello et al, 1988), this error can be eliminated by including a separate correction term in the momentum equation to account for this difference. Instead of correcting both the friction and gravity pressure gradient terms for mixed convection effects, the gravity correction term is included in the friction term. In doing so, a modified friction factor is defined which includes the gravity pressure gradient correction term. In a similar way consideration must be given to the Nusselt (Stanton) numbers for heat and mass transfer processes. The first attempts of such description were described in papers by Kornienko 1995, 1997b for round tube and flat channel, and for subchannel geometry, accordingly.

Moreover, it is important to take into account thermo-physical properties, the contribution of inner sources (sinks) of the substance, flows with injection (suction), two-phase flows and turbulent longitudinal flow through pin bundles. As a rule, the previous analytical descriptions and models (Petukhov, 1987; Novikov and Voskresensky, 1977; Kutateladze and Leont'ev, 1985; Lyon, 1951) were developed for solving certain specific problems and so they are limited by certain sets of assumptions which make possible applications more limited.

Of significant scientific and practical value is the extraction of the analytical and approximate solutions of these problems (or even revealing the conditions for existence of such solutions) that can be implemented only by a generalized formulation of the coefficients sought. Here, the use of the basic three dimensional drift flux model (Ishii, 1975) differential conservation law equations and boundary conditions provide the required completeness of the physical effects taking into account the analytical relationships sought for the above mentioned coefficients.

On the whole, the method proposed is a generalization of the Lyon's integral derivation not only for the heat transfer but for friction and mass-transfer coefficients as well. Besides, the approach expands of the above mentioned methods analytical descriptions with the account of the influence of additional effects, such as substance radial transfer mechanisms (injection and suction flows in their number), contributions of inner sources (or sinks) of the substance and two-phase flows (Kornienko, 1995, 1996, 1997b), effects of transient and azimuthal substance transfer.

The wall friction, heat and mass transfer coefficients local formulation is correlated with averaging of the variables within the boundaries of a small area normal to the channel wall. The local formulation is tightly bound to the subchannel geometry formulation. This work was performed to obtain standard and modified analytical description of the friction factor and Nusselt (Stanton) numbers with take into account the above mentioned separate effects. The general integral descriptions are obtained in non-dimensional forms in providing these results.

## 2. UNIFIED REPRESENTATION OF PARAMETER PROFILES IN CHANNEL AND SUBCHANNEL CROSS SECTION

Thermal hydraulic design studies of clustered rod bundles with axial flows are carried out using lumped parameter computer codes with segmenting the fuel assembly geometry into a large number of coolant subchannels, see Fig.1. The use of one-dimensional systems of the conservation law equations in such codes requires creating closure relationships for wall friction, heat and mass-transfer coefficients in a wide range of physical phenomena and

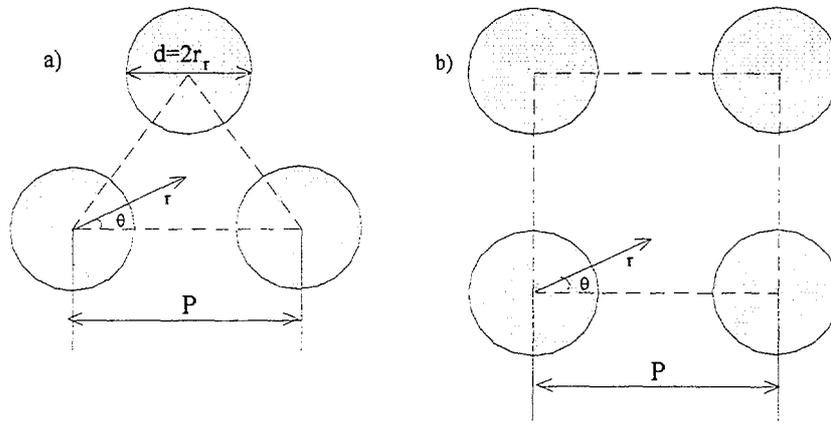


Fig1. Cross-section of an equilateral a) and square b) rod array. P - pitch,  $r_r$  - rod radius.

operation conditions for one- and two-phase coolant flows. These closure relationships appearing in the mathematical models depend on both molecular transport properties and the averaging procedures. In general, these terms will depend on the flow geometry, geometric arrangement of the phases, conventionally termed the flow regime, and thus criteria for flow regimes determination (based on predicted quantities) will also be needed.

Experimental data (Ibragimov et al, 1978; Bobkov et al, 1984 and others) shows that velocity and temperature distributions along the normal to wall (even for a complicated subchannel such as cell with dense packaging pin bundles all over) is described with good precision by relationships for round tube. Thus, the hypothesis is valid about universal velocity and temperature distributions in the complicated form channels in just the same way as the round tubes. Practically, one can define thermally and hydraulically interconnected channels (subchannels) connected to a particular fuel element and considering a cluster with an infinite number of fuel rods, see Fig.1.

According to the phenomenological theory of hydrodynamics, heat and mass transfer (Petukhov, 1987; Novikov and Voskresensky, 1977; Reynolds, 1974) used for describing substance, heat and momentum fluxes well-known Fick's, Fourier's and Newton's transfer gradient hypotheses are applied.

For longitudinal flows through unbaffled assemblies of fuel rods with constant cross-sections, the main flow characteristics vary both in axial, radial and azimuthal directions. In particular, flows in the frames of the boundary layer (see Fig.2.) various models may be considered for this type of problem. Then, for fully stabilized flows in channels with a constant cross-section, changes of local value of the S-variable (by which we mean axial velocity, enthalpy or concentration) prove to be identical at every normal to wall of the subchannel and at least linear along the channel, Reynolds (1974). The main assumption is the validity of gradient formulation of substance transfer along the normal to the wetted perimeter.

To unify the designations used and to reduce transpositions in deriving relationships for parameter profiles, as well as friction, heat and mass transfer coefficients, it is helpful to use the concepts of generalized substance transfer coefficients (see Reynolds, 1974) and the gradient transfer model. The mathematical similarity descriptions of the three transfer processes mentioned above (see the top line in TABLE I) make it possible to introduce a formally generalized equation, where the substance flux J is expressed by the transfer characteristic  $\gamma_T$  and the gradient of the transfer potential S normal to wall as:

$$J = \rho \gamma_T \frac{\partial S}{\partial y}, \quad (1)$$

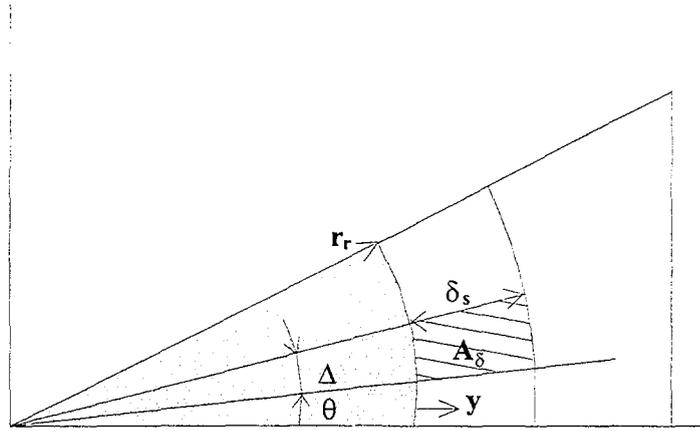


Fig2. Momentum, thermal (or concentrations) boundary layers near the rod of the bundle, not to scale.

$r_r$  - rod radius,  $\theta$  - azimuthal angle,  $\Delta$  - increment of the azimuthal angle,  
 $y$  - coordinate from rod surface and normal to it,  
 $\delta_s$  - substance (momentum, thermal (or concentration)) boundary layer thickness,  
 $A_\delta$  - annular cross sector area.

where  $\gamma_T = \gamma + \varepsilon_t$  - total (molecular ( $\gamma$ ) + molar ( $\varepsilon_t$ )) substance transfer characteristic (coefficient), namely, viscosity, thermal conductivity or diffusion coefficient (see the 1-st line in TABLE I). It should be emphasized that in accordance with Boussinesque's idea of expressing the turbulent friction formula in the form of Newton's laminar law, the turbulent heat and mass fluxes are recasted to the form which generalizes Fourier and Fick laws.

After scaling variables in equation (1) with respect to their wall values and after integrating along radius  $Y$  we can obtain the profile of variation for any of the potentials under consideration in the channel cross-section if the substance flux and (molecular + molar) transfer characteristics are known. This is expressed by the formula:

$$S_w^+ - S^+ = Pe_{s*} \int_0^Y \frac{\bar{J}}{\bar{\rho} \bar{\gamma}_T} dY \quad (2)$$

A detailed description of the substance fluxes and the key to decode the designations are quite obvious from the first six lines of TABLE I.

Using definitions in TABLE I, one can easily reconstruct specific relationships for the profiles of axial velocity, enthalpy (temperature) and concentrations from integral (2), see the 2-nd line.

They prove to be a generalization of the relationships obtained by Petukhov (1987) for velocity and temperature profiles that are not limited by assuming that viscous stress distribution is linear.

### 3. GENERALIZED SUBSTANCE TRANSFER COEFFICIENT

In accordance with the definitions of the wall friction factor and heat- and mass transfer coefficients (Petukhov, 1987; Reynolds, 1974) each of them is a function of the averaging pattern in the cross-section flow profile parameters (velocity, temperature, and concentration, see the 5-th line in TABLE I). For example, local heat-transfer coefficient is

$$\alpha = \frac{dQ_w}{(T_w - T_f) dF} = \frac{q_w}{T_w - T_f}, \quad (3)$$

TABLE I. FORMULATIONS OF LOCAL SUBSTANCE TRANSFER FLUXES, NUMBERS AND COEFFICIENTS FOR SUBCHANNEL BOUNDARY LAYER MODEL

	Substance flux	Momentum flux	Heat flux	Mass flux
	$J = \rho \gamma_T \frac{\partial S}{\partial y}$	$\tau = \rho \nu_T \frac{\partial w}{\partial y}$	$q = -\rho k_T \frac{\partial h}{\partial y}$	$N = -\rho D_T \frac{\partial c}{\partial y}$
1	Transfer coefficient $\gamma_T = \gamma + \epsilon_T$ ; ( $\tilde{\gamma}_T = \gamma_T / \gamma_w$ )	$\tilde{\nu}_T = \tilde{\nu} + \tilde{\nu}_T$	$\tilde{k}_T = k + \tilde{\nu}_T Pr / Pr_T$	$\tilde{D}_T = \tilde{D} + \tilde{\nu}_T Sc / Sc_T$
2	Variable $S \rightarrow$ (w, h, (T), c)			
	$S_w^+ - S^+ = \frac{S}{S_*} = Pe_{S_*} \int_0^{\tilde{Y}} \frac{\tilde{J}}{\tilde{\rho} \tilde{\gamma}_T} dY'$	$w^+ = Re_* \int_0^{\tilde{Y}} \frac{\tilde{\tau}}{\tilde{\rho} \tilde{\nu}_T} dY'$ , for $w_w = 0$	$h_w^+ - h^+ = Pe_* \int_0^{\tilde{Y}} \frac{\tilde{q}}{\tilde{\rho} \tilde{k}_T} dY'$	$c_w^+ - c^+ = Pe_{D_*} \int_0^{\tilde{Y}} \frac{\tilde{N}}{\tilde{\rho} \tilde{D}_T} dY'$
3	Substance of friction $S_{*0} = J_{*0} / (\rho_{*0} w_{*0})$	$w_{*0}^2 = \tau_{*0} / \rho_{*0}$	$h_{*0} = q_{*0} / (\rho_{*0} w_{*0})$	$c_{*0} = N_{*0} / (\rho_{*0} w_{*0})$
4	Transfer number $Pe_{s0} = \langle \rho w \rangle \delta_s / (\rho_{*0} \gamma_{*0})$ $Pe_{s*0} = w_{*0} \delta_s / \gamma_{*0}$	$Re_{0} = \langle \rho w \rangle \delta_T / (\rho_{*0} \nu_{*0})$ $Re_{*0} = w_{*0} \delta_T / \nu_{*0}$	$Pe_{0} = \langle \rho w \rangle \delta_q / (\rho_{*0} k_{*0})$ $Pe_{*0} = w_{*0} \delta_q / k_{*0}$	$Pe_{D0} = \langle \rho w \rangle \delta_N / (\rho_{*0} D_{*0})$ $Pe_{D*0} = w_{*0} \delta_N / D_{*0}$
5	Friction, heat or mass-transfer coefficient	$\zeta_{0} = \frac{8 \tau_{*0}}{\langle \rho \rangle \langle \bar{w} \rangle^2} = \frac{8 \tau_{*0}}{\langle \rho w \rangle \bar{w}_{0}}$	$\alpha_{0} = q_{*0} / (\bar{T}_{*0} - \bar{T}_{0}^+)$ $St_{0} = \alpha_{0} / \langle \rho w \rangle C_p$	$\alpha_{N0} = \frac{N_{*0}}{c_{*0} - \bar{c}_{0}^+}$ ; $St_{D0} = \frac{\alpha_{N0}}{\langle \rho w \rangle}$
6	Reynolds flux $Rm_{s0} = J_{*0} / \eta (S_{*0} - \bar{S}_{0}^+)$	for $w_w = 0$ , $Rm_{T0} = \tau_{*0} / \bar{w}_{0}$	$Rm_{q0} = q_{*0} / (\bar{h}_{*0} - \bar{h}_{0}^+)$	$Rm_{N0} = N_{*0} / (c_{*0} - \bar{c}_{0}^+)$
7	Substance number $Nu_{s0} = Pe_{s*0} / (\tilde{\eta} (S_{*0}^+ - \bar{S}_{0}^+))$	for $w_w = 0, \tilde{\eta} = 1, \sigma = 0$ $Nu_{T0} = Re_{*0} / \bar{w}_{0}^+ = \zeta_{0} Re_{0} / 8$	for $\sigma = 1, \tilde{\eta} = C_{p*0} / \bar{C}_{p0}$ $Nu_{q0} = Pe_{*0} / ((T_{*0}^+ - \bar{T}_{0}^+) \tilde{\eta})$	for $\sigma = 1, \tilde{\eta} = 1$ $Nu_{N0} = Pe_{D*0} / (c_{*0}^+ - \bar{c}_{0}^+)$
8	Stanton number $\frac{1}{St_{s0}} = \tilde{\eta} Pe_{s0} \int_0^{\theta+\Delta l} \int_0^{\theta} \frac{\rho w^{\sigma}}{\langle \rho w \rangle} dY d\theta$ $\left[ \int_0^{\tilde{Y}} \frac{\tilde{J}}{\tilde{\rho} \tilde{\gamma}_T} dY' \right] (1 + 2\tilde{\delta}_s Y) dY d\theta$	for $w_w = 0, \tilde{\eta} = 1, \sigma = 0$ $\frac{1}{\zeta_{0}} = \frac{Re_{0}^{\theta+\Delta l}}{8} \int_0^{\theta} \frac{\rho}{\langle \rho \rangle} dY d\theta$ $\left[ \int_0^{\tilde{Y}} \frac{\tilde{\tau}}{\tilde{\rho} \tilde{\nu}_T} dY' \right] (1 + 2\tilde{\delta}_T Y) dY d\theta$	for $\sigma = 1, \tilde{\eta} = C_{p*0} / \bar{C}_{p0}$ $\frac{1}{St_{q0}} = \tilde{\eta} Pe_{0} \int_0^{\theta+\Delta l} \int_0^{\theta} \frac{\rho w}{\langle \rho w \rangle} dY d\theta$ $\left[ \int_0^{\tilde{Y}} \frac{\tilde{q}}{\tilde{\rho} \tilde{k}_T} dY' \right] (1 + 2\tilde{\delta}_q Y) dY d\theta$	for $\sigma = 1, \tilde{\eta} = 1$ $\frac{1}{St_{D0}} = Pe_{D0} \int_0^{\theta+\Delta l} \int_0^{\theta} \frac{\rho w}{\langle \rho w \rangle} dY d\theta$ $\left[ \int_0^{\tilde{Y}} \frac{\tilde{N}}{\tilde{\rho} \tilde{D}_T} dY' \right] (1 + 2\tilde{\delta}_N Y) dY d\theta$
9	Relative transfer laws $\tilde{St}_{s0} = \frac{St_{s0}}{St_{s00}} = \frac{Pe_{00} Pe_{*0} S_{*0}^+ - \bar{S}_{0}^+}{Pe_{0} Pe_{*0} S_{*0}^+ - \bar{S}_{0}^+}$	for $w_w = 0, \sigma = 0, \tilde{\eta} = 1$ . $\tilde{\zeta}_{0} = \frac{\zeta_{0}}{\zeta_{00}} = \frac{Re_{00}^2 Re_{*0}^2 \langle \bar{\rho} \rangle}{Re_{0}^2 Re_{*0}^2 \langle \bar{\rho} \rangle_0}$	for $\sigma = 1, \tilde{\eta} = C_{p*0} / \bar{C}_{p0}$ . $\tilde{St}_{s0} = \frac{St_{0}}{St_{00}} = \frac{Pe_{00} Pe_{*0} h_{*0}^+ - \bar{h}_{0}^+}{Pe_{0} Pe_{*0} h_{*0}^+ - \bar{h}_{0}^+}$	for $\sigma = 1, \tilde{\eta} = 1$ . $\tilde{St}_{D0} = \frac{St_{D00}}{St_{D00}} = \frac{Pe_{D00} Pe_{D*0} c_{*0}^+ - \bar{c}_{0}^+}{Pe_{D0} Pe_{D*0} c_{*0}^+ - \bar{c}_{0}^+}$

quantities  $q_w$  and  $T_w$  are taken for surface element  $dF$ . But the Newton's law does not determine the selection of the calculation temperature of fluid  $T_f$ . In general case of heat transfer by convection the temperature of fluid varies in the space considered. This requires a conventional solution on the fluid temperature to be used for calculation purposes, i.e. to be used in the equation describing Newton's law. It is common practice to use various values of temperature in calculations. For instance, with fluid flowing in tubes, the mean (or bulk-averaged, in western terminology) fluid enthalpy (or temperature) over the cross-section of flow is used

$$\bar{T}_f = \frac{\int_0^A \rho w T_f dA}{\int_0^A \rho w dA} = \frac{\int_0^{\pi r} \int_0^{\pi r} \rho w T_f 2\pi r dr d\theta}{\int_0^{\pi r} \int_0^{\pi r} \rho w 2\pi r dr d\theta} = \frac{\int_0^{\pi l} \int_0^{\pi l} \rho w T_f 2\pi R dR d\theta}{\int_0^{\pi l} \int_0^{\pi l} \rho w 2\pi R dR d\theta} = \frac{\langle \rho w T_f \rangle}{\langle \rho w \rangle}. \quad (4)$$

This definition gives the following usual average heat-transfer coefficient

$$\bar{\alpha} = \frac{\bar{q}_w}{\bar{T}_w - \bar{T}_f}. \quad (5)$$

For local and subchannel geometry in boundary layer frame (see Fig.2) we may take into account the azimuthal coordinate effect by formula for mean (or termed bulk in western countries) fluid temperature over the annular sector

$$\bar{T}_{f\theta} = \frac{\int_0^{A_s} \rho w T_f dA}{\int_0^{A_s} \rho w dA} = \frac{\int_{\theta}^{\theta+\Delta} \int_{r_r}^{r_r+\delta_q} \rho w T_f 2\pi(r_r+y) dy d\theta}{\int_{\theta}^{\theta+\Delta} \int_{r_r}^{r_r+\delta_q} \rho w 2\pi(r_r+y) dy d\theta} = \frac{\int_{\theta}^{\theta+\Delta} \int_0^1 \rho w T_f (1+2\tilde{\delta}_q Y) dY d\theta}{\int_{\theta}^{\theta+\Delta} \int_0^1 \rho w (1+2\tilde{\delta}_q Y) dY d\theta} = \frac{\langle \rho w T_f \rangle}{\langle \rho w \rangle}, \quad (6)$$

where  $\tilde{\delta}_q = \frac{\delta_q}{2r_r}$ ;  $Y = \frac{y}{\delta_q}$ , with definition of the local heat transfer coefficient

$$\bar{\alpha}_\theta = \frac{\bar{q}_{w\theta}}{\bar{T}_{w\theta} - \bar{T}_{f\theta}}, \quad (7)$$

and with the following relation between average and local heat transfer coefficients

$$\bar{\alpha}_\theta = \bar{\alpha} \frac{\bar{q}_{w\theta}}{\bar{q}_w} \frac{\bar{T}_w - \bar{T}_f}{\bar{T}_{w\theta} - \bar{T}_{f\theta}}. \quad (8)$$

Subscript  $\theta$  denotes an azimuthal direction. The following nomenclature is used below for the sake of brevity:  $S$  - assumes the value of enthalpy -  $h$  (or temperature -  $T$ ) and concentration -  $c$ , when  $\sigma=1$ .

$$S_{w\theta} - \bar{S}_\theta = \frac{\int_{\theta}^{\theta+\Delta} \int_{r_r}^{r_r+\delta_s} \rho w^\sigma (S_w - S) 2\pi(r_r+y) dy d\theta}{\int_{\theta}^{\theta+\Delta} \int_{r_r}^{r_r+\delta_s} \rho w^\sigma 2\pi(r_r+y) dy d\theta} = \frac{\int_{\theta}^{\theta+\Delta} \int_0^1 \rho w^\sigma (S_w - S) (1+2\tilde{\delta}_s Y) dY d\theta}{\int_{\theta}^{\theta+\Delta} \int_0^1 \rho w^\sigma (1+2\tilde{\delta}_s Y) dY d\theta} = \frac{\langle \rho w^\sigma (S_w - S) \rangle}{\langle \rho w^\sigma \rangle}. \quad (9)$$

The module of the  $|S_{w\theta} - \bar{S}_\theta|$  is required for axial velocity  $w$ . The mean velocity of flux over the annular sector is

$$\langle \rho w \rangle = \frac{\int_{\theta}^{\theta+\Delta} \int_0^{\delta_s} \rho w 2\pi(r_r + y) dy d\theta}{\int_{\theta}^{\theta+\Delta} \int_0^{\delta_s} 2\pi(r_r + y) dy d\theta} = \frac{1}{A_{\delta_s}} \int_{\theta}^{\theta+\Delta} \int_0^1 \rho w (1 + 2\tilde{\delta}_s Y) dY d\theta, \quad (10)$$

where  $A_{\delta_s} = \Delta(1 + \tilde{\delta}_s)$ .

Thus when there is no sliding on the wall (for  $w_w=0$ ) the weighted mean velocity is defined with a density-weighted value

$$\bar{w}_\theta = \frac{\langle \rho w \rangle}{\langle \rho \rangle}, \quad (11)$$

where mean density over the annular sector is

$$\langle \rho \rangle = \frac{\int_{\theta}^{\theta+\Delta} \int_0^{\delta_s} \rho 2\pi(r_r + y) dy d\theta}{\int_{\theta}^{\theta+\Delta} \int_0^{\delta_s} 2\pi(r_r + y) dy d\theta} = \frac{1}{A_{\delta_s}} \int_{\theta}^{\theta+\Delta} \int_0^1 \rho (1 + 2\tilde{\delta}_s Y) dY d\theta. \quad (12)$$

For the sake of uniformity of these definitions it is expedient to use the Reynolds (1974) flux concept (see, the 6-th line in TABLE I). Then, substance flux can be expressed, in a general form by the formula:

$$J_{w\theta} = \eta Rm_{s\theta} (S_{w\theta} - \bar{S}_\theta). \quad (13)$$

The choice of the sign depends on specific flow conditions, and it is considered to be positive by default. Multiplier  $\eta$  appears for the Reynolds heat flux as the averaged specific heat capacity of the flow for the heat transfer coefficient defined on the basis of Newton-Rikhman's law for friction and mass transfer coefficient considered to be  $\eta=1$ .

By definition the generalized substance transfer coefficient  $St_{s\theta}$  is expressed by the relation

$$St_{s\theta} = \frac{Rm_{s\theta}}{\langle \rho w \rangle}, \quad (14)$$

using axial velocity of flux  $\langle \rho w \rangle$ .

Elimination of  $Rm_{s\theta}$  from equations (13) and (14) results in the following relationship for the generalized substance transfer coefficient:

$$St_{s\theta} = \frac{J_{w\theta}}{\langle \rho w \rangle \eta (S_{w\theta} - \bar{S}_\theta)} = \frac{J_{w\theta} \langle \rho w \rangle^\sigma}{\langle \rho w \rangle \langle \eta (S_{w\theta} - \bar{S}_\theta) \rho w \rangle^\sigma}. \quad (15)$$

The physical meaning of  $St_{s\theta}$  becomes clear from eq. (14) and (15), which define the generalized substance transfer coefficient (momentum, heat and mass), as a measure of radial-to-axial substance transfer ratio.

#### 4. BASIC INTEGRAL RELATIONSHIP FOR GENERALIZED SUBSTANCE TRANSFER COEFFICIENT

As a result of scaling variables in equation (15), we obtain:

$$\frac{1}{St_{s\theta}} = \langle \tilde{\rho} w^+ \rangle \tilde{\eta} (S_w^+ - \bar{S}_\theta^+), \quad (16)$$

where  $\langle \tilde{\rho} w^+ \rangle = \frac{\langle \rho w \rangle}{\rho_{w\theta} w_{* \theta}} = \frac{Pe_{s\theta}}{Pe_{s^* \theta}}$ ,  $\tilde{\eta} = \begin{cases} 1 - \text{for friction and mass transfer,} \\ C_{p w \theta} / \bar{C}_{p \theta} - \text{for heat transfer,} \end{cases}$

$$\bar{C}_{p \theta} = \frac{(h_{w\theta} - \bar{h}_{f\theta})}{(T_{w\theta} - \bar{T}_{f\theta})} = \frac{\int_{\bar{T}_\theta}^{T_{w\theta}} C_p dT}{\bar{T}_\theta}. \quad (17)$$

Substituted integral relationship for the substance profile (2) taking account of eq. (9) or (11) into relationship (16) render the integral form of the generalized coefficient of substance transfer

$$\frac{1}{St_{s\theta}} = Pe_{s\theta} \tilde{\eta} \int_{\theta}^{\theta+\Delta 1} \int_{0 < \rho w^\sigma >} \frac{\rho w^\sigma}{\rho \tilde{\gamma}_T} \left( \int_0^Y \frac{\tilde{J}}{\rho \tilde{\gamma}_T} dY' \right) (1 + 2\tilde{\delta}_s Y) dY d\theta. \quad (18)$$

Thus  $St_{s\theta}$  is expressed by means of the local substance flux  $J$  profile, local physical properties  $\eta$ ,  $\rho$ ,  $w$ , and total (molecular + molar) transfer characteristics  $\gamma_T$ . It should be stressed that eq. (18) is universal, i.e. inserting appropriate fluxes and substance characteristics transforms it into the friction, heat- and mass transfer coefficients. This can be easily verified by using definitions given in TABLE I.

However, unlike (Petukhov, 1987), integral relationships for the coefficients mentioned above (see 7-th and 8-th lines of TABLE I) are not limited by assuming a linear distribution of viscous stresses along the normal to wall.

Introducing the "standard" (reference) condition concept (the lower index "o"), one can construct a generalized relative integral form for the substance transfer coefficients considered, namely, wall friction, heat and mass transfer factors:

$$\frac{St_{s\theta} Pe_{s\theta}}{St_{so\theta} Pe_{so\theta}} = \frac{\tilde{\eta}_o}{\tilde{\eta}} \int_{\theta}^{\theta+\Delta 1} \int_{0 < \rho w^\sigma >} \frac{(\rho w^\sigma)_o / (\rho w^\sigma)}{\rho \tilde{\gamma}_T / \rho \tilde{\gamma}_{To}} \left( \int_0^Y \frac{\tilde{J} / \tilde{J}_o}{\rho \tilde{\gamma}_T / \rho \tilde{\gamma}_{To}} dY' \right) (1 + 2\tilde{\delta}_s Y) dY d\theta. \quad (18a)$$

It is clear that in order to obtain a specific form for the particular transfer law one should have full information about the density distributions, mass velocity, the substance flux being transferred and also about turbulent transfer coefficients.

Relative laws of substance transfer can be written in an alternative form based on eq.(16):

$$\tilde{St}_s = \frac{St_s}{St_{so}} = \frac{< \tilde{\rho} w^+ > \tilde{\eta}_o (S_{w\theta}^+ - \bar{S}_{o\theta}^+)}{< \tilde{\rho} w^+ > \tilde{\eta} (S_{w\theta}^+ - \bar{S}_{\theta}^+)} = \frac{Pe_{o\theta} Pe_{*o\theta} \tilde{\eta}_o S_{wo\theta}^+ - \bar{S}_{o\theta}^+}{Pe_{\theta} Pe_{*o\theta} \tilde{\eta} S_{w\theta}^+ - \bar{S}_{\theta}^+}. \quad (18b)$$

The final expression of the wall friction, heat and mass transfer laws is given in the 9-th line of TABLE I.

## 5. SUBSTANCE FLUX DISTRIBUTION

To have a possibility to extend the method being developed to two-phase flows it is convenient to use the conservation laws in a form equally acceptable both for one-phase and two-phase flows. For this purpose it appears expedient to describe the two-phase flow on the basis of the drift flux model of Ishii (1975). TABLE II shows the drift flux model modified in accordance with the purposes of the present paper. The law of the light phase propagation is described by the convective diffusion equation. The conservation laws equations are used in a non-conservative form. Here for the sake of the two-phase flows conditions we should regard density  $\rho$ , mass velocity  $\rho u$  and enthalpy  $h$  as their two-phase characteristics, defined in the right part of TABLE II. The terms with subscript "d" that take into account the light phase drift are added to the corresponding turbulent substance fluxes with the subscript "t". The additional acceleration due to vapor formation at the phase interface can be written together with the hydrostatic component. Thus we obtain the form which is identical both for the one-phase and two-phase flows.

TABLE II. NON-CONSERVATIVE (TRANSPORTABLE) FORMS OF CONSERVATION LAW EQUATIONS, TWO-PHASE FLOW DESCRIPTION IS BASED ON DRIFT FLUX MODEL OF ISHII (1975).

Field equations of:	Definitions of parameters
general form: $\rho \frac{\partial S}{\partial t} + \rho \bar{u} \cdot \nabla S = -\nabla \cdot (\tilde{J}) + I_v,$	
mixture continuity: $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{u}) = 0,$	$\rho = (\varphi \rho)_g + (\varphi \rho)_f; \quad \rho \bar{u} = (\varphi \rho \bar{u})_g + (\varphi \rho \bar{u})_f;$ <p>where <math>\varphi_g = 1 - \varphi_f</math>.</p>
convective diffusion: $\rho \frac{\partial c}{\partial t} + \rho \bar{u} \cdot \nabla c = -\nabla \cdot (\bar{N}_T) + \tilde{I},$	$C = (\varphi \rho)_g / \rho, \quad \bar{N}_T = \bar{N}_T + \bar{N}_d;$ $\bar{N}_d = c(\varphi \rho)_g \bar{u}_{gf}; \quad \bar{u}_{gf} = \bar{u}_g - \bar{u}_f.$
mixture energy: $\rho \frac{\partial h}{\partial t} + \rho \bar{u} \cdot \nabla h = -\nabla \cdot (\bar{q}_T) + q_v,$	$h = [(\varphi \rho h)_g + (\varphi \rho h)_f] / \rho; \quad \bar{q}_T = \bar{q}_T + \bar{q}_d;$ $\bar{q}_d = c \rho_f \bar{u}_{gf} (h_g - h_f).$
mixture momentum: $\rho \frac{\partial \bar{u}}{\partial t} + \rho \bar{u} \cdot \nabla \bar{u} = \nabla \cdot (\tilde{\tau}_T) - \nabla P + \rho \bar{g}_T.$	$\bar{u} = [(\varphi \rho \bar{u})_g + (\varphi \rho \bar{u})_f] / \rho; \quad \tilde{\tau}_T = \tilde{\tau}_T + \tilde{\tau}_d;$ $\tilde{\tau}_d = \frac{c}{1-\varphi} \rho_f \bar{u}_{gf} \bar{u}_{gf}; \quad \rho \bar{g}_T = \rho \left( \bar{g} + \frac{\tilde{I}}{\rho} \bar{u}_{gf} \right).$

Further, we shall confine our to consideration to transient flow conditions for cylindrical coordinates (z-r- $\theta$ ) geometry. Using the boundary layer approximation we can write the conservation law equations presented in TABLE II in a generalized and unified form by means of the single substance transfer equation:

$$\frac{1}{r} \frac{\partial}{\partial y} (r J_{y\theta}) = \rho w \frac{\partial S}{\partial z} + \rho v \frac{\partial S}{\partial y} + \rho v_\theta \frac{1}{r} \frac{\partial w}{\partial \theta} - \frac{1}{r} \frac{\partial J_\theta}{\partial \theta} - I_{v\theta} + \rho \frac{\partial S}{\partial t}, \quad (19)$$

where  $w$  - axial,  $v$  - radial,  $v_\theta$  - azimuthal velocities,  $J_\theta$  - azimuthal substance flux. In the present paper the source term  $I_{v\theta}$  unites the pressure gradient and hydrostatic component in the motion equation, whereas in the energy and mass transfer equation it is the heat and mass source (sink).

After scaling the variables in eq. (19) we obtain

$$\frac{1}{1+2\tilde{\delta}_s Y} \frac{\partial(1+2\tilde{\delta}_s Y)\tilde{J}_{y\theta}}{\partial Y} = \tilde{\rho} w^+ \frac{\partial S^+}{\partial \tilde{z}} + \tilde{\rho} v^+ \frac{\partial S^+}{\partial Y} + \frac{\tilde{\rho} v_\theta^+}{1+2\tilde{\delta}_s Y} \frac{\partial S^+}{\partial \theta} - \frac{1}{1+2\tilde{\delta}_s Y} \frac{\partial \tilde{J}_\theta}{\partial \theta} - \tilde{I}_{v\theta} + \frac{\tilde{\rho}}{Sr^*} \frac{\partial S^+}{\partial \tilde{t}}, \quad (20)$$

where  $\tilde{I}_{v\theta} = I_{v\theta} \frac{\delta_s}{J_{w\theta}}$ ,  $\tilde{z} = \frac{z}{\delta_s}$ ,  $\tilde{\delta}_s = \frac{\delta_s}{2r}$ ,  $Y = \frac{y}{\delta_s}$ ,  $\tilde{J}_\theta = \frac{J_\theta}{J_{w\theta}}$ ,  $Sr^* = \frac{t_0 W^*}{\delta_s}$ ,  $\tilde{t} = \frac{t}{t_0}$ ,

$$t_i \ll t_0 < t_{tr}$$

Here  $t_{tr}$  - transport time,  $t_i$  - turbulent pulsation period.

Having integrated the latter equation first with the variable upper limit  $Y$  and then up to the boundary layer  $Y=1$  and having combined the integrals obtained we can derive the equation for the local substance flux:

$$\tilde{J}_{y\theta} = \left[ 1 - Y(1 + \tilde{\delta}_s Y) \left( \frac{1}{1 + \tilde{\delta}_s} + \langle Sz \rangle - \langle Sz \rangle_Y + \langle Sn \rangle - \langle Sn \rangle_Y + \langle Si \rangle - \langle Si \rangle_Y + \right. \right. \\ \left. \left. \tilde{\delta}_s \left( \langle S\theta \rangle - \langle S\theta \rangle_Y + \langle \frac{\partial \tilde{J}_\theta}{\partial \theta} \rangle - \langle \frac{\partial \tilde{J}_\theta}{\partial \theta} \rangle_Y \right) - \langle \tilde{I}_{v\theta} \rangle + \langle \tilde{I}_{v\theta} \rangle_Y \right) \right] \frac{1}{1 + \tilde{\delta}_s Y}, \quad (21)$$

here we use the following designations for the sake of brevity:

$$\langle Sz \rangle = \frac{1}{A_{\delta_s}} \int_{\theta}^{\theta+\Delta 1} \int_0^Y \tilde{\rho} w^+ \frac{\partial \mathcal{S}^+}{\partial z} (1 + 2\tilde{\delta}_s Y) dY d\theta \quad (22)$$

$$\langle Sz \rangle_Y = \frac{1}{A_Y} \int_{\theta}^{\theta+\Delta Y} \int_0^Y \tilde{\rho} w^+ \frac{\partial \mathcal{S}^+}{\partial z} (1 + 2\tilde{\delta}_s Y) dY d\theta, \quad (23)$$

$$\langle Sn \rangle = \frac{1}{A_{\delta_s}} \int_{\theta}^{\theta+\Delta 1} \int_0^Y \tilde{\rho} v^+ \frac{\partial \mathcal{S}^+}{\partial Y} (1 + 2\tilde{\delta}_s Y) dY d\theta, \quad (24)$$

$$\langle Sn \rangle_Y = \frac{1}{A_Y} \int_{\theta}^{\theta+\Delta Y} \int_0^Y \tilde{\rho} v^+ \frac{\partial \mathcal{S}^+}{\partial Y} (1 + 2\tilde{\delta}_s Y) dY d\theta, \quad (25)$$

$$\langle S\theta \rangle = \frac{1}{A_{\delta_s}} \int_{\theta}^{\theta+\Delta 1} \int_0^Y \tilde{\rho} v_{\theta}^+ \frac{\partial \mathcal{S}^+}{\partial \theta} dY d\theta, \quad (26)$$

$$\langle S\theta \rangle_Y = \frac{1}{A_Y} \int_{\theta}^{\theta+\Delta Y} \int_0^Y \tilde{\rho} v_{\theta}^+ \frac{\partial \mathcal{S}^+}{\partial \theta} dY d\theta, \quad (27)$$

$$\langle Si \rangle = \frac{1}{Sr^* A_{\delta_s}} \int_{\theta}^{\theta+\Delta 1} \int_0^Y \tilde{\rho} \frac{\partial \mathcal{S}^+}{\partial r} (1 + 2\tilde{\delta}_s Y) dY d\theta, \quad (28)$$

$$\langle Si \rangle_Y = \frac{1}{Sr^* A_Y} \int_{\theta}^{\theta+\Delta Y} \int_0^Y \tilde{\rho} \frac{\partial \mathcal{S}^+}{\partial r} (1 + 2\tilde{\delta}_s Y) dY d\theta, \quad (29)$$

$$\langle \frac{\partial \tilde{I}_{\theta}}{\partial \theta} \rangle = \frac{1}{A_{\delta_s}} \int_{\theta}^{\theta+\Delta 1} \int_0^Y \frac{\partial \tilde{I}_{\theta}}{\partial \theta} dY d\theta, \quad (30)$$

$$\langle \frac{\partial \tilde{I}_{\theta}}{\partial \theta} \rangle_Y = \frac{1}{A_Y} \int_{\theta}^{\theta+\Delta Y} \int_0^Y \frac{\partial \tilde{I}_{\theta}}{\partial \theta} dY d\theta, \quad (31)$$

$$\langle \tilde{I}_{v\theta} \rangle = \frac{1}{A_{\delta_s}} \int_{\theta}^{\theta+\Delta 1} \int_0^Y \tilde{I}_v (1 + 2\tilde{\delta}_s Y) dY d\theta, \quad (32)$$

$$\langle \tilde{I}_{v\theta} \rangle_Y = \frac{1}{A_Y} \int_{\theta}^{\theta+\Delta Y} \int_0^Y \tilde{I}_v (1 + 2\tilde{\delta}_s Y) dY d\theta. \quad (33)$$

It should be remembered that  $A_{\delta_s} = \Delta(1 + \tilde{\delta}_s)$ ;  $A_Y = \Delta \cdot Y(1 + \tilde{\delta}_s Y)$ .

Relationship (21) describing the radial distribution of the substance flux being transferred, and equations (2) and (18) enable us to calculate radial profiles and generalized substance transfer coefficients. As one can see from equation (21), the contribution of each of the effects under consideration (convective axial, radial and azimuthal directions transfer, inertial term, sources (sinks), etc.) to the substance flux being transferred can be described as a correction for the linear distribution. It is obvious that when the boundary layer is very thin,  $\tilde{\delta}_s \ll 1$ . Then, we see just a small correction for linear flux distribution  $\tilde{J}_{y\theta}$  accounting for the influence of above-mentioned transversal profile terms.

## 6. ANALYTICAL RELATIONSHIPS FOR SUBCHANNEL WALL FRICTION FACTOR, HEAT AND MASS TRANSFER COEFFICIENTS

Assuming that the axial pressure gradient does not change in the annular sector, we can obtain the following relationship for the wall friction factor from equations (21) and (18):

$$\frac{8}{\zeta_{\theta}} = \text{Re}_{\theta} \int_{\theta}^{\theta+\Delta 1} \int_0^Y \frac{\rho}{\langle \tilde{\rho} \rangle} \left( \int_0^Y \left[ 1 - Y(1 + \tilde{\delta}_r Y) \right] \left[ \frac{1}{1 + \tilde{\delta}_r} - \frac{\langle \tilde{\rho} \rangle}{Fr^*_{\theta}} - \langle Wz \rangle - \langle Vn \rangle - \langle Wi \rangle - \tilde{\delta}_r \left( \langle V\theta \rangle - \langle \frac{\partial \tilde{I}_{\theta}}{\partial \theta} \rangle \right) \right] dY \right) d\theta$$

$$+ \frac{\langle \tilde{\rho} \rangle_Y}{Fr_{*\theta}} + \langle W_z \rangle_Y + \langle V_n \rangle_Y + \langle W_i \rangle_Y + \tilde{\delta}_\tau \left( \langle V_\theta \rangle_Y - \left\langle \frac{\partial \tilde{\tau}_\theta}{\partial \theta} \right\rangle_Y \right) \left. \right\} \frac{dY}{\tilde{\rho} \tilde{v}_T (1 + 2\tilde{\delta}_\tau Y)} \Bigg] (1 + 2\tilde{\delta}_\tau Y) dY d\theta \quad (34)$$

where integrals  $\langle W_z \rangle$ ,  $\langle V_n \rangle$ ,  $\langle V_\theta \rangle$ ,  $\langle W_i \rangle$  are relationships (22)-(29) in substituting velocity for variable  $S$  and

$$\langle \tilde{\rho} \rangle_Y = \frac{1}{A_Y} \int_{\theta}^{\theta+\Delta Y} \int_0^Y \tilde{\rho} (1 + \tilde{\delta}_\tau Y) dY d\theta, \quad (35)$$

$$Fr_{*\theta} = \frac{\tau_{w\theta}}{\rho_{w\theta} g r_w}. \quad (36)$$

It is true to say that equation (34) generalizes the integral obtained by Petukhov-Popov (Petukhov, 1987) for the friction factor not only with respect to the functional way of taking into account the profile density. This equation takes into account the axial  $\langle W_z \rangle$ , and radial  $\langle V_n \rangle$ , and azimuthal  $\langle V_\theta \rangle$ , and inertial  $\langle W_i \rangle$  flow accelerations, generalizing at the same time two-phase flow model by Sato et al (1981).

Assuming as above that the axial enthalpy (and concentration) gradients are not the function of the radial coordinate, we can obtain the following expression from eq. (21) and (18):

$$\frac{1}{St_\theta} = Pe_\theta \tilde{\eta} \int_{\theta}^{\theta+\Delta 1} \int_0^Y \frac{\rho_w}{\langle \rho_w \rangle} \left( \int_0^Y \left[ 1 + \langle H_n \rangle_Y - \langle H_i \rangle_Y - \langle \tilde{q}_v \rangle_Y + \tilde{\delta}_q (\langle H_\theta \rangle_Y + \left\langle \frac{\partial \tilde{q}_\theta}{\partial \theta} \right\rangle_Y) - (1 - \langle H_n \rangle - \langle H_i \rangle + \langle \tilde{q}_v \rangle - \tilde{\delta}_q (\langle H_\theta \rangle + \left\langle \frac{\partial \tilde{q}_\theta}{\partial \theta} \right\rangle)) \right] \frac{\rho_w (1 + 2\tilde{\delta}_q Y) dY}{\langle \rho_w \rangle} \right) \frac{dY}{\tilde{\rho} k_T (1 + 2\tilde{\delta}_q Y)} \Bigg] (1 + 2\tilde{\delta}_q Y) dY d\theta, \quad (37)$$

where integrals  $\langle H_n \rangle$ ,  $\langle H_\theta \rangle$ ,  $\langle H_i \rangle$  are identical to relationships (24)-(29) when  $h$  is substituted for  $S$  and

$$\langle \tilde{q}_{v\theta} \rangle = \frac{1}{A_{\delta_q}} \int_{\theta}^{\theta+\Delta 1} \int_0^Y \tilde{q}_{v\theta} (1 + \tilde{\delta}_\tau Y) dY d\theta, \quad (38)$$

$$\langle \tilde{q}_{v\theta} \rangle_Y = \frac{1}{A_Y} \int_{\theta}^{\theta+\Delta Y} \int_0^Y \tilde{q}_{v\theta} (1 + \tilde{\delta}_\tau Y) dY d\theta. \quad (39)$$

The relationship for the mass transfer coefficient  $St_{D\theta}$  is functionally fully identical to eq. (37), differing only by  $\tilde{\eta}=1$  factor. The comparison of equation (37) obtained with the relationships found in references (Petukhov, 1987; Novikov and Voskresensky, 1977; Kutateladze and Leont'ev, 1985; Lyon, 1951) enables us to conclude that it is actually a generalization of Lyon's integral for flows with allowance for injection (suction), the inner heat sources (sinks), as well as azimuthal substance transfer effects. This is confirmed by the fact that the "conformity principle" is met including the limiting cases, for instance:

- (1) when transient (or inertial) term  $\langle H_i \rangle \rightarrow 0$  the local time variation is excluded and we obtain Kornienko (1997a) type relationship, which describes the heat transfer for boundary layer model accounting for the azimuthal transfer effects;
- (2) when  $\langle H_i \rangle \rightarrow 0$ ,  $V_\theta \rightarrow 0$  or/and  $\partial h^+ / \partial \theta \rightarrow 0$ , and  $\partial q_\theta / \partial \theta \rightarrow 0$  ( $\theta_{eff} \rightarrow 0$ ) the effect of azimuthal heat transfer is excluded and we obtain Kornienko (1995, 1996) type relationship, which describes the heat transfer for the flow in axial symmetry round tube;
- (3) when  $\langle H_i \rangle \rightarrow 0$ ,  $\theta_{eff} \rightarrow 0$  and  $v \rightarrow 0$ , the effect of the radial transfer is excluded, and we obtain Novikov and Voskresensky (1977) type relationship, which describes heat transfer for the flow with inner heat sources;

- (4) when  $\langle Hi \rangle \rightarrow 0$ ,  $\theta_{eff} \rightarrow 0$ ,  $\nu \rightarrow 0$  and  $q_\nu \rightarrow 0$  we obtain Petukhov-Popov's (Petukhov, 1987) relationship;
- (5) when  $\langle Hi \rangle \rightarrow 0$ ,  $\theta_{eff} \rightarrow 0$ ,  $\nu \rightarrow 0$  and  $q_\nu \rightarrow 0$  and the physical properties are constant, we obtain the classical Lyon's integral (Lyon, 1951).

The analytical expressions obtained above are nonlinear integral dimensionless equations. For their completion they require the relevant model representations for the physical phenomena included into consideration (beginning with the substance turbulent transfer models in one- and two-phase non-equilibrium flows up to the radial and axial transfer models). The analytical expressions concerned also require the development of corresponding numerical methods for solving equations like Sato et al (1981).

However, after several additional simplifications and assumptions eliminating nonlinearity it is possible to obtain quadrature solutions, retaining the generalized and heuristic properties of the integral forms obtained above, for example Kornienko (1997a).

## 7. EXAMPLES OF REDUCING GENERAL ANALYTIC RELATIONSHIPS TO PARTICULAR ONES

A detailed comparison of conclusions resulting from methods of closure relationships considered is very time and labour-consuming as it involves detailed descriptions of a numerous assumptions and a lot of additional extensive calculations, exceeding the frames of this work. That is why only some examples of obtaining new analytical relationships for wall friction and heat transfer coefficients are considered in this chapter; they are also compared with examples available from the literature.

The general idea is to introduce concepts, like (Kutateladze and Leont'ev, 1985), "standard" flow (subscribed by "o") and of a flow perturbed by some physical effect or process. Moreover, the conservation hypothesis of integral momentum and heat transfer characteristics in boundary layer zones and turbulent core is used for standard and perturbed flows.

### 7.1. Relative Laws for Friction and Heat Transfer

A relative law for the wall friction coefficient can be obtained from (18a) or directly from TABLE I (line 9, column 2) as follows:

$$\left( \frac{\zeta_\theta}{\zeta_{o\theta}} \right)_{Re_{*o}=idem} = \left[ \int_0^Y \frac{\tilde{\rho}_o}{\tilde{\rho}} \left( \int_0^Y \frac{\tilde{\rho}/\tilde{\rho}_o (\tilde{v}_\tau/\tilde{v}_{\tau o})}{\tilde{\tau}_\theta/\tilde{\tau}_{o\theta}} dY' \right) (1 + 2\tilde{\delta}_\tau Y) dY \right]^2. \quad (40)$$

Following L. Prandtl's model for describing the turbulent viscous stresses results in a friction relative law generalization, which is close in form to Kutateladze-Leont'yev (1985) relationship. So it follows:

$$\tau_t = \rho \left( \ell \frac{dw}{dy} \right)^2. \quad (41)$$

Using our designations, it can be written:

$$\frac{\tilde{v}_\tau}{\tilde{v}_{\tau o}} = \frac{Re_{*o}}{Re_{*o\theta}} \frac{d}{dR} \left( \frac{w^+}{w_o^+} \right). \quad (42)$$

Then, using designation  $\omega = w^+/w_o^+$ , expression is obtained:

$$\left( \frac{\zeta_\theta}{\zeta_{o\theta}} \right)_{Re_{*o}=idem} = \left[ \int_0^Y \frac{\tilde{\rho}_o}{\tilde{\rho}} \left( \int_0^Y \frac{\tilde{\rho}/\tilde{\rho}_o d\omega}{\tilde{\tau}_\theta/\tilde{\tau}_{o\theta}} \right) \right]^2 (1 + 2\tilde{\delta}_\tau Y) dY, \quad (43)$$

which reflects one of fundamental properties of turbulence, i.e. wall flow conservation when viscosity influence disappears.

It is clear that in so doing all the limiting transitions of Kutateladze-Leont'yev (1985) model are fulfilled and the limiting relative friction law depends only on parameters characterizing the compressibility and stress profile, being not dependent on Re.

Relative law for heat and mass transfer coefficients is derived from (11) or directly from TABLE I (line 8, column 3(4)) as follows:

$$\frac{St_\theta}{St_{o\theta}} \frac{Pe_\theta}{Pe_{o\theta}} = \frac{\tilde{\eta}_o}{\tilde{\eta}} \int_0^{\theta+\Delta} \int_0^1 \frac{(\rho w)_o / (\rho w)}{\left( \int_0^Y \frac{\tilde{\rho} / \tilde{\rho}_o (\tilde{k}_T / \tilde{k}_{T_o})}{\tilde{q} / \tilde{q}_o} dY' \right)} (1 + 2\tilde{\delta}_q Y) dY d\theta. \quad (44)$$

Transformations with the heat conductivity relative coefficient  $\tilde{k}_T / \tilde{k}_{T_o}$  under condition  $k_w \ll Pr / Pr_t * v_w$  result in

$$\frac{\tilde{k}_T}{\tilde{k}_{T_o}} = \frac{Pe_{*o\theta}}{Pe_{*o\theta}} \frac{d \left( \frac{w^+}{w_o^+} \right) Pr_{to}}{dR \left( \frac{w_o^+}{w_o^+} \right) Pr_t}. \quad (45)$$

Then relative law of heat and mass transfer will be expressed follows:

$$\left( \frac{Nu_\theta}{Nu_{o\theta}} \right)_{Re,\theta=idem} = \frac{\tilde{\eta}_o}{\tilde{\eta}} \int_0^{\theta+\Delta} \int_0^1 \frac{(\rho w)_o / (\rho w)}{\left( \int_0^Y \frac{\tilde{\rho} / \tilde{\rho}_o (Pr_{to} / Pr_t) d\omega}{\tilde{q} / \tilde{q}_o} \right)} (1 + 2\tilde{\delta}_q Y) dY d\theta. \quad (46)$$

In case there is a temperature and velocity fields similarity eq. (37) for  $\sigma=0$  and  $\eta=1$  turns into eq. (34). The relationship for the mass transfer relative law under the same assumptions has the form of equations (35) and (37) for  $\eta=1$ ,  $\sigma=1$  after substituting  $N$  for  $q$  and  $Sc$  for  $Pr$ .

## 7.2. Hydrostatic Force Effects on the Wall Friction Factor

If we neglect all the terms (except hydrostatic term) in the general relationship (21) in the shear stress form, we can take results in the following expression:

$$\tau_{zy\theta} = \tau_{zw\theta} \frac{1 - \frac{Y(1 + \tilde{\delta}_\tau Y)}{1 + \tilde{\delta}_\tau} \left[ 1 \mp \frac{1 + \tilde{\delta}_\tau}{Fr_{*o\theta}} (\langle \rho \rangle - \langle \rho \rangle_Y) \right]}{1 + 2\tilde{\delta}_\tau Y}. \quad (47)$$

For the thin boundary layer when  $\tilde{\delta}_\tau \ll 1$ , we have

$$\tilde{\tau}_{zy\theta} = 1 - Y \left[ 1 \mp \frac{1}{Fr_{*o\theta}} (\langle \tilde{\rho} \rangle - \langle \tilde{\rho} \rangle_Y) \right]. \quad (48)$$

The upper and lower signs at Froude number term in the both eqs. (47) and (48) should be used for upward and downward flows, respectively. Eqs. (47, 48) demonstrate the functional form and influence of density transversal variation on the linear distribution of shear stresses in the boundary layer approximation.

It is not difficult to derive specific relationships for friction factor and Stanton number (when  $Pr \approx 1$ ) similar to those published by Kornienko (1996, 1997a).

## 8. ON INTERPRETATION OF THE LOCAL CLOSURE RELATIONSHIP FORMULATIONS FOR FLOW IN ANNULAR CHANNEL

The relationships reported are useful as initial formulas in construction of the annular channel wall friction, heat and mass transfer coefficients taking into account additional (separate) effects represented by the integrals (22)-(33). However, as there is a large variety of the boundary conditions of the first and second kind on the inner and outer wall (from constant and equal to each other values up to periodical and unequal to each other values on every wall). They involve a large number of transpositions beyond the scope of this article. Therefore, it is appropriate at this point to bring only the brief interpretation of the above relationships for annular channels.

Relationship (21) for substance flux is suitable for its profile description in the inner and outer regions of annular channel. It is obvious that for annular channel the substance flux profile is non-linear even when we neglect all of the individual effects expressed by the integrals (22)-(33). This has an important influence on the local wall friction, heat and mass transfer coefficients introduced in the 8-th line of Table I.

Flow in concentric annulus and designations of appropriate variables are shown in Fig.3. For axial symmetry concentric annular channel we may neglect azimuthal transfer effects. The general ideas of the construction of the closure relationships for annular channels go back to Dwyer's (1963) method. But now we can take into account the number of separate effects from the list of integrals (22) - (33).

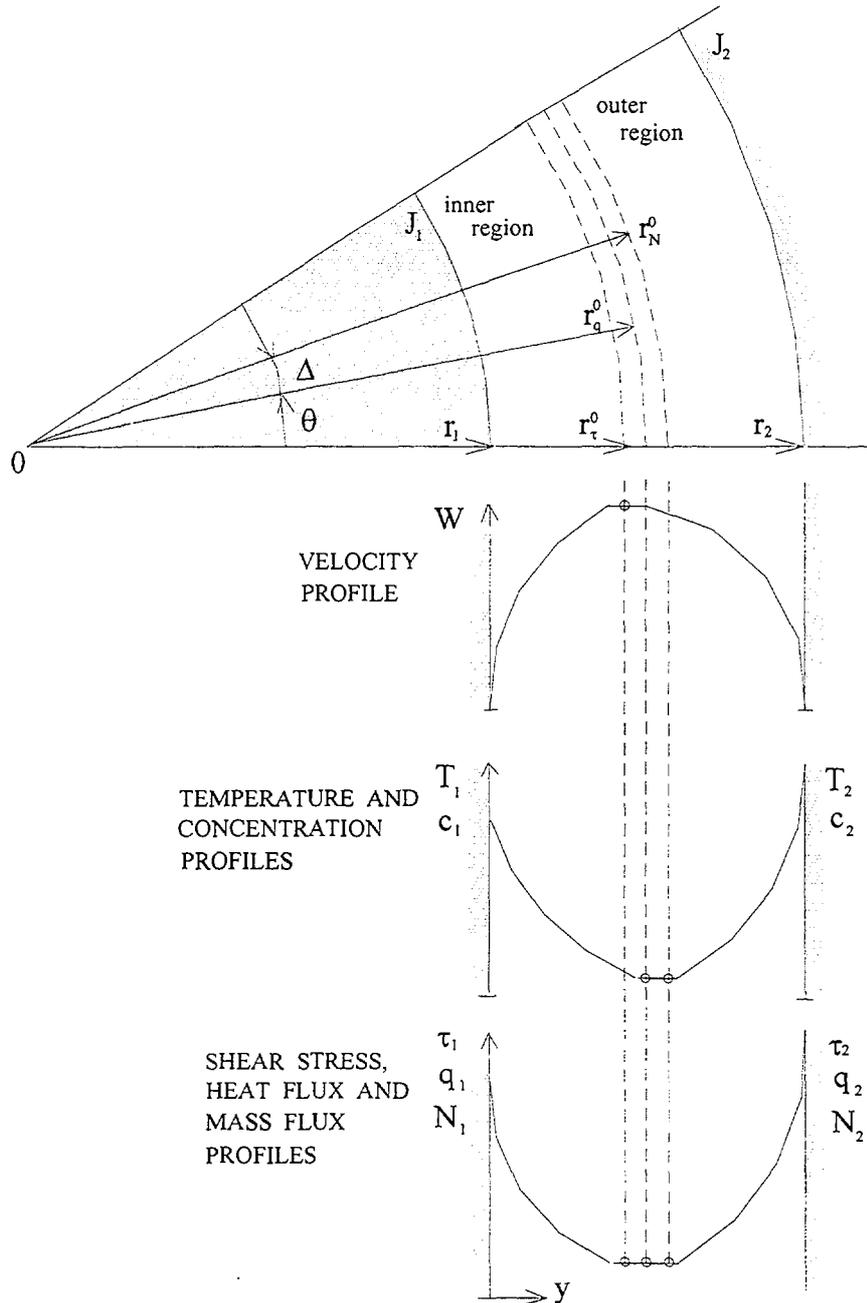


Fig.3. Graphical representation the case of unequal shear stresses, heat and mass fluxes from the walls of concentric annulus to fluid flowing therein.

Expression (47) is valid for the annular channel shear stress profile, taking into account the hydrostatic term. However, as the boundary layer thickness should be take the distance from the wall to the line of maximum velocity

$$\delta_\tau = r_\tau^0 - r_1. \quad (49)$$

The heat flux profile has a similar form. The distance from the wall to the minimum temperature line is:

$$\delta_q = r_q^0 - r_1. \quad (50)$$

Using the balance of the substance fluxes in general form, it is not difficult to derive the equation for boundary between the inner and outer region in the concentric annular channel

$$\tilde{r}_s^2 = \frac{A_{in}\tilde{r}_{12}^2 + A_{ou}F_s}{A_{in} + A_{ou}F_s}, \quad (51)$$

where relative extremum velocity (temperature or concentration) radius is:

$$\tilde{r}_s = \frac{r_s^0}{r_2} = \frac{r_1 + \delta_s}{r_2} = \frac{r_1(1 + 2\tilde{\delta}_s)}{r_2} = \tilde{r}_{12}(1 + 2\tilde{\delta}_s), \quad (52)$$

boundary parameters

$$F_s = \frac{J_1 r_1}{J_2 r_2} = \left\{ F_\tau = \frac{\tau_1 r_1}{\tau_2 r_2}, \text{ or } F_q = \frac{q_1 r_1}{q_2 r_2}, \text{ or } F_N = \frac{N_1 r_1}{N_2 r_2} \right\}. \quad (53)$$

Relationships  $A_{in}$  and  $A_{ou}$  - are integrals in the form of eqs. (22), (24), (26), (28), (30) and (32), for inner and outer regions of the annular channel.

In particular, we can derive from the momentum balance, neglecting all of the separate effects (excepting hydrostatic term), the following relationship for the boundary line between inner and outer velocity regions

$$\tilde{r}_\tau^2 = \frac{\left( \frac{dP}{dz} - \langle \rho \rangle_{in} g \right) \tilde{r}_{12}^2 + \left( \frac{dP}{dz} - \langle \rho \rangle_{ou} g \right) F_\tau}{\frac{dP}{dz} - \langle \rho \rangle_{in} g + \left( \frac{dP}{dz} - \langle \rho \rangle_{ou} g \right) F_\tau}. \quad (54)$$

From the heat balance, neglecting all of the separate effects (except that of inner heat sources), we have relationship for boundary line between inner and outer temperature region

$$\tilde{r}_q^2 = \frac{\left( \langle \rho w \frac{\partial h}{\partial z} \rangle_{in} + \langle q_v \rangle_{in} \right) \tilde{r}_{12}^2 + \left( \langle \rho w \frac{\partial h}{\partial z} \rangle_{ou} + \langle q_v \rangle_{ou} \right) F_q}{\langle \rho w \frac{\partial h}{\partial z} \rangle_{in} + \langle q_v \rangle_{in} + \left( \langle \rho w \frac{\partial h}{\partial z} \rangle_{ou} + \langle q_v \rangle_{ou} \right) F_q}. \quad (55)$$

It is not difficult to verify that these equations take the same form as the Dwyer's (1963) equations when we neglect the hydrostatic term in eq. (54) and inner heat sources in eq.(55).

Using methods Maubach (1972) and Kornienko (1997a) one can get the wall friction factor for annular channel in "two-zone model" approximations, as the harmonic mean form:

$$\zeta = \frac{1}{\left( \sqrt{\frac{\langle \rho \rangle (\beta^2 - \alpha^2)}{\zeta_1 \langle \rho \rangle_{in} \alpha (1 - \alpha)}} \frac{\beta^2 - \alpha^2}{1 - \alpha^2} + \sqrt{\frac{\langle \rho \rangle (1 - \beta^2)}{\zeta_2 \langle \rho \rangle_{ou} (1 - \alpha)}} \frac{1 - \beta^2}{1 - \alpha^2} \right)^2} \quad (56)$$

where Maubach designations are used for  $\tilde{r}_{12} = \alpha$  and  $\tilde{r}_\tau = \beta$ .

## CONCLUSION

A simple and descriptive approach to construct generalized three-dimensional integral relationships for local and subchannel wall friction, heat and mass transfer coefficients has

been suggested. This approach is based on the boundary layer approximations using Reynolds' flux concept and a generalized substance transfer coefficient.

With this approach, one can create integral analytical forms for the wall friction factor, heat and mass transfer coefficients accounting for the contribution of various complementary effects. It is precisely these transversal varied profiles that are complementary effects for the one-dimensional model. They include not only density (in the mixed convection problem), but also other components at the momentum, heat and mass transfer processes, as well as their sources and sinks in the channel flow cross section.

Unlike the well known Kutateladze-Leont'yev (1985) relationships for the limiting friction, heat and mass transfer laws, and unlike Petukhov-Popov's relationship (Petukhov, 1987), the integral forms deduced in this paper are typified by a more general character and are characterized by an additive form of notation of the effects under consideration. This is significant for the criteria assessments of the contribution of the effect in question. Besides, it is the weighted value-mean integral difference that is important, rather than the absolute value of this effect.

The general integral relationships developed are recommended as a basis for the construction of new phenomenological models of local and subchannel wall friction factor, heat and mass transfer coefficients.

## NOMENCLATURE

$C_p$	specific heat capacity			<b>Greek symbols</b>
$C$	concentration, $(\varphi\rho)_g / \rho$	$\alpha$		heat transfer coefficient
$D$	diffusion coefficient	$\alpha_N$		mass transfer coefficient
$g$	gravitational acceleration	$\Gamma$		mass source
$h$	specific enthalpy	$\nu$		kinematic viscosity
$k$	thermal conductivity	$\rho$		density, $\{\varphi\rho_g + (1-\varphi)\rho_f\}$
$N$	mass flux, $\{\rho w\}$	$\tau$		shear stress
$P$	pressure, pitch	$\varphi$		void fraction
$q$	heat flux	$\zeta$		friction factor
$R$	relative coordinate, $r / r_w$	$\theta$		azimuthal angle
$r$	radius, distance from axis	$\Delta$		increment of an azimuthal angle
$r_1$	inner radius of annulus	$\delta_s$		boundary layer thickness
$r_2$	outer radius of annulus			
$\tilde{r}_{12}$	annulus radius ratio $r_1/r_2$			
$S$	generalized variable, $(w, h(t), c)$	$D$	diffusion,	$s$ substance transfer,
$T$	temperature	$d$	drift,	$q$ heat transfer,
$\bar{u}$	local velocity	$f$	fluid,	$N$ mass transfer,
$\bar{u}_{gf}$	phase difference velocity	$g$	gas{vapor},	$\tau$ momentum transfer,
$v$	radial velocity	$t$	turbulent,	$v$ local sources,
$v_\theta$	azimuthal velocity	$T$	total,	$in$ inner region
$y$	distance from the rod wall	$w,r$	wall,	$ou$ outer region
$Y$	relative coordinate, $y/\delta_s$	$Y$	variable upper limit of integration	
$w$	axial velocity			
$z$	axial coordinate			

	Symbols		Dimensionless groups
		Fr	Froude number
		Pe	Peclet number
*	friction	Pr	Prandtl number
+	friction scales	Re	Reynolds number
~	relative	St	Stanton number.
-	weighted mean (bulk) value		
<>	area average		
↔	annular sector area average		
⊗	tensor		

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