XA0055273

the

abdus salam

united nations
educational, scientific
and cultural
organization

international atomic
energy agency

international

centre

for theoretical

physics

A SELF-TUNING EXACT SOLUTION
AND THE NON-EXISTENCE OF HORIZONS
IN 5d GRAVITY-SCALAR SYSTEM

Chuan-Jie Zhu

preprint

31 / 34

United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency

THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

A SELF-TUNING EXACT SOLUTION AND THE NON-EXISTENCE OF HORIZONS IN 5d GRAVITY-SCALAR SYSTEM

Chuan-Jie Zhu[1]
Institute of Theoretical Physics, Chinese Academy of Sciences,
P.O. Box 2735, Beijing 100080, People's Republic of China[2]
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

Abstract

We present an exact thick domain wall solution with naked singularities to five dimensional gravity coupled with a scalar field with exponential potential. In our solution we found exactly the special coefficient of the exponent as coming from compactification of string theory with cosmological constant. We show that this solution is self-tuning when a 3-brane is included. In searching for a solution with horizon we found a similar exact solution with fine-tuned exponent coefficient with an integration constant. Failing to find a solution with horizon we prove the non-existence of horizons. These naked singularities actually can't be resolved by horizon. We also comment on the physical relevance of this solution.

MIRAMARE – TRIESTE

May 2000

[1] E-mail: zhucj@itp.ac.cn
[2] Permanent address.

1 Introduction

After the Randall-Sundurm (RS) [1, 2] suggestion of a new compactification to confine gravity to four-dimensions, there is an explosive activity of studying various generalizations. One possible application of the RS scenario is to solve the cosmological constant problem which is the key obstacle to make the models of particle physics that can be derived from string theory more realistic [3, 4]. Recently a simple self-tuning mechanism has been suggested in [5, 6, 7] to at least improve on the cosmological constant problem (see also [8] for an earlier mechanism involving extra dimensions) where all order standard model loop contributions are off-set by the parameters appearing in the solution to the five dimensional gravity-scalar system. The authors of [5, 6, 7] showed that one can find static solutions to the classical equations of motion for the coupled gravity-scalar system. However, all the solutions found in [5, 6, 7] (see also [9]) were obtained either by having a constant potential for the scalar or by making a simple ansatz. In [10] some integrable bulk potentials are obtained and a general and exact solution was obtained for the exponential potential by using a first order formalism [11, 12, 13, 14, 15, 16, 17, 18, 19]. Most of these solutions have naked singularities.

Leaving aside the physical interpretation of the naked singularity one would like to understand better these solutions. However the "explicit" solution given in [10] is not quite explicit and it is given only as an implicit function of the coordinate r (see below). Even if this is quite enough for numerical analysis they are not quite suitable for ananlytic calculations. Some solutions [12, 16] are quite explicit and reasonably simple. But these solutions are just invented purposely (to be solvable and simple) and the origin for the scalar potential is not clear. (The potentials in gauged supergravity are also quite complicated and the brane world is not yet realized as a BPS or non-BPS configuration of supersymmetric theory [20, 21, 22] (however see [23] for a cosine potential).) In this paper we will fill this gap and show that there does exist a simple and exact solution with the simple exponential scalar potential. Amazingly, the exponent for which we can find such a solution is exactly the value which comes from string theory as noted in [5].

The organization of this paper is as follows: in Section 2 we present full details of our exact solution. Here the solution is not obtained by using the first order formalism [11, 12, 13, 14, 15, 16, 17, 18, 19] as was done in [10]. Instead we cast the coupled system of equations into a simple form of (effectively) two first order differential equations, given as eqs. (11) and (12). From here we found the special value which made the equations solvable by elementary method and function. This value $c = 1$ ($a = \sqrt{\frac{2}{3}}$) corresponds exactly to the potential which was reduced from the compactification of string theory with a non-vanishing cosmological constant [5] ($a = \frac{4}{3}$ in their normalization, see eq. (1) below). We also show that this solution is self-tuning [5] when a 3-brane is included. In Section 3 we turn to a more general problem of finding solutions with horizons. Here the equations can be simplified as in Section 2 and we also found a first

2

integration, eq. (49). The equations obtained have the same structure as the equations without horizons. By using the same method we found a similar exact solution with fine-tuned exponent coefficient with an integration constant (eq. (50)). Failing to find a solution with horizon we prove the non-existence of horizons. The naked singularities cannot be resolved by horizons. We comment and conclude in Section 4.

2 The self-tuning exact solution

Our starting point is the following action for five-dimensional gravity coupled to a single real scalar with an exponential potential [5]:

$$S = \int d^D x \sqrt{|G|} \left(R - \frac{1}{2} (\nabla \phi)^2 - \Lambda\, e^{a\phi} \right). \tag{1}$$

Brane sources are not included at this moment but they can easily be added (see below). From this action we have the following equations of motion:

$$R_{MN} - \frac{1}{2} \nabla_M \phi \nabla_N \phi = \frac{\Lambda}{D-2} e^{a\phi}\, G_{MN}, \tag{2}$$

$$\nabla^2 \phi - a\, \Lambda\, e^{a\phi} = 0. \tag{3}$$

In this paper we will first study solutions with Poincaré-invariant $(D-1)$-dimensional space-time and we have the following ansatz for the metric:

$$ds^2 = e^{2A(r)} \left(\eta_{\mu\nu} dx^\mu dx^\nu \right) + dr^2, \tag{4}$$

where the function A depends only on r^3.

Substituting this ansatz into eqs. (2)-(3), these equations are given as follows (setting $D = 5$):

$$A'' + 4(A')^2 + \frac{\Lambda}{3} e^{a\phi} = 0, \tag{5}$$

$$4A'' + 4(A')^2 + \frac{1}{2} (\phi')^2 + \frac{\Lambda}{3} e^{a\phi} = 0, \tag{6}$$

$$\phi'' + 4A'\phi' - a\, \Lambda\, e^{a\phi} = 0. \tag{7}$$

where prime denotes differentiation with respect to r, and we have assumed that the scalar field also depends only on r.

By simple algebra the above three equations can be recast into the following form

$$A'' + \frac{1}{6} (\phi')^2 = 0, \tag{8}$$

$$\left(A'' + \frac{1}{3a} \phi'' \right) + 4 \left(A' + \frac{1}{3a} \phi' \right) A' = 0, \tag{9}$$

$$A'' + 4(A')^2 + \frac{\Lambda}{3} e^{a\phi} = 0, \tag{10}$$

---

[3]We use the mostly positive convention for the metric.

By doing some simple rescaling and setting $\phi = \sqrt{6c}\,\Phi$ and $a = \sqrt{2c/3}$, the above equations are simplified to the following form:

$$A'' + c\,(\Phi')^2 = 0, \tag{11}$$

$$(A'' + \Phi'') + 4(A' + \Phi')\,A' = 0, \tag{12}$$

$$A'' + 4(A')^2 + \frac{\Lambda}{3}\,e^{2c\Phi} = 0, \tag{13}$$

Adding together (11) and (12) gives the following equation

$$(2A'' + \Phi'') + (2A' + \Phi')^2 + (c-1)(\Phi')^2 = 0. \tag{14}$$

Now we observe that for $c = 1$ the above equation simplifies and we can solve it to obtain

$$2A' + \Phi' = \frac{1}{r - r_0}, \tag{15}$$

where r_0 is an arbitrary (integration) constant.

Now we use the above relation in eq. (11) to eliminate A to get the following equation for Φ (remember $c = 1$):

$$\Phi'' - 2\,(\Phi')^2 + \frac{1}{(r - r_0)^2} = 0. \tag{16}$$

To solve this equation we first find a special solution by trying the ansatz $\Phi' = a_1/(r - r_0)$ to get $a_1 = -1$. Setting

$$\Phi' = \tilde{\Phi}' - \frac{1}{r - r_0}, \tag{17}$$

the equation for $\tilde{\Phi}$ is given as follows

$$\tilde{\Phi}'' + \frac{4}{r - r_0}\,\tilde{\Phi}' - 2\,(\tilde{\Phi}')^2 = 0, \tag{18}$$

which can easily be solved, first by dividing both sides by Φ' and then combining the first two terms into a total derivative to yield

$$\frac{1}{\tilde{\Phi}'(r - r_0)^4} = \frac{2/3}{(r - r_0)^3} + \text{const.}. \tag{19}$$

Substituting the above result back to eq. (17) we finally get

$$\Phi(r) = \frac{1}{2}\left(\ln|r - r_0| - \ln|(r_1 - r_0)^3 - (r - r_0)^3|\right) + \Phi_0, \tag{20}$$

$$A(r) = \frac{1}{4}\left(\ln|r - r_0| + \ln|(r_1 - r_0)^3 - (r - r_0)^3|\right) + A_0, \tag{21}$$

where r_0, r_1, Φ_0 and A_0 are integration constants. Substituting these results back into eq. (13) fixes the integration constant Φ_0:

$$\Phi_0 = \frac{1}{2}\ln\frac{9}{|\Lambda|} = \ln 3 - \frac{1}{2}\ln|\Lambda|. \tag{22}$$

For $\Lambda > 0$ the consistency of the equation ($A'' + 4(A')^2 \leq 0$) requires $0 < r - r_0 < r_1 - r_0$ for $r_1 > r_0$ and $r_1 - r_0 < r - r_0 < 0$ for $r_1 < r_0$. It is always possible to set $r_0 = 0$ by using

4

the translation invariance along the fifth dimension. We also assume $r_1 > 0$ as the other case can be obtained from this one by a reflection operation $r \to -r$. In this case our solution is interpolating between two naked singularities at $r = 0$ and $r = r_1$.

On the other hand for $\Lambda < 0$ the consistency of the equation requires the following inequality (after setting $r_0 = 0$):

$$\frac{r}{r^3 - r_1^3} > 0. \tag{23}$$

For $r_1 > 0$ this gives two regions: $r > r_1$ and $r < 0$. For $r_1 < 0$ the two regions are $r > 0$ and $r < r_1$. In both cases, the five dimensional spacetime consists of two disconnected pieces. Each piece has a naked singularity at $r = \pm r_0$ and extends to $\pm \infty$.

To get some idea of how the solution looks like, in Fig. 1 we show the profile of the volume of the 4d spacetime along the fifth dimension r. The height is the relative volume of the 4d space-time which is given by $e^{4A(r)} = |r(1 - r^3)|$. The middle part is the region $\Lambda > 0$. The disconnected left and right parts are the two regions for $\Lambda < 0$.
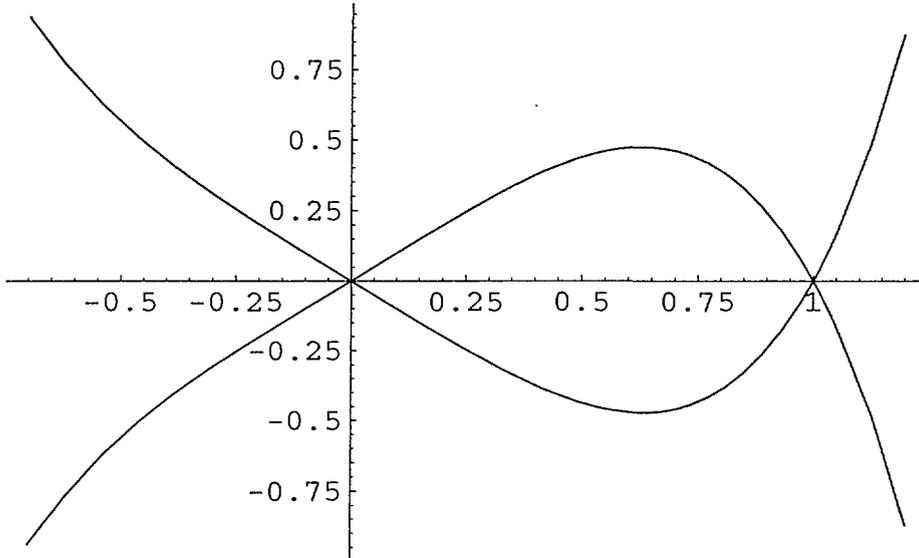

Figure 1: The profile of 4d spacetime volume along the 5th dimension.

As one can see from Fig. 1, this solution is not symmetric between the two naked singularities. The singularity ar $r = 0$ is at weak coupling $g = e^\phi = 0$, and the singularity at $r = R$ is at strong coupling $g = +\infty$. We will discuss the possible physical interpretation of this asymmetry in Section 4.

With this exact (bulk) solution at hand one can also study the self-tuning mechanism of refs. [5, 6, 7] by putting a 3-brane along the 5th dimension, say at $r = 0$. As the self-tuning mechanism is not evident here, we will analyse it in some details. For definiteness we will consider the case

5

of $\Lambda < 0$. The solution on the two sides of the 3-brane is given as follows[4]:

$$r > 0,$$

$$A(r) = \frac{1}{4}\left(\ln(r + r_1) + \ln((r + r_1)^3 - R_1^3)\right) + A_1, \tag{24}$$

$$\Phi(r) = \frac{1}{2}\left(\ln(r + r_1) - \ln((r + r_1)^3 - R_1^3)\right) + \Phi_0, \tag{25}$$

$$r < 0,$$

$$A(r) = \frac{1}{4}\left(\ln(r_2 - r) + \ln((r_2 - r)^3 - R_2^3)\right) + A_2, \tag{26}$$

$$\Phi(r) = \frac{1}{2}\left(\ln(r_2 - r) - \ln((r_2 - r)^3 - R_2^3)\right) + \Phi_0, \tag{27}$$

where r_1, r_2, R_1, R_2, A_1 and A_2 are constants and Φ_0 is given by eq. (22). We take r_1 and r_2 to be positive and they satisfy the following inequalities: $r_1 > R_1$ and $r_2 > R_2$. (R_1 and R_2 are not necessarily positive and can take any (real) values.) One can check that these are solutions to the bulk equations in their respective valid regions.

The matching conditions are as follows [5][5]

$$\Phi'(r)|_{r=0+} - \Phi'(r)|_{r=0-} = \frac{bV}{4}e^{\frac{3b}{2}\Phi(0)}, \tag{28}$$

$$A'(r)|_{r=0+} - A'(r)|_{r=0-} = -\frac{V}{6}e^{\frac{3b}{2}\Phi(0)}, \tag{29}$$

$$\Phi(r)|_{r=0+} = \Phi(r)|_{r=0-} \equiv \Phi(0), \tag{30}$$

$$A(r)|_{r=0+} = A'(r)|_{r=0-}. \tag{31}$$

One can always use the last equation to solve A_1 and the other constant A_2 is an overall constant. Noticing the particular combinations $2A' \pm \Phi'$ we obtain the following relations from the first three equations in the above:

$$\frac{1}{r_1} + \frac{1}{r_2} = -\frac{V}{4}\left(\frac{4}{3} - b\right)e^{\frac{3b}{2}\Phi(0)}, \tag{32}$$

$$\frac{3r_1^2}{r_1^3 - R_1^3} + \frac{3r_2^3}{r_2^3 - R_2^3} = -\frac{V}{4}\left(\frac{4}{3} + b\right)e^{\frac{3b}{2}\Phi(0)}, \tag{33}$$

$$\Phi(0) - \Phi_0 = \frac{1}{2}(\ln r_1 - \ln(r_1^3 - R_1^3)) = \frac{1}{2}(\ln r_2 - \ln(r_2^3 - R_2^3)). \tag{34}$$

Because of $r_{1,2} > 0$, we must have $V < 0$ and $-\frac{4}{3} < b < \frac{4}{3}$ in order to have solutions. From eq. (34) one can solve R_2 in terms of $r_{1,2}$ and R_1 uniquely. By using eq. (34), Eq. (33) can be simplified as follows:

$$\frac{3r_1}{r_1^3 - R_1^3}(r_1 + r_2) = -\frac{V}{4}\left(\frac{4}{3} + b\right)e^{\frac{3b}{2}\Phi(0)}, \tag{35}$$

which can be combined with eq. (32) to yield

$$r_2 = \frac{(4 + 3b)}{3(4 - 3b)}\frac{r_1^3 - R_1^3}{r_1^2} > 0. \tag{36}$$

---

[4]This solution doesn't confine gravity because $A(r) \to +\infty$ as $r \to \pm\infty$. We use it here just to show the self-tuning mechanism.

[5]Please do the rescaling $\phi \to \frac{3}{2}\Phi$ and set $a = \frac{4}{3}$.

Substituting this result into eq. (32) and using eq. (34) we have

$$\frac{3}{r_1^{(1+\frac{3b}{4})}(r_1^3 - R_1^3)^{(1-\frac{3b}{4})}} \left[\left(1 + \frac{(4+3b)}{3(4-3b)}\right) r_1^3 - R_1^3 \right] = -\frac{V}{4} \left(\frac{4}{3} - b\right) e^{\frac{3b}{2}\Phi_0}. \tag{37}$$

For $-\frac{4}{3} < b < \frac{2}{3}$ and taking any value of $R_1 > 0$, as r_1 varies from R_1 to $+\infty$, the left-hand side of the above equation varies from $+\infty$ to 0. So for any value (> 0) of the right-hand side, we can find a value of $r_1 > 0$ which solves the above equation. This will give a unique solution with r_2 and R_2 which depend on an arbitrary R_1 and the various parameters appearing in the action: V, b and Λ. So we have a flat 4 dimensional spacetime which is self-tuning. Presumably other regions of the parameters will be covered by different choices of pasting the bulk solutions [5].

As a final note, we point out that we can also get the solution (II) obtained in [5] by taking the limit $\Lambda \to 0$. Here we must adjust the integration constant $r \equiv R$ appropriately with Λ so that one can take the limit smoothly. An expansion of the solution

$$\Phi(r) = \frac{1}{2}\left(\ln|r| - \ln|R^3 - r^3|\right) + \Phi_0, \tag{38}$$

$$A(r) = \frac{1}{4}\left(\ln|r| + \ln|R^3 - r^3|\right) + A_0, \tag{39}$$

around $r = 0$ gives the solution (2.37) and (2.38) with the positive sign in [5], and an expansion around $r = R$ gives the solution with the minus sign there.

3 The non-existence of solutions with horizon

It is important to understand better the physical significance of the singularities in our solution. Normally these naked singularities would be discarded as unphysical, but in some instances there are reasons to believe that considering these singularities may be meaningful [16, 18].

One possible resolution of these naked singularities is to cover them with horizons which have been studied in the appendix of [18]. Here we will prove the non-existence of such solutions with (generic) exponential potential. We will comment on other interpretations of these singularities in Section 4.

As in the Schwarzschild solution in general relativity we make the following ansatz for the 5 dimensional metric [18]:

$$
\begin{aligned}
ds^2 &= -e^{2A_0(r)}(dt)^2 + e^{2A_1(r)}((dx^1)^2 + (dx^2)^2 + (dx^3)^2) + e^{2(A_1(r)-A_0(r))}(dr)^2 \\
&= e^{2A_1(r)}\left(-h(r)(dt)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2\right) + \frac{(dr)^2}{h(r)},
\end{aligned}
\tag{40}
$$

where $h(r) = e^{2A_0(r)-2A_1(r)}$. Horizon appears at the point where the function $h(r)$ has a simple zero.

By using this ansatz we obtain the following equations of motion from eqs. (2)-(3):

$$A_0'' + 2A_0'(A_0' + A_1') + \frac{\Lambda}{3}e^{a\phi+2(A_1-A_0)} = 0, \tag{41}$$

7

$$A_1'' + 2\,A_1'\,(A_0' + A_1') + \frac{\Lambda}{3}\,e^{a\phi + 2(A_1 - A_0)} = 0, \tag{42}$$

$$A_0'' + 3A_1'' + 2\,A_0'(A_0' + A_1') + \frac{1}{2}\,(\phi')^2 + \frac{\Lambda}{3}\,e^{a\phi + 2(A_1 - A_0)} = 0, \tag{43}$$

$$\phi'' + 2\,\phi'\,(A_0' + A_1') - a\,\Lambda\,e^{a\phi + 2(A_1 - A_0)} = 0, \tag{44}$$

A similar rescaling of the field ϕ and some simple algebras can bring the above equations to the following equivalent form:

$$A_1'' + c\,(\Phi')^2 = 0, \tag{45}$$

$$(A_1'' + \Phi'') + 2(A_1' + \Phi')\,(A_0' + A_1') = 0, \tag{46}$$

$$(A_0'' - A_1'') + 2(A_0' - A_1')\,(A_0' + A_1') = 0, \tag{47}$$

$$A_1'' + 2\,A_1'(A_0' + A_1') + \frac{\Lambda}{3}\,e^{2c\Phi + 2(A_1 - A_0)} = 0, \tag{48}$$

Now we can use eq. (46) and eq. (47) to get

$$A_0' - A_1' = d\,(A_1' + \Phi'), \tag{49}$$

where d is an integration constant. By using this relation we can eliminate A_0' from eqs. (45)-(46) to arrive at a similar system of equations as discussed in the previous section. We will not go through the various steps as we did in the previous section, as we have intentionally presented the full details arriving at the special value and solving the resulting equations there. The final result is that if we choose d carefully (a fine-tuning) we can actually use the method in Section 2 to obtain an exact solution. We have

$$d = \frac{2(c - 1)}{2 - c}, \tag{50}$$

$$\Phi(r) = \frac{(2 - c)}{2c}\,\ln|r| - \frac{1}{2}\,\ln\left|r_1^{(\frac{4}{c} - 1)} - r^{(\frac{4}{c} - 1)}\right| + \Phi_0, \tag{51}$$

$$A_0(r) = -\frac{(c^2 - 6c + 4)}{4c}\,\ln|r|$$
$$+ \frac{(2 - c)}{4}\,\ln\left|r_1^{(\frac{4}{c} - 1)} - r^{(\frac{4}{c} - 1)}\right| + a_0, \tag{52}$$

$$A_1(r) = \frac{(2 - c)^2}{4c}\,\ln|r| + \frac{c}{4}\,\ln\left|r_1^{(\frac{4}{c} - 1)} - r^{(\frac{4}{c} - 1)}\right| + a_1. \tag{53}$$

Note that the above solution is not valid for $c = 2$, for this will give $d = \infty$ which is not a well defined mathematical operation. In this special case we have $A_1' = -\Phi'$. This kind of ansatz was used in [5] and has been well studied there.

With the above special solution at hand one can easily see that there is no horizon. The function $h(r)$ is given as

$$h(r) \equiv e^{2(A_0(r) - A_1(r))} = (\text{const.}) \times r^{\frac{-(c^2 - 5c + 4)}{2c}}\,(r_1^{(\frac{4}{c} - 1)} - r^{(\frac{4}{c} - 1)})^{1 - c}, \tag{54}$$

which can only have possible zeroes at $r = 0$ and/or $r = r_1$. These two points are also the singularities of the metric (curvature). So it is not a horizon.

Actually one can prove that there is no exact solutions with horizons. This proof doesn't depend on the above special solution. The crucial observation is eq. (49). Assuming we do have a solution with a horizon at $r = r_h$:

$$h(r) = e^{2(A_0(r) - A_1(r))} = a_3((r - r_h) + O((r - r_h)^2)).$$ (55)

Then we have

$$A_0' - A_1' = \frac{1/2}{r - r_h} + \cdots,$$ (56)

around $r = r_h$. If we require that r_h is different from any singularities of $A_1(r)$, $A_1(r)$ is well behaved around $r = r_h$ and so is A_1':

$$A_1'(r) = A_1'(r_h) + A_1''(r_h)(r - r_h) + \cdots.$$ (57)

By using eq. (49) we have the following expansion for Φ':

$$\Phi'(r) = \frac{1}{2d} \frac{1}{r - r_h} + \cdots,$$ (58)

but this expansion is in sharp contradiction with eq. (45) ($c \neq 0$). So we proved that there is no solution with horizon for the five dimensional gravity coupled with a scalar with exponential potential. For a discussion with a generic potential with more scalar fields, please see [18].

4 Comment and conclusion

Five dimensional gravity (with or without matters) is an interesting subject which was resurrected many times. The original Kaluza-Klein idea involves only 5 dimensions. But non-abelian gauge field requires more dimensions. With the advent of superstring and M theory, one can view the extra dimensions as a dynamical thing: the radii of the extra dimensions could depend on coupling constant. Recent development with AdS/CFT correspondence [24] gives a new look at the fifth dimension: it is identified with the energy scale of the 4d field theory [25, 26] and the Hamiltonian-Jocobi equation was used to derive the renormalization group flow equations [27, 28] (see [29, 30, 31, 11, 32] for related works and, for example, [33, 34, 35, 36, 37] for later developments). In RS scenarios the compactification and the confining of gravity to 4 dimensions was achieved by an exponential warped factor and one has the freedom to put 3-branes at some points along the fifth dimension by fine-tuning various parameters [1, 2]. According to [5, 6, 7] this fine-tuning is not required because the parameters in the solution will adjust themselves (a self-tuning mechanism). But here we have a plethora of different solutions and some solutions have naked singularities. The puzzle is: what principle should we use to select the right solution?

As we said in the introduction, solutions with singularities cannot be simply discarded as unphysical. Some solutions with singularities have physical interpretations in type IIB superstring theory (see [11, 38], for example). In fact the solution we found in eqs. (20)-(21) was argued as

some solution-independent behaviour [29] and corresponds to the deformation of non-conformal field theory [39, 32]. We may associate the two naked singularities to two different field theories (or putting it differently: the two naked singularities are resolved by two well-defined field theories). The problem with this interpretation is that it is difficult to think that these two non-conformal field theories can be related by a renormalization group flow. The different behaviour of the coupling constants between the two singularities offer a possible way out: either the 5d gravity description breaks down at one of the singularities or these two non-conformal field theories are in fact strong-weak dual pairs. It is interesting to study the higher embedding of these naked singularities.

For $\Lambda > 0$ our exact solution has an interesting behaviour: as r varies from 0 to r_1, $A(r)$ (so is $e^{4A(r)}$) first increases, reaches a maximum, and then decreases, all within a finite interval of r. In the original RS scenario this behaviour was introduced by hand by introducing 3-brane. Here we have no 3-branes. We can only attribute this behaviour to the inclusion of scalar field. In the AdS/CFT correspondence [24], scalar fields are interpreted as deformations of four dimensional field theory. The profile of these scalar fields are just running coupling constants. This correspondence is valid in the strong coupling. Recent suggestions to extend this correspondence to weak coupling seem to be in contradiction with our exact solution. A general programme of Hamiltonian-Jacobi equation/Renormalization group flow equation correspondence seems only an approximation.

Acknowledgments

References

[1] L. Randall and R. Sundrum, *An alternative to compactification*, Phys. Rev. Lett. **83** (1999) 4690, hep-th/9906064.

[2] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, Phys. Rev. Lett. **83** (1999) 3370, hep-ph/9905221.

[3] S. Weinberg, *The cosmological constant problem*, Rev. Mod. Phys. **61** (1989) 1-23, and references therein.

[4] E. Witten, *The cosmological constant from the viewpoint of string theory*, hep-ph/0002297.

[5] S. Kachru, M. Schulz and E. Silverstein, *Self-tuning flat domain walls in 5d gravity and string theory*, hep-th/0001206;

[6] S. Kachru, M. Schulz and E. Silverstein, *Bounds on curved domain walls in 5d gravity*, hep-th/0002121.

[7] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, *A small cosmological constant from a large extra dimension*, Phys. Lett. B480 (2000) 193-199, hep-th/0001197.

[8] V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. **125B** (1983) 139.

[9] D. Youm, *Solitons in brane worlds*, hep-th/9911218; *Solitons in brane worlds II*, hep-th/0001018; *A note on solitons in brane worlds*, hep-th/0001166.

[10] C. Csáki, J. Erlich, C. Grojean and T. Hollowood, *General properties of the self-tuning domain wall approach to the cosmological constant croblem*, hep-th/0004133.

[11] D. Z. Freedman, S. S. Gubser, K. Pilch, and N. P. Warner, *Renormalization group flows from holography supersymmetry and a c-theorem*, hep-th/9904017; *Continuous distributions of D3-branes and gauged supergravity*, hep-th/9906194.

[12] K. Skenderis and P.K. Townsend, *Gravitational stability and renormalization-group flow*, Phys. Lett. **B468** (1999), hep-th/9909070

[13] A. Chamblin and G. W. Gibbons, *Nonlinear supergravity on a brane without compactification*, Phys. Rev. Lett. 84 (2000) 1090-1093, hep-th/9909130.

[14] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *The supergravity dual of N = 1 superYang-Mills theory*, hep-th/9909047.

[15] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, *Modeling the fifth dimension with scalars and gravity*, hep-th/9909134.

[16] M. Gremm, *Four-dimensional gravity on a thick domain wall*, hep-th/9912060; *Thick domain walls and singular spaces*, hep-th/0002040.

[17] C. Csáki, J. Erlich, T. J. Hollowood and Y. Shirman, *Universal aspects of gravity localized on thick branes*, hep-th/0001033.

[18] S. S. Gubser, *Curvature singularities: the good, the bad, and the naked*, hep-th/0002160.

[19] For review see: M. Cvetič and H. Soleng, *Superstring domain walls*, Phys. Rept. **282** (1997) 159, hep-th/9604090.

[20] K. Behrndt and M. Cvetič, *Supersymmetric domain-wall world from D=5 simple gauged supergravity*, hep-th/9909058; *Anti-de Sitter vacua of gauged supergravities with 8 supercharges*, hep-th/0001159.

[21] R. Kallosh, A. Linde and M. Shmakova, *Supersymmetric multiple basin attractors*, J. High Energy Phys. **11** (1999) 010, hep-th/9910021.

[22] R. Kallosh and A. Linde, *Supersymmetry and the brane world*, J. High Energy Phys **02** (2000) 005, hep-th/0001071.

[23] K. Behrndt, *A non-singular infra-red flow from D=5 gauged supergravity*, hep-th/0005185.

[24] Juan M. Maldacena, *The large N limit of superconformal field theories and supergravity*, Adv. Theor. Math. Phys. **2** (1998) 231-252 or Int. J. Theor. Phys. **38** (1999) 1113-1133, hep-th/9711200; S. S. Gubser, I. R. Klebanov and A. M. Polyakov, *Gauge theory correlators from non-critical string theory*, Phys. Lett. **B428** (1998) 105-114, hep-th/9802109; E. Witten, *Anti-de Sitter space and holography*, Adv. Theor. Math. Phys. **2** (1998) 253-291, hep-th/9802150; For review see: O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, *Large N field theories, string theory and gravity*, Phys. Rept. 323 (2000) 183-386, hep-th/9905111.

[25] L. Susskind and Edward Witten, *Holographic bound in anti-de Sitter space*, hep-th/9805114.

[26] A. W. Peet and J. Polchinski, *UV/IR relations in AdS dynamics*, Phys. Rev. **D59** (1999) 065011, hep-th/9809022.

[27] H. Verlinde, *Holography and compactification*, hep-th/9906182.

[28] J. de Boer, E. Verlinde and H. Verlinde, *On the holographic renormalization group*, hep-th/9912012; E. Verlinde and H. Verlinde, *RG-flow, gravity and the cosmological constant*, hep-th/9912018.

[29] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *Novel local CFT and exact results on perturbations of N = 4 super Yang-Mills from AdS dynamics*, J. High Energy Phys. **12** (1998) 022, hep-th/9810126; *Confinement and condensates without fine tuning in supergravity duals of gauge theories*, J. High Energy Phys. **05** (1999) 026, hep-th/9903026.

[30] A. Khavaev, K. Pilch and N. P. Warner, *New vacua of gauged N = 8 supergravity in five dimensions*, hep-th/9812035.

[31] A. Karch, D. Lust and A. Miemiec, *New N = 1 superconformal field theories and their supergravity description*, Phys. Lett. **B454** (1999) 265, hep-th/9901041.

[32] S. S. Gubser, *Dilaton-driven confinement*, hep-th/9902155; *Non-conformal examples of AdS/CFT*, hep-th/9910117.

[33] J. Khoury and H. Verlinde, *On open/closed string duality*, hep-th/0001056.

[34] V. Sahakian, *Comments on D branes and the renormalization group*, J. High Energy Phys. **05** (2000) 011, 0002126.

[35] C. S. Chan, P. L. Paul and H. Verlinde, *A note on warped string compactification*, hep-th/0003236.

[36] H. Verlinde, *Supersymmetry at large distance scales*, hep-th/0004003.

[37] Feng-Li Lin and Yong-Shi Wu, *The c-functions of noncommutative Yang-Mills tTheory from holography*, hep-th/0005054.

[38] C. V. Johnson, A. W. Peet and J. Polchinski, *Gauge theory and the excision of repulson singularities*, Phys. Rev. **D61** (2000) 086001, hep-th/9911161; J. Polchinski and M. J. Strassler, *The string dual of a confining four-dimensional gauge theory*, hep-th/0003136.

[39] A. Kehagias and K. Sfetsos, *On running couplings in gauge theories from type-IIB supergravity*, Phys.Lett. **B454** (1999) 270-276, hep-th/9902125; S. Nojiri and S. D. Odintsov, *Two-boundaries AdS/CFT correspondence in dilatonic gravity*, Phys. Lett. **B449** (1999) 39-47, hep-th/9812017.