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STABILIZATION EFFECT OF A STRONG HF ELECTRICAL FIELD ON BEAM-PLASMA INTERACTION IN A RELATIVISTIC PLASMA WAVEGUIDE

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Abstract

The influence effect of a strong HF electrical field on the excitation of surface waves by an electron beam under the development of instability of low-density electron beam passing through plane relativistic plasma is investigated. Starting from the two fluid plasma model we separate the problem into two parts. The "temporal" (dynamical) part enables us to find the frequencies and growth rates of unstable waves. This part within the redefinition of natural (eigen) frequencies coincide with the system describing HF suppression of the Buneman instability in a uniform unbounded plasma. Natural frequencies of oscillations and spatial distribution of the amplitude of the self - consistent electrical field are obtained by solving a boundary - value problem ("spatial" part) considering a specific spatial distribution of plasma density. Plasma electrons are considered to have a relativistic velocity. It is shown that a HF electric field has no essential influence on dispersion characteristics of unstable surface waves excited in a relativistic plasma waveguide by a low-density electron beam. The region of instability only slightly narrowing and the growth rate decreases by a small parameter and this result has been reduced compared to nonrelativistic plasma. Also, it is found that the plasma electrons have not affected the solution of the space part of the problem.

1. Introduction

Investigations of the beam-plasma instability of a bounded electron beam in an unbounded plasma usually deal with the behavior of the growth rate as a function of the parameters of the problem for one or two oscillation modes which have the largest growth rate. This approach has permitted considerable insight into the important role of transverse transport of the oscillation energy with the group velocity out of the beam region in determining the behavior of the beam-plasma interaction [1-5].

The control of instabilities, in particular their suppression, could be achieved by using an intense high-frequency (HF) fields (e.g., [6-13]). Based on existing experimental and theoretical results, it may be deduced that the effect of an intense electromagnetic radiation on a plasma may produce a qualitative change in its main properties (see e.g., [14-18] a refs. therein). In particular, the plasma dispersion characteristics can change to some extent: the absorption of an external (pump) electromagnetic field energy by plasma particles increases in comparison to collisional absorption, the possibility exists for parametric excitation of plasma waves, and for stabilization of many plasma instabilities.

The stabilization effect of a uniform HF electric field on a two-stream (Buneman) instability in uniform (or nonuniform) unbounded (or bounded) plasma has been investigated by Aliev et al. [18] and Demechenko et al. [19]. They obtained the dispersion equation for characteristic frequencies of electrostatic oscillations excited by the relative motion of electrons and ions in a HF electric field. The presence of a pump wave strongly modifies the dispersion equation of the Buneman instability. Consequently, the growth rate of instability is reduced in comparison with the growth rate at vanishing external field amplitude.

The method described is used for the solution of the effect of a HF electric field and the instability of a low-density electron beam passing through a plasma waveguide in the presence of a HF electric field are investigated in [19].

Contrary to recent works [19], we take into consideration the influence of relativistic plasma electrons on the beam-plasma interaction in a plasma waveguide pumped by HF electric field.

2. Separation Method in the Problem of a Beam-Plasma Interaction in Bounded Relativistic Plasma under the Effect of HF Electric Field.

Let us consider a cold electron beam propagation a long a nonuniform plane relativistic plasma waveguide. The vector of the external HF field $\vec{E}_p = \vec{E}_0 \sin(\omega t)$ is along the $z-$axis. The " separation " method has been described [19] in application to the problem of parametric excitation of a surface waves in a cold isotropic plasma. Here we shall follow this paper. Representing the perturbations of velocity, density and electrical potential in the form $\delta\vec{V}_\alpha, \delta n_\alpha, \Phi \sim \exp i(kz-\omega t)$ the linearized set of hydrodynamical equations together with Poisson equation can be reduced to the form

$$\frac{\partial^2 v_{\alpha_1}}{\partial t^2} - i\Delta_\gamma \frac{\partial v_{\alpha_1}}{\partial t} = -\frac{m_e p^2}{e^2} e^{iA_\alpha} \sum_{\beta=e,i} v_{\beta_1} \frac{e_\beta^2}{m_\beta} e^{-iA_\beta} \quad , \tag{1}$$

where $\alpha = e, i \; or b$ and p is a separation constant, and v_{α_1} is the temporal part of the density perturbations ($v_\alpha = v_{\alpha_1}(t) v_{\alpha_2}(x)$), $\Delta_\gamma = k u_0 (1-\gamma)$, $\gamma = (1-\frac{u_0^2}{c^2})^{-1/2}$,

$$A_\alpha = (ku_{\alpha_0})t - a_\alpha \sin(\omega_0 t), and \; a_\alpha \equiv \frac{e_\alpha k E_0}{m_\alpha \omega_0^2} \approx a_e \tag{2}$$

Assuming that the ions is at rest ($u_{i_0} \equiv 0$) and that the frequency of the HF field much larger than the eigen frequencies of the excited surface waves ($\omega_0 >> \omega_{SW} \sim \omega_{P_e}$), we may use the method of averaging.

Introducing the explicit form of $A_\alpha(a_b = a_e >> a_i)$, and a new variable $w_i = \frac{m_e}{m_i}\frac{\alpha_i}{\alpha_e} v_{i_1}$, we find the final form of the equations which describes the dynamical (temporal) part of our problem

$$\left.\begin{aligned}
&\frac{d^2 v_{b_1}}{dt^2} + \varepsilon_b p^2 (v_{b_1} + v_{e_1} e^{ik(u_{b0}-u_{e0})t} + w_i e^{ik(u_{b0}-u_{i0})t - ia\sin(\omega_0 t)}) = 0 \\
&\frac{d^2 v_{e_1}}{dt^2} - i\Delta_\gamma \frac{d v_{e_1}}{dt} + p^2 (v_{e_1} + v_{b_1} e^{-ik(u_{b0}-u_{e0})t} + w_i e^{ik(u_{b0}-u_{i0})t - ia\sin(\omega_0 t)}) = 0 \\
&\frac{d^2 w_i}{dt^2} + p^2 \frac{m_e}{m_i}(w_i + e^{ia\sin(\omega_0 t)}(v_{e_1} e^{-ik(u_{e0}-u_{i0})t} + v_{b_1} e^{-ik(u_{b0}-u_{i0})})) = 0
\end{aligned}\right\} \qquad (3)$$

where, $\varepsilon_b = n_{0b} / n_0$.

Here we shall confine our analysis for the influence of the HF electric field on the dispersion characteristics of unstable surface waves excited in a plasma waveguides by an electron beam.

Using the Jacobi - Anger formula $e^{\pm ia\sin(\omega_0 t)} = \sum_{m=-\infty}^{\infty} J_m(a)\, e^{\pm im\omega_0 t}$, we obtain from equations (3)

$$\left.\begin{aligned}
&\frac{d^2 <v_{b_1}>}{dt^2} + \varepsilon_b p^2 (<v_{b_1}> + e^{i(ku_0)t}[<v_{e_1}> + J_0(a) <w_i>]) = 0 \\
&\frac{d^2 <v_{e_1}>}{dt^2} - i\Delta_\gamma \frac{d <v_{e_1}>}{dt} + p^2 (<v_{e_1}> + e^{i(ku_0)t}[<v_{b_1}> + J_0(a) <w_i>]) = 0 \\
&\frac{d^2 <w_i>}{dt^2} + \frac{m_e}{m_i} p^2 (<w_i> + e^{-i(ku_0)t} J_0(a)[<v_{b_1}> + <v_{e_1}>]) = 0
\end{aligned}\right\} \qquad (4)$$

where, $(<v_{\alpha_1}>, <w_i>) = \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0} (v_{\alpha_1}, w_i)\, dt$,

and $J_m(a)$ are the Bessel functions.

The system of equations (4) coincide (if $\gamma = 1$, $n_{0b} = 0$, $\omega_{P_e}^2 \to p^2$, $\omega_{P_i}^2 \to (m_e / m_i) p^2$) with the system describing the HF stabilization of the two - stream instability in a uniform (or nonuniform) unbounded (or bounded) plasma [18] or [19].

If there is no externally injected beam ($\varepsilon_b = (n_{0b} / n_0) = 0$) then the system of equations (4) coincide with the system describing the HF stabilization of the Buneman instability in a nonuniform bounded relativistic plasma [20].

3. Solution of the time - dependent equations.

Let us now assume that an electron beam of low density ($\varepsilon_b = n_{0b} / n_0 << 1$) is passing through a quasineutral plasma with the velocity $\vec{u}_{b_0}$. We shall also suppose that both plasma components are at rest ($\vec{u}_{e_0} = \vec{u}_{i_0} = 0$). According to equations (4), plasma oscillations are then described by the dispersion equation

$$(\omega^2 - \omega_{LF}^2)[(\omega - ku_b)^2(\omega(\omega + \Delta_\gamma) - \omega_{HF}^2) - \varepsilon_b p^2 \omega(\omega + \Delta_\gamma)] = 0\,, \tag{5}$$

where

$$\omega_{LF}^2(p) = \frac{m_e}{m_i} p^2 (1 - J_0^2(a)),\; \omega_{HF}^2(p) = p^2 (1 + \frac{m_e}{m_i} J_0^2(a)) \tag{6}$$

In the case when $\vec{E}_0 = 0$ and $\gamma = 1$ equation (5) agrees with the dispersion equation which describes the unstable oscillations that excited in a uniform unbounded plasma by a low - density electron beam [21].

(a) Non - resonant case ($ku_b \approx \omega_{HF}$)

We have from (5)

$$\omega = ku_b \pm \frac{\sqrt{\varepsilon_b}\, p[ku_b(ku_b + \Delta_\gamma)]^{1/2}}{[ku_b(ku_b + \Delta_\gamma) - \omega_{HF}^2(p)]^{1/2}}. \tag{7}$$

Under the conditions

$$p^2(1 + \frac{m_e}{m_i} J_0^2(a)) > ku_b(ku_b + \Delta_\gamma) > 0, \tag{8}$$

The roots of equation (7) are complex and one of them corresponds to an instability with the growth rate

$$\gamma_{NR} = \frac{\sqrt{\varepsilon_b}\, p[ku_b(ku_b + \Delta_\gamma)]^{1/2}}{[\omega_{HF}^2(p) - ku_b(ku_b + \Delta_\gamma)]^{1/2}}. \tag{9}$$

(b) Resonant case ($ku_b \approx \omega_{HF}$)

The frequency of unstable oscillations can be represented in the form $\omega = \omega_{HF}(p) + \Delta\omega$, where

$$\mathrm{Re}\,\Delta\omega = -\frac{\varepsilon_b^{1/3}\, p(1 + \frac{m_e}{m_i} J_0^2(a))^{1/6}}{2\,[2 + \frac{\Delta_\gamma}{p(1 + \frac{m_e}{m_i} J_0^2(a))^{1/2}}]^{1/3}}, \tag{10}$$

$$\gamma_R = \mathrm{Im}\,\Delta\omega = \frac{\sqrt{3}}{2} \frac{\varepsilon_b^{1/3}\, p(1 + \frac{m_e}{m_i} J_0^2(a))^{1/6}}{[2 + \frac{\Delta_\gamma}{p(1 + \frac{m_e}{m_i} J_0^2(a))^{1/2}}]^{1/3}}, \tag{11}$$

where p is determined by the equation of the space part of the problem.

It follows from expressions (8) - (11) that the HF electric field has no essential influence on the dispersion characteristics of unstable surface waves excited in a plasma waveguides by a low - density electron beam. The region of instability only slightly narrows and the growth rate decreases by a small factor.

The results obtained are in a full agreement with the conclusion that an external HF field may have a stabilizing effect on the electron beam-plasma interaction in uniform (or nonuniform) plasma [19, 20]. The relativistic plasma reduced the growth rate.

We conclude that the growth rate of the electron beam-plasma interaction decreases more in a relativistic plasma than in a nonrelativistic plasma [19].

4. Solution of the spatial part of the problem.

The main feature of the expressions (9), (10) and (11) is the existence of a separation constant p which enables us to take into consideration the plasma boundaries.

To find an explicit expression for the constant p it is necessary to solve the following differential equation (for detail see [22]):

$$\frac{d}{dx}(\varepsilon(p,x)\frac{d\Phi_2}{dx}) - k^2\varepsilon(p,x)\Phi_2 = 0. \qquad (12)$$

where, $\varepsilon(p,x) = 1 - \frac{\omega_{P_e}^2}{p^2}$

This equation is the same equation in nonrelativistic plasma waveguide [19] i.e., the relativistic plasma waveguide have no effect on the space part of the problem.

By the solution of equation (12) and using the continuity condition of Φ_2 and $\varepsilon d\Phi_2 / dx$ at the points $x = \pm l/2$ we get

$$\tanh(kl) = -\frac{2\varepsilon_0(p)}{\varepsilon_0^2(p)+1}. \tag{13}$$

Equation (13) can be satisfied only for $\varepsilon_0(p) < 0$ $(\omega_{P_0} > p)$. In the case of a "thick" plasma layer $(kl >> 1)$, as would be expected, equation (13) coincides with the dispersion equation $\varepsilon_0(p) = -1$ for surface oscillations of a semi - bounded plasma $(p = \omega_{P_e} / \sqrt{2})$. For a "thin" plasma layer $(kl << 1)$, equation (13) is solvable only if $|\varepsilon_0| >> 1$. Then it takes the form $(kl)\varepsilon_0(p) = -2$, from which we find

$$p = \omega_{P_e} (\frac{kl}{2})^{1/2} . \tag{14}$$

The separation constant p is now determined by equation (14).

5. Conclusions

The paper deals with parametric excitation of the potential surface waves in a bounded nonuniform relativistic plasma by monochromatic HF electrical field. It is shown that the problem can be reduced to the solution of the "temporal" (parametric) and "stationary" (spatial) parts.

The effect of HF electric field on the excitation of surface waves by an electron beam under the development of instability of low-density electron beam passing through a relativistic plasma is considered. The "temporal" (dynamical) part enables us to find the frequencies and growth rates of unstable waves. This part within the redefinition of natural (eigen) frequencies coincide with the system describing HF suppression of the Buneman instability in a uniform unbounded plasma. Natural frequencies of oscillations and spatial distribution of the amplitude of the self - consistent electrical field are obtained by solving a boundary - value problem ("spatial" part) considering a specific spatial distribution of plasma density. The plasma electrons are considered to have a relativistic velocity.

The method was used for the solution of the stabilization effect of a strong HF electric field on beam-plasma interaction in a plane relativistic plasma waveguide. We solved the "temporal" (time-dependent) equations and obtained the corresponding dispersion equation (5) in plane geometry, which was analyzed for two cases: Nonresonance instability ($ku_b \approx \omega_{HF}$), and resonance one ($ku_b \approx \omega_{HF}$). In both cases the frequency growth rates of the oscillations are obtained (relations (9) and (11)). The separation constant p is obtained from relation (14), the results are compared with the case when the external electric field is absent ($\vec{E}_0 = 0$) and a nonrelativistic plasma ($\gamma = 1$).

In conclusion it is shown that a HF electric field has no essential influence on dispersion characteristics of unstable surface waves excited in a relativistic plasma waveguide by a low-density electron beam. The region of instability only slightly narrowing and the growth rate decreases by a small parameter and this result has been reduced compared to nonrelativistic plasma. Also, it is found that the plasma electrons have not affected the solution of the space part of the problem.

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