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Two-Fluid and Parallel Compressibility Effects in Tokamak Plasmas

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The MHD, or single fluid, model for a plasma has long been known to provide a surprisingly good description of much of the observed nonlinear dynamics of confined plasmas, considering its simple nature compared to the complexity of the real system. On the other hand, some of the supposed agreement arises from the lack of the detailed measurements that are needed to distinguish MHD from more sophisticated models that incorporate slower time scale processes. At present, a number of factors combine to make models beyond MHD of practical interest. Computational considerations still favor fluid rather than particle models for description of the full plasma, and suggest an approach that starts from a set of fluid-like equations that extends MHD to slower time scales and more accurate parallel dynamics.

This paper summarizes a set of two-fluid equations for toroidal (tokamak) geometry that has been developed and tested as the MH3D-T code [1] and some results from the model. The electrons and ions are described as separate fluids. The code and its original MHD version, MH3D [2], are the first numerical, initial value models in toroidal geometry that include the full 3D (fluid) compressibility and electromagnetic effects. Previous nonlinear MHD codes for toroidal geometry have, in practice, neglected the plasma density evolution, on the grounds that MHD plasmas are only weakly compressible and that the background density variation is weaker than the temperature variation. Analytically, the common use of toroidal plasma models based on aspect ratio expansion, such as reduced MHD, has reinforced this impression, since this ordering reduces plasma compressibility effects.

For two-fluid plasmas, the density evolution cannot be neglected in principle, since it provides the basic driving energy for the diamagnetic drifts of the electrons and ions perpendicular to the magnetic field. It also strongly influences the parallel dynamics, in combination with the parallel thermal conductivity. The true parallel plasma dynamics are driven by additional kinetic processes that are not included in the fluid picture, but the basic fluid effects remain and should be understood first.

The two-fluid code is part of a larger project, the M3D, or Multi-Level 3D project [3] for toroidal plasmas. Its goal is to develop a comprehensive suite of simulation models that cover a range of physics from simple to complex, starting from the fluid and progressing toward kinetic models. In addition to the new physics that it describes beyond MHD, the present two-fluid code provides a good base for adding additional, non-fluid effects in a fully electromagnetic and toroidal model. The use of gyrokinetic particle simulation to provide an improved closure for the ion fluid, combined with an electron fluid, is currently under development.

Two-Fluid Equations

The two-fluid equations for toroidal (tokamak) geometry that comprise the MH3D-T code [1] are based on the drift ordering [4], which has been generalized to arbitrary perturbation size. This ordering contains time scales slower than MHD ($\partial/\partial t \sim \delta v_{th}$, where δ is a small parameter and v_{th} is the ion thermal velocity), while retaining validity beyond the strictly collisional regime, but it is not rigorous nonlinearly. Electrons and ions are treated as separate fluids. The model includes the ion gyroviscous force, the Hall terms and electron pressure gradient in Ohm's law, and equations for the electron and ion temperature evolution, with parallel and perpendicular thermal conductivities. As a first approximation, the electrons are treated as massless and the pressures assumed to be isotropic.

The fluid velocities can be written exactly in terms of the generalized diamagnetic velocities as $\mathbf{v}_i = \mathbf{v} + \mathbf{v}_{di}$ and $\mathbf{v}_e = \mathbf{v} + \mathbf{v}_{*e} - \mathbf{J}_{\|}/en_e$, where $\mathbf{v}_{*j} \equiv \mathbf{B} \times \nabla p_j/(q_j n_j B^2)$. The perpendicular ($\perp \mathbf{B}$) component of $\mathbf{v}$ is the guiding center velocity of the electrons and ions in the perpendicular direction. The generalized ion 'diamagnetic' velocity $\mathbf{v}_{di} \equiv \mathbf{J}_{\perp}/(en_e) + \mathbf{v}_{*e}$ contains the polarization drift.

In terms of $\mathbf{v}$, the two-fluid equations can be written in rationalized units as

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -(\mathbf{v}_{di}\cdot\nabla)\mathbf{v}_{\perp} + \frac{\mathbf{J}\times\mathbf{B}}{\rho} - \frac{\nabla p}{\rho} + \mu\nabla_{\perp}^2\mathbf{v} + \frac{\mathcal{V}_{gv}}{\rho} - \frac{\nabla\cdot\mathbf{\Pi}_{i\|}}{\rho} \quad (1)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E} \quad (2)$$

$$\mathbf{E} = -\mathbf{v}\times\mathbf{B} + \eta\mathbf{J}^* - \frac{\nabla_{\|}p_e}{en} - \frac{\nabla\cdot\mathbf{\Pi}_{e\|}}{en}, \quad (3)$$

where $\mathbf{J}^* = \mathbf{J} + (3/2)en\mathbf{v}_{*Te}$. Here ρ is the plasma mass and $\mathcal{V}_{gv}$ represents the parallel vorticity-related part of the ion gyroviscous force $\nabla\cdot\mathbf{\Pi}_i^{gv}$, $\mathcal{V}_{gv} = -[(\nabla_{\perp} + 2\hat{\mathbf{b}}\nabla_{\|})X + (p_i/\Omega_{ci})(\hat{\mathbf{b}}\times\nabla_{\perp})\nabla_{\|}\mathbf{v}_{i\|}]$ with $X \equiv -(p_i/2\Omega_{ci})\,\hat{\mathbf{b}}\cdot\nabla\times\mathbf{v}_{i\perp}$. The continuity and temperature equations ($p_j = n_j T_j$, $n_e = n_i \equiv n$) are, with $\nabla\cdot\mathbf{J} = 0$,

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}_e) = 0 \quad (4)$$

$$\frac{\partial T_j}{\partial t} + \mathbf{v}_j\cdot\nabla T_j = -\hat{\Gamma}_j T_j \nabla\cdot\mathbf{v}_j + \nabla\cdot n\kappa_{\perp j}\nabla_{\perp}T_j + \nabla\cdot n\kappa_{\|j}\nabla_{\|}T_j - \hat{\Gamma}_j\nabla\cdot(T_j\mathbf{v}_{*Tj}), \quad (5)$$

for electrons and ions $j = e, i$, where $\hat{\Gamma}_j = \Gamma_j - 1$, Γ_j being the ratio of specific heats, and $\mathbf{v}_{*Tj}$ is the diamagnetic drift based on the temperature gradient. The neoclassical, collisional parallel viscous forces $\nabla\cdot\mathbf{\Pi}_{j\|}$, that extend the fluid model into the long parallel mean free path regime [5] are currently being tested. Writing the equations in terms of $\mathbf{v}$ introduces a few approximations in the model equations and in particular in the form chosen for the ion gyroviscous stress tensor, discussed in [1]. These should be minor for the results described here.

Parallel Compressibility

Tests of the two-fluid model showed a number of unexpected effects due to the newly introduced parallel dynamics; they are equally important in MHD.

The continuity equation introduces the possibility of sound wave propagation along the magnetic field whenever the steady state condition $\nabla \cdot n\mathbf{v}_i$ is not satisfied. This propagation condition differs from that given by the pressure equation in the absence of density evolution, whenever there is a difference between the temperature and density equilibration along the magnetic field lines (e.g., different $\kappa_{\parallel}$'s) or when the equilibrium density and temperature profiles are not simply related (e.g., by adiabaticity, $p \propto n^{\Gamma}$). A confined plasma typically encompasses both these differences. Usually the electron, and possibly the ion, temperature experiences faster parallel smoothing than the density, due to collisions, waves, or the thermal streaming of the electrons, while the density smoothing is mediated by sound waves. The parallel dynamics have much stronger effects in a torus than in a cylinder, due to the poloidal asymmetry of the flux surfaces, which also influences the poloidal plasma rotation and thereby the parallel flows through various forms of 'magnetic pumping' [6]. The plasma equilibrium profiles are also driven to deviate from a simple relation $p(n)$ relation by the different effective perpendicular conductivities and sources/sinks for the temperature and density.

The numerical results clearly illustrate the effects of the density evolution and the parallel thermal conductivity. For linear modes, the density evolution can be stabilizing or destabilizing. The strong stabilizing effect of the density equation on the 1/1 resistive mode in a torus with $R/a = 3$ has been shown for strongly and weakly ideal MHD unstable modes [9] [8]. A factor of 2 stabilization was seen for the resistive modes with weaker growth rates. The effect on the mode in a cylinder is also stabilizing, but much smaller.

The competition between the parallel thermal conductivity (stabilizing) and the density evolution (destabilizing) for a resistive ballooning MHD 3/2 mode in a torus with $R/a = 3$ and magnetic Reynold's number $S = 10^4$ is shown in Figure 1. The equilibrium density and temperature profiles are strongly peaked and the safety factor q varies from 1.1 at the magnetic axis to 5 at the edge. For the mode, the perturbed pressure and velocity stream function u contours (where $\mathbf{v} = \epsilon R \nabla u \times \hat{\phi} + \nabla_{\perp}\chi + v_{\phi}\hat{\phi}$) appear similar for the case with no density evolution ($\partial n/\partial t = 0$) and $\kappa_{\parallel} = 0$ and for the case with density evolution and relatively strong $\kappa_{\parallel}$. The latter is shown in (a). The growth rate for the $\partial n/\partial t = 0$, $\kappa_{\parallel} = 0$ case is $\gamma = 0.029$ in inverse Alfvén times $\tau_A = a/v_A$, $v_A \equiv (a/R)B_o/(4\pi n m_i)^{1/2}$. Turning on $\kappa_{\parallel}$ decouples perturbations on adjoining magnetic surfaces and stabilizes the mode, $\gamma < 0$, shown in (b). Turning on the density evolution from this state returns to the growing mode (a), but the growth rate is reduced by 1/3, to $\gamma = 0.010$.

Nonlinearly, the parallel dynamics described by the density equation also introduces important differences in the development of magnetic islands [1]. When the density evolves, the results show that both MHD and two-fluid islands can couple much more easily to islands of different toroidal mode number n as the original island grows than with a fixed density. The difference in relative island sizes between two-fluid and MHD with the same density equation were much smaller than those between an evolving or fixed density. The two-fluid islands of different mode number were somewhat more strongly coupled than in MHD when the density evolution was included, and they also had different relative phases and poloidal rotations. In the example, a 2/1 island was grown nonlinearly for a given amount of time, starting from the same initial size. With the density variation, it triggered large 3/2 and 4/3 islands, as well as the other $n = 1$ islands (3/1, 4/1, 5/1, etc.) expected from the usual toroidal mode coupling ($q_o > 1$). With an unvarying density, the companion $n = 1$ islands developed in the same amount of time, while the $n \neq 1$ islands remained very small.

Plasma Rotation

In a cylinder, plasma rotation in the poloidal direction is not strongly constrained in the steady state, due to the poloidal symmetry of the flux surfaces. A torus, however, resists poloidal rotation of the plasma, while the toroidal rotation is constrained to have the form $v_\phi = R\Omega(\psi)$. In addition to previously studied mechanisms for magnetic pumping, which require dissipation, such as an effective viscosity, to remove the rotational energy as a plasma fluid element is compressed and uncompressed in a poloidal circuit [6], the numerical simulation shows that the fluid parallel dynamics in a torus can also damp the ion poloidal rotation into toroidal motion and electromagnetic changes, even when the dissipation is small. These effects operate on both fast flows and flows on the order of the diamagnetic velocities.

For subsonic poloidal flows, $M_p \simeq v_\theta/(v_s r/Rq) < 1$, where v_s is the sound speed, poloidal motion can be converted into toroidal motion through changes in the radial electric field E_ψ over a relatively fast time scale, equivalent to several sound-wave transit times around the torus, when the parallel thermal conductivity $\kappa_{||}$ is relatively large. The magnitude of the effect depends on $\kappa_{||}$. Since the density steady state $\nabla \cdot n\mathbf{v} \neq 0$, the poloidal velocity v_θ oscillates on a fast scale and the oscillations experience a slow outward radial propagation for a longer time. The results support the analytic expectation that the equilibrium ion poloidal velocity is zero when the parallel viscosity and ion gyroviscous stress tensor are included [7].

In the case of a very large applied poloidal rotation, above a critical velocity with poloidal Mach number $M_p \gtrsim 1$, a much stronger effect can be generated [8]. The end result is a new MHD equilibrium with an apparent internal 'transport barrier,' a localized region of steepened temperature gradient around a certain radius, but it is not a transport process. Given a poloidal rotation source strong enough to drive a radial strong shear in the toroidal velocity, a new MHD equilibrium can be established, with a region of steepened temperature gradient just inside a very low order rational surface, while the poloidal velocity is greatly reduced inside this surface. This requires density evolution and a $\kappa_{||}$ large enough to keep $T \simeq T(\psi)$ roughly constant on a flux surface. It is a nondiffusive, axisymmetric process, where the magnetic flux surfaces shift in response to the toroidal velocity shear, due to conservation of the toroidal canonical momentum. Since the accompanying velocity oscillations propagate radially outward, the initial region of sharp toroidal velocity shear may also propagate outward until it reaches a low order rational surface, on which the field lines close on themselves in the fewest circuits around the torus (e.g., $q = 2$, when $q > 1$). The radial shape of the driving poloidal velocity is not important, since similar effects were triggered with velocities that increased toward the plasma edge and those that reached a maximum at mid-radius.

Two-Fluid Effects

Two-fluid effects can also have important stabilizing or destabilizing effects on mode stability. The most straightforward depend on the diamagnetic drifts of the electrons and ions and is described by the ratio of ω_{*j}/γ_o, where γ_o is the growth rate without two-fluid effects. For reconnecting modes, the ion ω_{*i} is generally stabilizing, while the electron ω_{*e} has a more complex effect, since it causes a complex radial shearing of the electron motion (i.e., the current) in the reconnection layer. In addition, the sound speed gyroradius $\rho_s = v_s/\Omega_{ci}$ is always destabilizing. The two-fluid code demonstrates the stabilizing effect of ω_{*i} on the 1/1 resistive mode in a cylinder and also the general physical mechanism of ω_{*i}-stabilization for reconnecting modes of all m [1], shown in Fig. 2. For the 1/1 resistive mode, the electron drift ω_{*e} is destabilizing. The ion ω_{*i} stabilizes the mode by causing the plasma mass inflow into the reconnection layer from $r < r_1$ to rotate poloidally relative to the reconnection X-point in the ω_{*e} direction. This results from the growth and gradual dominance of the perturbed $\tilde{v}_{*ir}$ part of the radial ion fluid velocity as ω_{*i} increases. When $\omega_{*e} = 0$, the rotation angle reaches approximately $\pi/2$ near $\omega_{*i}/\gamma_o \simeq 2$, the point where the mode reaches the maximum stabilization, i.e., the inflow is almost exactly out of phase with the reconnection (Fig. 2b). If $\omega_{*e} \neq 0$, the perturbed current J_z in the reconnection layer also shears poloidally, and the relative inflow rotation angle and stabilization effect are reduced. The poloidal direction of the velocity component $\tilde{v}_{*ir}$ is always exactly out of phase with the X-point, and has the same stabilizing effect at all m, for both cylinder and torus.

Another two-fluid diamagnetic effect is the splitting of the TAE (toroidal Alfvén eigenmode) frequency. The usual MHD TAE mode is degenerate, since two modes propagate exactly opposite to each other along the magnetic field lines at the same frequency, $\omega^2 = \omega_{MHD}^2$, $\omega_{MHD} = k_{\|}v_A$. When an ion diamagnetic drift exists, the modes separate, with the dispersion relation $\omega(\omega - \omega_{*i}) = \omega_{MHD}^2$. This has been demonstrated using MH3D-T, for a decaying mode [1]. Such modes can be excited by externally driven waves and have been seen on JT-60U and JET.

Two-fluid effects on nonlinear magnetic island evolution have been discussed under parallel compressibility, above.

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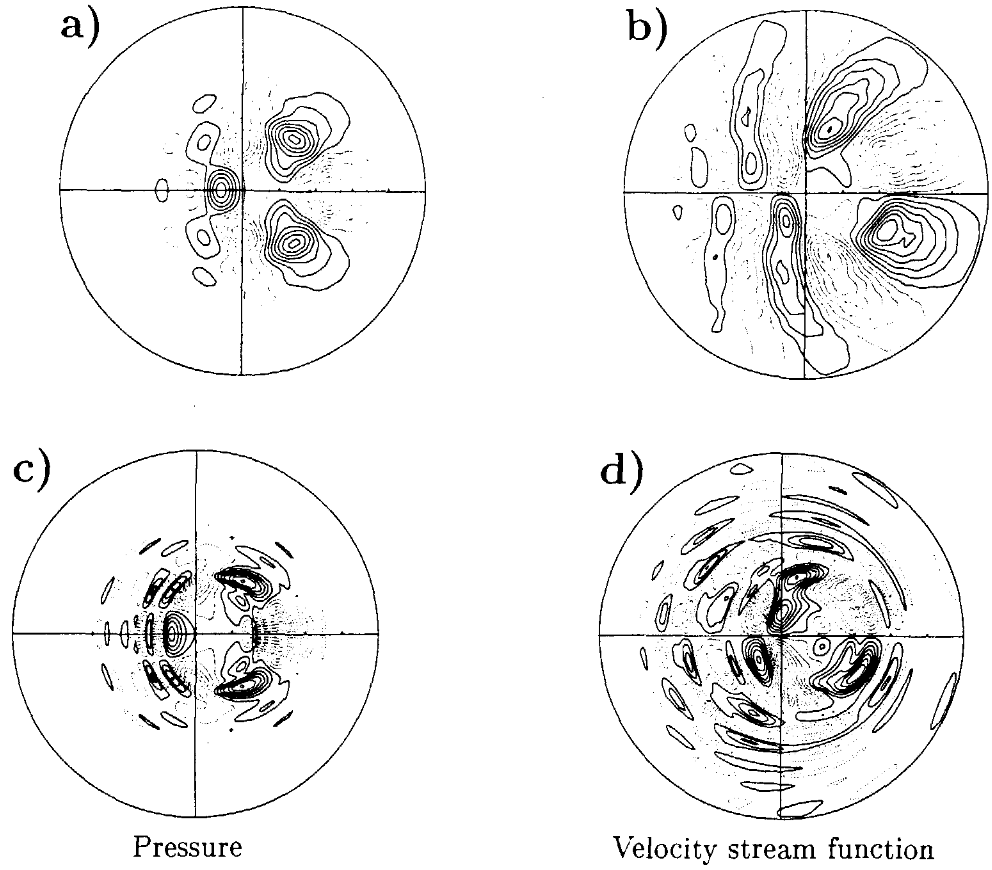

Figure 1. a) and b) Contours for resistive MHD 3/2 mode in a torus with strong $\kappa_{\|}$ and density evolution, $\gamma = 0.01$. The case with $\kappa_{\|} \simeq 0$ and $\partial n/\partial t = 0$ has similar contours, but larger $\gamma = 0.029$. c) and d) Case with large $\kappa_{\|}$ and $\partial n/\partial t = 0$ is stable, $\gamma < 0$.

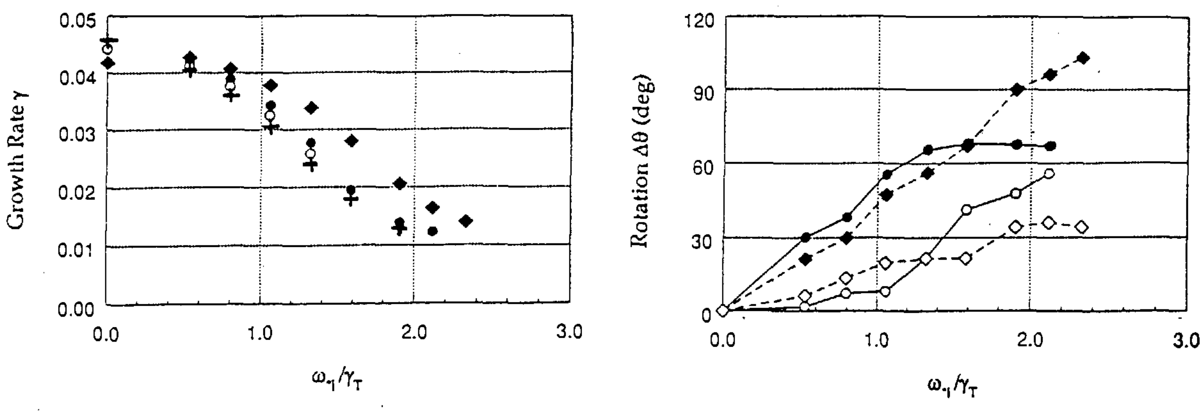

Figure 2. a) Growth rates for 1/1 resistive mode in a cylinder, showing the stabilizing effect of ω_{*i}, compared to the analytic dispersion relation (+) for $\omega_{*e} = -\omega_{*i}$ (open circles with $\kappa_{\|} = 0$ and solid circles with $\kappa_{\|} \neq 0$). The case $\omega_{*e} = 0$ is shown by diamonds. b) Relative poloidal rotation of direction of radial plasma inflow into reconnection layer from $r < r_1$, compared to the location of the reconnection X-point (ω_{*e}-direction). The ion fluid flow v_{ir} is given by the solid circles and diamonds, while the common flow v_r is given by the open symbols.