On Creating Transport Barrier by Radio-Frequency Waves

S. Sen[1,2], R. A. Cairns[3], B. Dasgupta[4] and G. Pantis[5]

1 Max Planck Institute for Plasmaphysics, EURATOM Association, 85748 Garching, Germany

2 Department of Physics, Flinders University of South Australia, GPO Box 2100, Adelaide 5001, Australia

3 School of Mathematical and Computational Sciences, University of St Andrews, St Andrews, Fife, KY16 9SS, UK

4 Saha Institute for Nuclear Physics, AF/1 Bidhannagar, Calcutta 700064, India

5 Department of Physics, University of Ioannina, Ioannina, Greece

ABSTRACT: The use of radio frequency (RF) waves in the range of Alfven frequency is shown to stabilise the drift-ballooning modes in the tokamak if the radial profile of the RF field energy is properly chosen. Stabilisation is achieved by the ponderomotive force arising due to the radial gradient in the RF field energy. The estimate of the RF power required for this stabilisation is found to be rather modest and hence should be easily obtained in the actual experiments. This result therefore shows that the use of the RF waves can create a transport barrier to reduce the loss of particle and energy from the plasma. The new improved mode produced by the RF is expected to have all the advantageous features of the enhanced reverse shear (ERS) modes and at the same time will, unlike the ERS plasma, be sustainable for unlimited period of time and hence should be an attractive choice for the reactor-grade self-sustaining plasma.

The economic viability of a tokamak as a leading contender for a fusion reactor crucially depends on the development of a magnetic configuration that has good confinement and stability and a large fraction of bootstrap current. Understanding and control of turbulent transport and of its underlying driving agents is therefore a prerequisite in this process. Recent discoveries of various enhanced performance operational regimes like the H-modes [1], the VH-modes [2] and the enhanced reverse shear modes (ERS-modes) [3-4] has opened up a new window for improved tokamak operation. It is usually believed that the ERS configuration can provide the characteristics desirable for a fusion reactor [5].

One of the remaining challenges for enhanced tokamak operation is the development and understanding of the basic physics involved in the process that leads to the transition to the improved confinement modes. While a sheared poloidal (toroidal) flow is found to be responsible for the H- (VH-) modes, a hollow q profile (hence normally a hollow current profile) is necessary for the ERS modes. Most tokamaks however operate with inductive current drive which in general produces a peaked current density profile at the magnetic axis because of the strong dependence of the plasma conductivity on the electron temperature. Only by noninductive current drive or transient techniques can a hollow current density profile be generated.

Recent experiments in the PRX-M indicate that if NBI heated plasma is further heated up by ion Bernstein waves (IBW) there is a significant increase in the core plasma temperature and hence in the confinement, the so-called CH-modes [6]. This raises interest in whether RF waves can be used to create the transport barrier to reduce the loss of particles and energy from the plasma. The final run of experiments in the TFTR were originally planned to investigate this key issue. From the theoretical point of view, there is already an indication that RF waves can stabilise ballooning type MHD instabilities [7]. However, no such investigation on the effect of the RF waves to our knowledge has been carried out for the drift-type microinstabilities, stabilisation of which is essential for the formation of the transport barrier.

In this work we suggest that the transport barrier may be created by suitably launching RF waves in the Alfven range of frequencies. We demonstrate that RF waves through ponderomotive force arising due to the radial gradient in the RF field energy stabilises drift-ballooning modes, which have been identified as the likely mechanism for anomalous transport in the plasma [8, 9]. We also demonstrate that the stabilisation can be achieved for rather modest values of the RF power and hence should be easily obtained in the actual experiments. The new improved mode produced by the RF will, unlike the ERS modes, be non-transient in nature and should be sustainable for unlimited period of time. This mode thus is free from the main shortcoming of the ERS plasma and should be an attractive choice for the fusion reactor.

A nonlocal stability analysis for the collisionless drift-ballooning modes is carried out on the basis of a simple two-fluid model. We use the usual (r, θ, ξ) coordinates, corresponding to the minor radial, poloidal and toroidal directions respectly and consider the long-wavelength $(k_\theta^2 \rho_s^2 \ll 1)$ drift waves for a large aspect-ratio circular tokamak. The perturbed potential can then be expressed as:

$$\phi(\vec{x}, t) \equiv \phi(\rho, \theta, \xi, t) = \phi(\rho) exp[i(n\xi - m_0\theta - \omega t)],$$

where $\rho = (r - r_0)$, r_0 is the radius of the reference mode rational surface, i.e., $m_0 = nq(r_0)$, $k_\theta = \frac{m_0}{r_0}$, and $\hat{s} = \frac{rq'}{q}$ at $r = r_0$. Here, for simplicity we will assume the ions to be cold and will ignore the electron temperature gradient. Using fluid descriptions the eigenvalue equation in the presence of RF ponderomotive force can be derived in a straightforward way [10-11]

$$[\rho_s^2(\frac{\partial^2}{\partial \rho^2} - k_\theta^2) + \frac{(\omega_e^* - \omega)}{\omega} - \frac{{\omega_e}^{*2}}{\omega^2}(\frac{\epsilon_c}{k_\theta \rho_s})^2 \times$$

$$(\frac{\partial}{\partial \theta} + ik_\theta \rho \hat{s})^2 - 2\epsilon_n \frac{\omega_e^*}{\omega}(cos\theta + \frac{isin\theta}{k_\theta}\frac{\partial}{\partial \rho}) - \frac{1}{2}\frac{k_\parallel}{\omega^2}\frac{d}{d\rho}|V_{RF}(\rho)|^2 -$$

$$\frac{\rho^2 \omega_e^*}{L_*^2}]\phi = 0 \qquad (1)$$

where $\rho_s^2 = \frac{C_S^2}{\omega_{ci}^2}, C_S^2 = \frac{T_e}{m_i}, \epsilon_n = q\epsilon_c = L_n/R$, $\omega_e^*(\rho)$ the diamagnetic drift frequency and L_* the density gradient variation scale length and typically of the order of density scale length L_n. $\vec{V}_{RF}(\rho)$ is the induced oscillating fluid velocity in the presence of an external RF field. The ponderomotive contribution due to the RF field enters the force balance equation through the term $mn < (\vec{V}_{RF}.\nabla)\vec{V}_{RF}^* >$ and can be simplified to $\frac{1}{2}mn\frac{d}{d\rho}|V_{RF}|^2$ assuming that the ponderomotive forces are radially symmetric [12].

The first term in equation (1) arises from the finite Larmor radius effect and the third from the ion-sound. The fourth is the effect of toroidal coupling whereas the last term introduces the radial variation of ω_e^* with $L_*^{-2} = (\frac{1}{\omega_e^*})(\frac{d^2\omega_e^*}{d\rho^2})$. We assume a simple general case of the variation of $\vec{V}_{RF}(\rho)$ with the radial distance such that $|V_{RF}(\rho)|^2 = \frac{|V_{RFO}|^2\rho}{L_{RF}} + \frac{|V_{RFO}|^2\rho^2}{L_{RF}^2}$, where $\frac{dV_{RF}}{d\rho} = \frac{V_{RFO}}{L_{RF}}$ and $\frac{1}{2}\frac{d^2V_{RF}}{d\rho^2} = \frac{V_{RFO}}{L_{RF}^2}$.

To reduce the 2 dimensional (2D) eigenmode problem to 1 dimensional (1D), we will apply the ballooning transformation. To determine the radial mode structure the solution of the fully two-dimensional eigenmode problem must be obtained. Within the framework of ballooning formalism, this needs solving the problem to higher order. The problem then separates into two distinctive radial length scales. To leading order the problem reduces to the usual 1D eigenmode equation (with radial variable appearing only as parameter) which determines the mode structure along the magnetic field lines. The next order equation then determines the radial mode structure. In the usual theory of high n ballooning modes [13] one maps the poloidal angle θ on to an extended coordinate χ with $-\infty < \chi < \infty$ and writes the perturbation in the form

$$\phi(\theta, x) = \sum_m e^{-im\theta} \int_{-\infty}^{\infty} e^{im\chi}\hat{\phi}(\chi, x)d\chi, \quad \text{where, } \hat{\phi} = A(x)F(\chi, x)\ exp[-ix(\chi + \chi_0)]$$

where χ_o is an arbitrary phase of the eikonal and $x = k_\theta\rho\hat{s}$. Here, $A(x)$ is assumed to vary on some scale intermediate between the equilibrium scale length and the perpendicular wavelength. Now to leading order (in $n^{-1/2}$ expansion) the ballooning equation becomes

$$[\frac{\omega_e^{*2}}{\omega^2}(\frac{\epsilon_c}{k_\theta\rho_s})^2\frac{\partial^2}{\partial\chi^2} + \frac{1}{2}\frac{1}{\omega^2 qR}\frac{d}{d\rho}|V_{RF}(\rho)|^2\frac{\partial}{\partial\chi} - (\omega_e^* - \omega)/\omega + 2\epsilon_n\frac{\omega_e^*}{\omega}(cos\chi + \hat{s}(\chi + \chi_o)sin\chi) +$$

$$(k_\theta\rho)^2(1 + \hat{s}^2(\chi + \chi_o)^2) + \frac{\rho^2\omega_e^*}{L_*^2\omega}]F = 0 \qquad (2)$$

The first derivative (with respect to χ) term can be removed by suitably changing the dependent variable (to ξ). Equation (2) then reduces to

$$[\frac{\omega_e^{*2}}{\omega^2}(\frac{\epsilon_c}{k_\theta\rho_s})^2\frac{\partial^2}{\partial\chi^2} - (\omega_e^* - \omega)/\omega + 2\epsilon_n\frac{\omega_e^*}{\omega}(cos\chi + \hat{s}(\chi+\chi_o)sin\chi) - \frac{1}{4}(\frac{1}{2}\frac{1}{\omega^2 qR}\frac{d}{d\rho}|V_{RF}(\rho)|^2)^2/\frac{\omega_e^{*2}}{\omega^2}(\frac{\epsilon_c}{k_\theta\rho_s})^2$$

$$+(k_\theta\rho)^2(1 + \hat{s}^2(\chi + \chi_o)^2) + \frac{\rho^2\omega_e^*}{L_*^2\omega}]\xi = 0 \qquad (3)$$

where,

$$\xi = Fexp[\frac{1}{4\omega^2 qR}(-\frac{|V_{RFO}|^2 x}{L_{RF}} - \frac{|V_{RFO}|^2 x^2}{2L_{RF}^2})] \qquad (4)$$

Now to explore the implication of the RF field profile on the radial structure and on the stability of the modes one needs the higher order ballooning theory. In the higher order theory χ_o is obtained from the equation $\frac{\partial\Omega}{\partial\chi_o}(x, \chi_0) = 0$ and the radial envelope function $A(x)$ satisfies

$$\frac{\partial^2\Omega}{\partial\chi_0^2}\cdot\frac{d^2A}{dx^2} + [2(\Omega - \Omega_0) - p_1 - 2p_2x^2]A = 0, \qquad (5)$$

where, $\Omega = \frac{1}{k_\theta^2 \rho_s^2 \hat{s}^2}[\frac{\omega_e^*}{\omega} - 1 - k_\theta^2 \rho_s^2]$, $\Omega_o = i\frac{\epsilon_c}{k_\theta^2 \rho_s^2 \hat{s}}$, $p_1 = \frac{1}{2}(\frac{V_{RFO}^2}{\omega^2 q R L_{RF}})^2 / \frac{\omega_e^{*2}}{\omega^2}(\frac{\epsilon_c}{k_\theta \rho_s})^2$, $p_2 = \frac{1}{k_\theta^4 \rho_s^2 \hat{s}^4}(\frac{1}{L_n^2} - (\frac{V_{RFO}^2}{C_s^2}\frac{L_n}{L_{RF}^2}\frac{1}{\rho_s k_\theta})^2)$, $x = k_\theta \rho \hat{s}$. Equation (5) is a simple Weber equation. When p_2 is positive and $\partial^2\Omega/\partial\chi_o^2 > 0$ ($\partial^2\Omega/\partial\chi_o^2 > 0$ is necessary in order that the mode be most unstable [13]), $A(x)$ is a localised Gaussian function. However, an important change is introduced by the ponderomotive contribution. Assuming $L_n \sim 3cm$, $L_{RF} \sim 1cm$ and taking $k_\theta \rho_s \sim 0.1$, we find that p_2 becomes negative for $V_{RFO}/C_s \sim 10^{-1}$ (which means that the RF power is only a small fraction of the thermal power. Note also that this is the RF power at the resonance, which is much larger than the overall average RF power). It is important to note that the length scale for the RF wave is difficult to give precisely since the intensity has a singularity in ideal MHD theory, and the actual width predicted will depend on what sort of resistive damping or finite Larmor radius effects are used to eliminate the singularity. It is also interesting to note that to consider the most unstable situation we have assumed L_n rather small, which however is expected in the region where transport barrier is formed. With p_2 negative $A(x)$ is then given by

$$A(x) = exp[-i\frac{1}{2}(\mid p_2 \mid / \mid \frac{\partial^2\Omega}{\partial\chi_0^2} \mid)^{1/2} x^2] \tag{6}$$

So, the mode envelope is now radially outgoing which is reminiscent of the equivalent slab problem [14]. The quadratic x variation of the RF field energy in the toroidal problem like the magnetic shear in the corresponding slab problem creates an *antiwell* in the radial direction (instead of the well formed by the diamagnetic frequency). The wave energy is therefore convected outward. The eigenvalue is given by

$$\Omega = \Omega_0 + \frac{p_1}{2} - i\frac{1}{2}(p_2 \mid \frac{\partial^2\Omega}{\partial\chi_0^2} \mid)^{1/2} \tag{7}$$

The negative imaginary contribution in the eigenvalue shows that the RF field profile *stabilises* toroidal drift waves which otherwise escape magnetic shear damping! The actual stability of the mode will be determined by the relative magnitude of the destabilising contribution (Ω_o) and the stabilising contribution of the third term in the RHS of equation (7). Choosing, as before, $L_n \sim 3cm, q = 3, \hat{s} \sim 1, R = 1.5m, k_\theta\rho_s \sim 0.1, \lambda_\theta \sim 1cm$, $L_\Omega[= (\frac{1}{\Omega}\frac{\partial\Omega}{\partial\chi_o})^{-1}] \sim 1 radian$, and $\Omega \sim 1$, it is easy to see that the complete stabilisation of the mode is actually possible. It is interesting to note that the sign and the magnitude of the linear variation of the RF energy has no direct effect on the stability. The linear term shifts the potential, but does not alter the quadratic structure. It also shifts the centre of the mode away from the x=0 rational surface. The main stabilising effect comes from the quadratic term which forms an additional *antiwell* which pushes the wave function away from x=0, thus increasing the stabilising effect and weakening the driving term simultaneously. Also important to note that the theory assumes that the RF wave is producing an oscillating fluid velocity, so this points towards a low frequency wave. One possibility is to use the Alfven resonance, which produces highly localised intensity maxima which coincide closely with flux surface [15].

We will now discuss the effect of the ponderomotive force on the turbulent transport coefficient by using the mixing length theory, according to which the turbulent diffusivity is given by

$$\chi_k = \gamma_k \triangle_k^2$$

where γ_k is the mode growth rate and $\triangle_k$ is the mode width. We have already seen that the growth rate of the linear mode is reduced in the presence of the RF field. It is also easy to see that the radial width of the modes related to the individual magnetic surfaces is also reduced. To see this we note that in deriving Equation (3) we have removed the first derivative (w.r. to χ) term by suitably changing the dependent variable (from F) to ξ. The relation between F and ξ is given by equation (4). So as evident from equation (4) the radial width of the individual mode is reduced due to the presence of the ponderomotive force. So, the corresponding transport is also expected to be reduced by the ponderomotive force. Coming to the global transport, at the first instant it seems difficult to reach a definitive conclusion about it. This is because, whereas the growth rate of the linear mode decreases, its global mode width [given by Equation (6)] increases. However, as has been mentioned by Sen [11] that there may be some additional processes (such as electron Landau damping and fluid resonance) determining a finite mode width rather than the outgoing wave situation which otherwise holds in this situation. Hence the corresponding global transport is also expected to be reduced in the presence of the ponderomotive force.

In summary, we have investigated the effect of the RF waves on the stability of the collisionless drift-ballooning modes, which have been identified as the key mechanism for the anomalous transport. Our full analytic stability analysis shows that RF waves in the Alfven range of frequencies can have a significant effect on the stability of the drift-ballooning modes if the radial profile of the RF field energy is properly chosen. This stabilisation is achieved through the ponderomotive force arising due to the radial gradients in the RF field energy and is different from the stabilisation mechanism due to RF-induced poloidal-shear-flow which is thought to be responsible in the case of IBW induced CH modes [6]. The estimate shows that stabilisation can be achieved for rather modest values of the RF power and should be easily obtained in the actual experiments. This result therefore shows that the use of RF waves can create a transport barrier to reduce the loss of particle and energy from plasma and as there is no time limit for how long RF power can be launched, a stationary, non-transient improved mode can be envisioned in the plasma core by this technique. This mode is expected to have all the advantageous features of the ERS plasma and at the same time will, unlike the ERS mode, be non-transient in nature and should in principle be sustainable for unlimited period of time.

Before concluding it is important to mention several simplifying assumptions that have been made which may restrict the scope of our results. Among the effects that we have not considerd here are the nonlinear contributions of the RF waves, in particular the sideband coupling contributions have been neglected. Contributions coming from these terms are however negligible for the limit considered by us $L_n > L_{RF}$ [7]. We have also assumed that the ponderomotive forces due to the RF field is radially symmetric. However, it has been pointed out by Appert et al. [15] that toroidal effects do not greatly effect the position of the Alfven resonance, so this is a reasonable assumption. Furthermore, we have neglected the modification of the equilibrium profiles in the presence of the RF waves so that our results are limited to the low RF power levels. Notwithstanding these restrictions, however, the basic stabilising mechanism from the RF ponderomotive force is quite unambiguously shown by our simple model and is not expected to be seriously modified by any of the above simplifications.

We would like to thank D. Biskamp, J. W. Connor, R. L. Dewar, D. Pfirsch and H. R. Wilson for useful discussions. Thanks are also due to the TEXTOR team in particular to G. Van Oost and R. Koch for discussions on the experimental feasibility of the theory developed in this work. S. Sen acknowledges the hospitality provided at the Max-Planck Institute for Plasma Physics, Garching and the financial support through the mobility contract No. 131-83-7 FUSC.

References

[1] F. Wagner et al., Phys. Rev. Lett., **49**, 1408 (1982)

[2] G. L. Jackson et al., Phys. Rev. Lett., **67**, 3098 (1991)

[3] F. M. Levinton et al., Phys. Rev. Lett., **75**, 4417 (1995)

[4] E. J. Strait et al., Phys. Rev. Lett., **75**, 4421 (1995)

[5] C. B. Forest et al., Phys. Rev. Lett., **77**, 3141 (1996)

[6] M. Ono et al., (IAEA-CN-60/A-3-I-7, 15th International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Sivelle, Spain, 1994), IAEA, Vienna (1996)

[7] D. A. D'Ippolito et al., Phys. Rev. Lett., **58**, 2216 (1987)

[8] D. Biskamp and A. Zeiler, Phys. Rev. Lett., **74**, 706 (1995)

[9] J. F. Drake et al., Phys. Rev. Lett., **77**, 494 (1996)

[10] W. Horton et al., Phys. Fluids, **21**, 1366 (1978)

[11] S. Sen, Plasma Phys. Controlled Fus., **37**, 95 (1995)

[12] S. Sen et al., Phys. Lett. A, **157**, 411 (1991)

[13] J W Connor, R J Hastie and J B Taylor, Proc. of Royal Society A, **365**, 1 (1979)

[14] L D Pearlstein and H L Berk, Phy. Rev. Lett., **23**, 220 (1969)

[15] K. Appert et al., Nuc. Fus., **22**, 903 (1983)