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Abstract

We show that the spin-orbit potential of the nuclear mean field destroys isoscalar superfluid correlations in self-conjugate nuclei. Using group theory and boson mapping techniques on a hamiltonian including single particle splittings and a $SO_{ST}(8)$ pairing interaction, we give analytical expressions for the spin-orbit dependence of some $N=Z$ properties such as the relative position of $T=0$ and $T=1$ states in odd-odd sytems or double binding-energy differences of even-even nuclei.

With the advent of radioactive beams an area of intense research in nuclear physics concerns the structure of exotic systems with roughly equal numbers of neutrons and protons ($N \approx Z$). These nuclei, close to the proton stability line and of importance in astrophysical nucleosynthesis, might in addition develop a proton-neutron superfluidity which is currently compared to pairing correlations between like nucleons via shell-model calculations [1] or mean field approximations [2]. In these studies, the main motivation is to identify what to expect in a regime where isoscalar and isovector Cooper pairs can coexist. In particular, there are many attemps to isolate the specific role of $T=0$ pairing on some features of the $N=Z$ line, such as binding-energy singularities or, as the mass increases, the transition from a $T=0$ to a $T=1$ ground state in odd-odd nuclei.

Our study is also focussed on such pairing competition but with particular attention to the shell structure of the space where the superfluid correlations take place. More precisely, it will be shown that the spin-orbit potential disfavours proton-neutron pairs with parallels spins ($T=0$ pairs). For this purpose, we consider a simple model hamiltonian H that incorporates single particle energies and different modes of pairing :

$$H = \sum_{\rho} \varepsilon_{\rho} \sqrt{2(2j+1)} \left[a_{\rho}^{+} \otimes \tilde{a}_{\rho} \right]_{0,0}^{0,0} + V_P \tag{1}$$

where $\rho=(n,l,j)$ denotes an orbit of a harmonic oscillator shell and $\varepsilon_\rho=\varepsilon^{(0)}_{n,l}-\frac{V_{so}}{2}\left[j(j+1)-l(l+1)-\frac{3}{4}\right]$ is its energy in presence of a spin-orbit potential $-V_{so}\,\vec{l}.\vec{s}$ $(V_{so}>0)$. In addition, V_P is the residual $SO_{ST}(8)$ pairing interaction [3] given by :

$$V_P=-\frac{g\Omega(1+x)}{2}A^+_{0,1}.\tilde{A}_{0,1}-\frac{g\Omega(1-x)}{2}A^+_{1,0}.\tilde{A}_{1,0} \tag{2}$$

with $\Omega=\sum_l(2l+1)$ the orbital degeneracy of the valence space and A^+_{S,M_S,T,M_T} the collective pairs $\frac{1}{\sqrt{2\Omega}}\sum_l\sqrt{2l+1}\left[a^+_{l,1/2,1/2}\otimes a^+_{l,1/2,1/2}\right]^{L=0,S,T}_{M_L=0,M_S,M_T}$. Moreover, g is the overall strength of the pairing force while x is a parameter that controls the relative importance of the $T=0$ and $T=1$ pairing. The interaction (2) has been used in recent calculations concerning for example the role of the isoscalar pairing channel in the Gamow-Teller β decay [4] or in the study of α-like cluster structures along the $N=Z$ line [5]. In addition, an analysis of realistic G-matrix interactions [6] has also shown that its pairing terms are only characterized by an $L=0$ character given precisely by equation (2).

The solution of the eigenvalue problem asociated with the full hamiltonian (1) in a large space, like the *fp* shell, requires substantial numerical effort that can be avoided via boson mapping techniques [7]. The main idea of these approaches is to map bi-fermion operators onto a boson algebra in such a way as to preserve the physics of the original fermion problem. For example, one mapping is given by the non hermitian Dyson expansion [8] :

$$a^+_{n_1}a^+_{n_2}\rightarrow\sum_\omega Y^\omega_{n_1,n_2}b^+_\omega-\sum_{\omega_1,\omega_2,\omega_3}Y^{\omega_1}_{n_1,n_3}Y^{\omega_2}_{n_2,n_4}Y^{\omega_3}_{n_3,n_4}b^+_{\omega_1}b^+_{\omega_2}b_{\omega_3} \tag{3.a}$$

$$a_{n_1}a_{n_2}\rightarrow\sum_\omega Y^\omega_{n_2,n_1}b_\omega \tag{3.b}$$

$$a^+_{n_1}a_{n_2}\rightarrow\sum_{\omega_1,\omega_2}Y^{\omega_1}_{n_1,n_3}Y^{\omega_2}_{n_2,n_3}b^+_{\omega_1}b_{\omega_2} \tag{3.c}$$

where b^+_ω denotes boson operators associated with collective fermion pairs $A^+_\omega=\frac{1}{2}\sum_{n_1,n_2}Y^\omega_{n_1,n_2}a^+_{n_1}a^+_{n_2}$.

Note that these boson images make use of all the two-fermion degrees of freedom and as a consequence the diagonalization does not become easier in the boson space. However, matters simplify if the hamiltonian can be expressed only in terms of collective pairs $A^+_{\underline{\omega}}$ that are algebraically closed $\left(\left[\left[A^+_{\underline{\omega}_1},A^+_{\underline{\omega}_2}\right],A^+_{\underline{\omega}_3}\right]=\sum_{\underline{\omega}_4}\Gamma^{\underline{\omega}_4}_{\underline{\omega}_1,\underline{\omega}_2,\underline{\omega}_3}A^+_{\underline{\omega}_4}\right)$. In this case, an exact boson reproduction of the collective eigenspectrum can be obtained using a Dyson expansion followed by the elimination of all terms that contain bosons other than b_{ω}^{+} (this is the so-called "skeletonisation theorem" [9]). In our case, the $SO_{ST}(8)$ dynamical invariance of V_P guarantees a perfect decoupling of all states built from A^{+}_{S,M_S,T,M_T} $[(S,T)=(0,1)$ or $(1,0)]$ operators. We can thus construct a perfect realization $(V_P)_B$ of the pairing interaction (2) in terms of a $U(6)$ algebra formed by $J=0,T=1$ bosons (s^{+}) and $J=1,T=0$ bosons (p^{+}) associated with the isovector $A^{+}_{S=0,M_S=0,T=1,M_T=\mu}$ and the isoscalar $A^{+}_{S=1,M_S=\nu,T=0,M_T=0}$ pairs respectively. After a naïve hermitization by an arithmetic mean of $(V_P)_B$ and $(V_P)_B^{+}$, one obtains [5] :

$$(V_P)_B^{Herm.}=-\frac{11g}{4}\hat{\mathcal{N}}+\frac{g}{4}\hat{\mathcal{N}}(\hat{\mathcal{N}}+5)-\left[\frac{g\Omega(1+x)}{2}+\frac{3gx}{2}\right]\hat{\mathcal{N}}_s+\frac{gx}{4}\hat{\mathcal{N}}_s(\hat{\mathcal{N}}_s+3)$$
$$-\left[\frac{g\Omega(1-x)}{2}-\frac{3gx}{2}\right]\hat{\mathcal{N}}_p-\frac{gx}{4}\hat{\mathcal{N}}_p(\hat{\mathcal{N}}_p+3)+\frac{g}{4}C_2[SU(4)]+\frac{gx}{4}(\hat{T}^2-\hat{S}^2) \qquad (4)$$

where $\hat{\mathcal{N}}$ is the total number of bosons, $\hat{\mathcal{N}}_s$ and $\hat{\mathcal{N}}_p$ the number operators of s and p bosons and $C_2[SU(4)]$ the quadratic invariant associated to the Wigner subgroup $SU(4)$ of the boson $U(6)$ group.

With non-zero single particle splittings, this boson analysis is not so simple. First, energetically favoured pairs are no longer given by A^{+}_{S,M_S,T,M_T} and it becomes necessary to bosonize the lowest eigenstates of the two-particle space. For relatively small values of the spin-orbit coupling, these eigenstates carry the quantum numbers $(J,T)=(0,1)$ and $(1,0)$ and will thus be associated with new s^{+} and p^{+} bosons. In addition, such pairs do not verify the closure hypothesis and so the simple truncation of the Dyson expansion to (s,p) bosons is not exact. However, it remains an excellent approximation for the lowest $J=0,T=0$ level in even-even $N=Z$ nuclei and for the first $J=0,T=1$ or $J=1,T=0$ state in odd-odd self-conjugate systems. This is shown in table 1 where a comparison is made with the exact fermionic results in the sd space. All the results concern $N=Z$ nuclei with a gap $\varepsilon_{1d}^{(0)}-\varepsilon_{2s}^{(0)}=1\,MeV$, a spin-orbit strength $V_{so}=2\,MeV$ and an asymmetry $x=0.8$ between $T=0$ and $T=1$ pairing.

To see the influence of the shell organization on the superfluid structure of heavy $N=Z$ nuclei, the (s,p) boson realization of the hamiltonian (1) has been systematically studied in the fp shell for different values of the spin-orbit strength V_{so} and of the asymmetry x between the two pairing modes. In all calculations we use $\varepsilon_{2p}^{(0)} - \varepsilon_{1f}^{(0)} = 1\,MeV$ and a total pairing strength $g = 0.7\ MeV$ close to the value deduced from the KB3 interaction by Dufour & Zuker [6]. As a first result, the energy difference beween the lowest $J = 0, T = 1$ and $J = 1, T = 0$ states in $N = Z$ odd-odd nuclei is shown in figure 1. If isoscalar superfluid correlations are dominant $(x < 0)$ or if both pairings are of equal strength $(x = 0)$, the $J = 0, T = 1$ level is favoured by the the spin-orbit coupling and progressively more so as the particle number increases. On the other side, if isovector pairing is more important $(x > 0)$, the spin-orbit coupling has a negligible effect on the relative position of $T = 0$ and $T = 1$ states in odd-odd $N = Z$ nuclei. These conclusions are also confirmed by the barchart representations of figure 2 concerning the pair structure of the lowest $J = 0, T = 1$ and $J = 1, T = 0$ states in ^{50}Mn $(N = Z = 25)$. Again, it appears clearly that the number of proton-neutron pairs with parallels spins decreases when a spin-orbit term is introduced. In fact, a similar result holds also for the $J = 0, T = 0$ ground state of even-even $N = Z$ systems.

To have a better understanding of the effects induced by the spin-orbit mean-field coupling on nuclear superfluid properties at low isospin, an analytical study can be done assuming that the influence of the $\vec{l}.\vec{s}$ potential can be treated in a perturbative way. In this case, assuming that s^+ and p^+ bosons always correspond to the $SO_{ST}(8)$ pairs A^+_{S,M_S,T,M_T}, the boson interaction remains given by the two body terms of (4) while the one-body part becomes $\varepsilon_{0,1}\hat{\mathcal{N}}_s + \varepsilon_{1,0}\hat{\mathcal{N}}_p$ with single energies $\varepsilon_{S,T}$ obtained from perturbation theory on unperturbed states A^+_{S,M_S,T,M_T}. Up to second order in V_{so}, it can be shown that :

$$\varepsilon_{0,1} = -\frac{g\Omega(1+x)}{2} - \frac{8\alpha V_{so}^2}{g\Omega(1+x)},\ \alpha = \sum_l l(l+1) \tag{5}$$

and

$$\varepsilon_{1,0} = -\frac{g\Omega(1-x)}{2} - \frac{8\beta V_{so}^2}{3g\Omega^2(1-x)},\ \beta = \sum_l \frac{l(l+1)\left(4l^2+4l-1\right)^2}{(2l+1)^3} \tag{6}$$

If, finally, the two pairing channels are assumed to have nearly the same intensity, we obtain a boson hamiltonian H_B that separates into an $SU(4)$ invariant part H_o and an hermitian perturbation V depending on the superfluidity asymmetry parameter x and on the strength V_{so} of the spin-orbit coupling :

$$H_B = H_o + V \tag{7}$$

with

$$H_o = -g\left(\frac{11}{4} + \frac{\Omega}{2}\right)\hat{\mathcal{N}} + \frac{g}{4}\hat{\mathcal{N}}\left(\hat{\mathcal{N}} + 5\right) + \frac{g}{4}C_2[SU(4)] \tag{8}$$

and :

$$V=\frac{gx(\Omega+3)}{2}\left(\hat{\mathcal{N}}_p-\hat{\mathcal{N}}_s\right)+\frac{gx}{4}\hat{\mathcal{N}}_s\left(\hat{\mathcal{N}}_s+3\right)-\frac{gx}{4}\hat{\mathcal{N}}_p\left(\hat{\mathcal{N}}_p+3\right)-\frac{8\alpha V_{so}^2\,(1-x)}{g\Omega}\hat{\mathcal{N}}_s-\frac{8\beta V_{so}^2\,(1+x)}{3g\Omega^2}\hat{\mathcal{N}}_p \tag{9}$$

Using the results given in [5] concerning the expansion of $SU(4)$ states on a basis that preserves the numbers $\left(\mathcal{N}_s,\mathcal{N}_p\right)$, it then becomes straightforward to obtain the expectation value of V in the eigenstates of H_o of interest. For example, in an odd-odd $N=Z$ nucleus, one finds that the difference ΔE between the lowest $J=0, T=1$ and $J=1, T=0$ is, in first order, given by :

$$\begin{aligned}\Delta E &= E(J=0,T=1)-E(J=1,T=0)\\ &= gx-\frac{gx}{4}(\mathcal{N}+3)(\Omega+3)+\frac{gx}{8}(\mathcal{N}+3)^2-\frac{2V_{so}^2}{g\Omega}(\mathcal{N}+3)\left[\left(\alpha-\frac{\beta}{3\Omega}\right)-x\left(\alpha+\frac{\beta}{3\Omega}\right)\right]\end{aligned} \tag{10}$$

This expression explains the parabolic behavior of ΔE versus the number $\mathcal{N}$ of bosons that was observed in the numerical results of figure 1. It also shows how the spin-orbit coupling acts to fix the isospin of the ground state in odd-odd $N=Z$ structures : with equal $T=0$ and $T=1$ pairing, $(x=0)$ equation (10) becomes

$$\Delta E=-\frac{2V_{so}^2(\mathcal{N}+3)}{g\Omega}\left(\alpha-\frac{\beta}{3\Omega}\right)<0 \text{ since } \alpha>\frac{\beta}{3\Omega} \tag{11}$$

and as a consequence, we find that a small spin-orbit potential always breaks the $SU(4)$ symmetry, where $T=0$ and $T=1$ levels would be degenerate, in favour of the isovector state and with a linear mass dependence.

Finally, the boson hamiltonian (7) also allows to give a new insight in the problem of mass anomalies along the $N=Z$ line. In particular, such singularities appear in the double binding energy differences [10] defined, for example, in even-even nuclei by [11] :

$$\delta V_{np}(N,Z)=\frac{1}{4}\left[B(N,Z)-B(N,Z-2)-B(N-2,Z)+B(N-2,Z-2)\right] \tag{12}$$

where B is the binding energy. The experimental values of this indicator are reproduced in figure 3 where one observes an enhancement of δV_{np} at $N=Z$ and which can be interpretated as a remnant of $SU(4)$ symmetry [10]. Our purpose is here to look at the evolution of δV_{np} in the context of the boson hamiltonian (7) treated by perturbation theory. In first order, the ground-state energy $B(\mathcal{N},T)$ of an even-even nucleus of $\mathcal{N}$ bosons and isospin T is found to be :

$$B(\mathcal{N},T) = -g\left(\frac{11}{4}+\frac{\Omega}{2}\right)\mathcal{N} + \frac{g}{4}\mathcal{N}(\mathcal{N}+5) + \frac{g}{4}T(T+4) - \frac{4V_{so}^2}{g\Omega}\left[\frac{T(\mathcal{N}+3)}{T+3}\left(\alpha-\frac{\beta}{3\Omega}\right)+\mathcal{N}\left(\alpha+\frac{\beta}{3\Omega}\right)\right] \quad (13)$$

where only the case of equal $T=0$ and $T=1$ pairing has been considered. For the double binding energy differences, one obtains :

$$\delta V_{np}(\mathcal{N},T\neq 0) = -\frac{6V_{so}^2}{g\Omega}\frac{\mathcal{N}+2}{(T+2)(T+3)(T+4)}\left(\alpha-\frac{\beta}{3\Omega}\right)\ (N\neq Z \text{ nuclei}) \quad (14)$$

$$\delta V_{np}(\mathcal{N},T=0) = -\frac{3g}{8}+\frac{2V_{so}^2(\mathcal{N}+3)}{g\Omega}\left(\alpha-\frac{\beta}{3\Omega}\right)\ (N=Z \text{ nuclei}) \quad (15)$$

We thus obtain a singularity at $N=Z$ reduced by the spin-orbit coupling and which behaves linearly in an harmonic oscillator shell with a smaller slope as larger spaces are considered. All these points are observed to a good extent in the experimental values of δV_{np} (figure 3). This shows that many features of δV_{np} are explained in terms of a $SU(4)$ breaking induced by the spin-orbit potential.

In conclusion, we have studied in this letter the effects of a spin-orbit coupling on the competition between the $T=0$ and $T=1$ superfluidity in exotic nuclear systems at low isospin. Using a schematic hamiltonian solved by a skeletonized Dyson boson expansion, a clear destruction by the spin-orbit potential of isoscalar proton-neutron correlations in $N=Z$ nuclei has been found. Its consequences can qualitatively explain, for the first time, the observed change from $T=0$ to $T=1$ in the ground state isospin of odd-odd self-conjugate nuclei. They also shed light on the deviations from $SU(4)$ symmetry of the mass singularites in $N=Z$ systems.

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References

[1] Poves A, Martinez-Pinedo G, Phys. Lett. B 430 (1998) 203
[2] Goodman A L, Phys. Rev. C 60 (1999) 014311
[3] Flowers B H, Szpikowski S, Proc. Phys. Soc. 84 (1964) 673
[4] Engel J, Pittel S, Stoitsov M, Vogel P, Dukelsky J, Phys. Rev. C 55 (1997) 1781

[5] Van Isacker P, Dukelsky J, Pittel S, Juillet O, J. Phys. G 24 (1998) 1261

[6] Dufour M, Zuker A P, Phys. Rev. C 54 (1996) 1641

[7] Klein A, Marshalek E R, Rev. Mod. Phys. 63 (1991) 375

[8] Dyson J F, Phys. Rev. 102 (1956) 1217

[9] Vincent C M, Henry L J, Kim G K, Phys. Rev. C 39 (1989) 1583

[10] Van Isacker P, Warner D D, Brenner D S, Phys. Rev. Lett. 74 (1995) 4607

[12] Zhang J Y, Casten R F, Brenner D S, Phys. Lett. B 227 (1989) 1

<u>Table 1</u>

Comparison between the first shell model levels associated with the hamiltonian (1) in the *sd* shell and those obtained in its (s, p) boson realization.

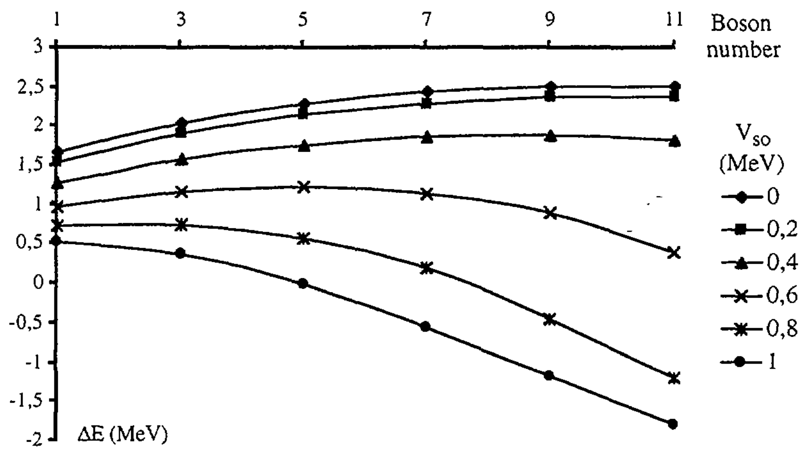
Number $\mathcal{N}$ of bosons	J	T	Shell model energy	(s, p) boson model energy
1	0	1	-1.061609	-1.061609
	1	0	-0.048448	-0.048448
2	0	0	-2.46956	-2.470745
3	0	1	-2.88023	-2.888593
	1	0	-1.84713	-1.836192
4	0	0	-3.64679	-3.652977
5	0	1	-3.37266	-3.426017
	1	0	-2.33107	-2.362323
6	0	0	-3.4779	-3.545329

Figure 1

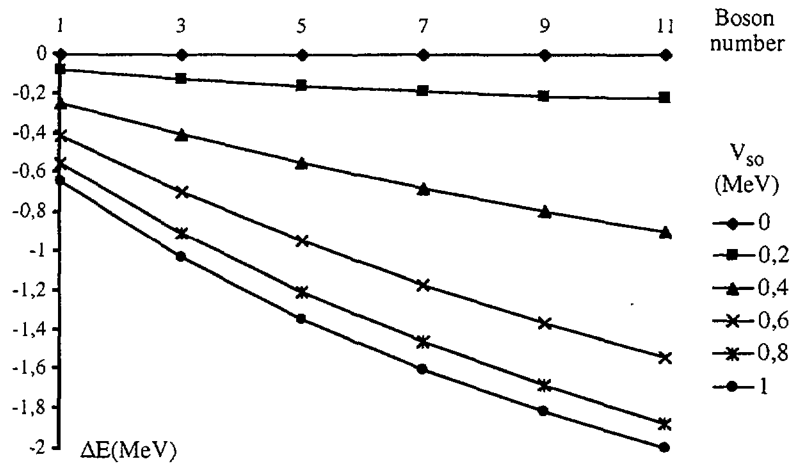
Evolution of the difference energy $\Delta E = E(J=0, T=1) - E(J=1, T=0)$ in odd-odd $N = Z$ nuclei of the fp shell.

V_{so} is the spin-orbit strength and x the asymmetry between the two pairing modes

$x = -0.25$



$x = 0$



$x = 0.25$

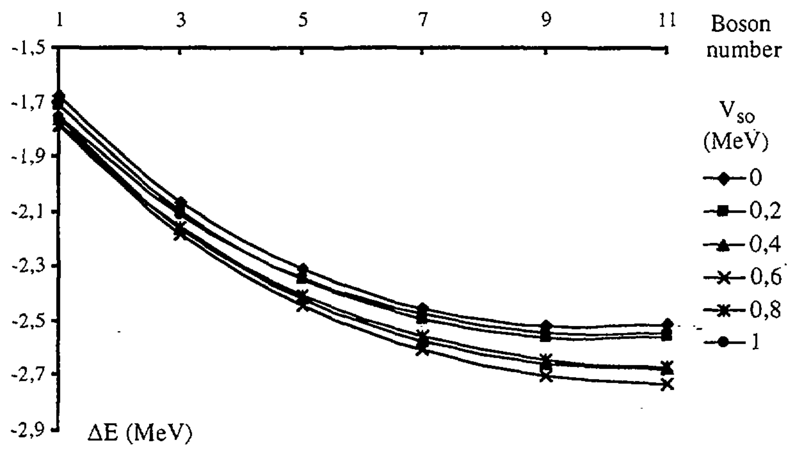
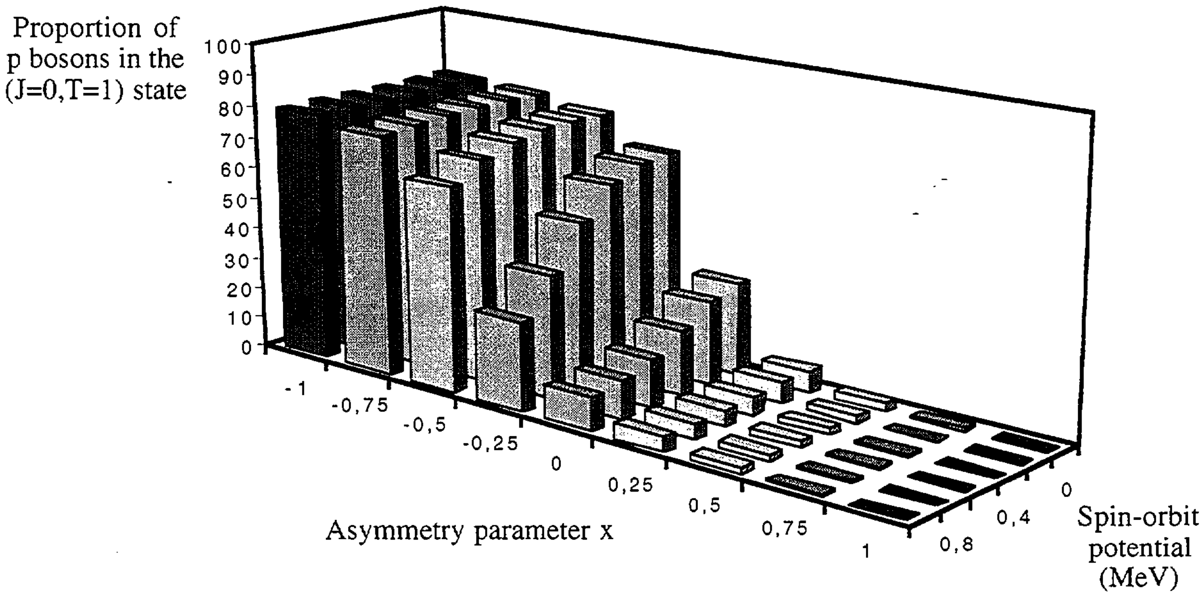
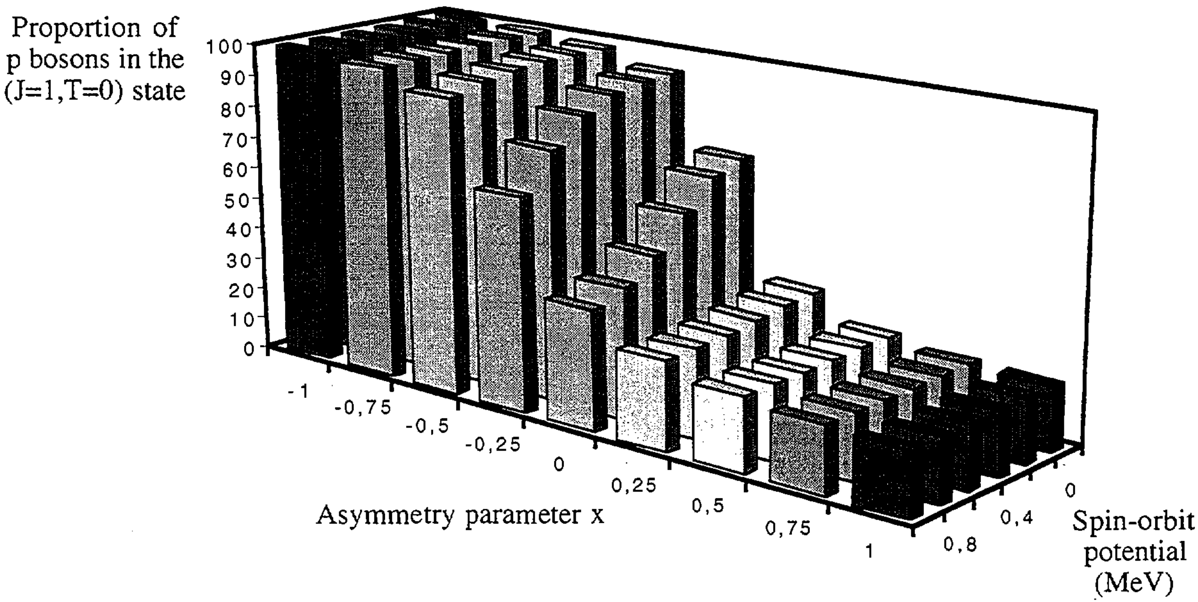


Figure 2

Pair structure of the first $(J = 1, T = 0)$ and $(J = 0, T = 1)$ states in ^{50}Mn $(N = Z = 25)$ as a function of the spin-orbit strength and the relative intensity of the two pairing modes





<u>Figure 3</u>

Experimental values of the double binding energy difference δV_{np} for all known even-even nuclei with atomic number Z <50