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FREE ELECTRON LASER SMALL SIGNAL DYNAMICS AND INCLUSION OF ELECTRON-BEAM ENERGY PHASE CORRELATION

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SUMMARY

We analyze the problems associated with the generation of coherent radiation by an e-beam, traversing an undulator magnet, with an initial energy-phase correlation. The mechanisms of the process are explained and the role played by the bunching is clarified. The effect of the correlation on the stimulated part of the emission is also discussed.

(FREE ELECTRON LASER, ENERGY-PHASE CORRELATIONS, BUNCHING, COHERENT EMISSION, TWISS PARAMETERS)

RIASSUNTO

In questo lavoro esaminiamo le problematiche associate all'emissione di radiazione coerente da parte di un fascio di elettroni correlato in fase ed energia, propagantesi in un magnete ondulatore.
Si spiegano i meccanismi del processo ed il ruolo giocato dal meccanismo di "bunching" è chiarito. L'effetto della correlazione sulla parte stimolata del processo è anche discusso.

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FREE ELECTRON LASER SMALL SIGNAL DYNAMICS AND INCLUSION OF ELECTRON-BEAM ENERGY PHASE CORRELATION

1. INTRODUCTION

The use of a prebunched electron-(e-)beam, to enhance the generation of coherent power in the undulator has been the subject of a recent number of theoretical and experimental investigations.1

The interest for this type of mechanism is justified by the fact that a suitable e-beam prebunching can be exploited as an equivalent seed to be amplified in spectral regions where conventional lasers are not available. 2,3 For high energy e-beam, and thus for devices operating at short wave-lengths, the most convenient prebuncher is an external laser or the FEL itself. In the case of low energy beams and long wave-length, the R.F. accelerating system may act as prebuncher.

It has recently been suggested in Ref. 4 that a further enhancement of the coherently generated power can be obtained by exploiting energy-phase-correlated e-beams. In this paper we discuss this aspect of the problem, by reformulating the e-beam dynamics with the inclusion of an initially correlated longitudinal phase-space distribution.

Denoting by (ν,ζ) the longitudinal FEL phase-space canonical variables, where ν is the dimensionless energy and ζ the electron field relative phase,5 we will say that an energy phase correlation exhists whenever

$$<\nu\zeta>\neq 0 \tag{1}$$

where the brackets denote average on the longitudinal distribution.

To discuss the problem on general grounds we parametrize the initial distribution by using the following Courant-Snyder form 6

$$f(\nu,\zeta)=\frac{1}{2\pi\Sigma_\nu}\exp\left\{-\frac{1}{2\Sigma_\nu}\left[\beta_\nu(\nu-\bar{\nu})^2+2\alpha_\nu(\nu-\bar{\nu})\zeta+\gamma_\nu\zeta^2\right]\right\} \tag{2a}$$

where (see also appendix) Σ_ν is linked to the longitudinal emittance and $(\gamma_\nu,\alpha_\nu,\beta_\nu)$ are the Twiss parameters, satisfying the relation

$$\gamma_\nu\beta_\nu-\alpha_\nu^2=1 \tag{2b}$$

The energy distribution function will be therefore given by

$$g(\nu)=\int_{-\infty}^{+\infty}f(\nu,\zeta)d\zeta=\frac{1}{\sqrt{2\pi}\,\sigma_\nu}\exp\left[-\frac{(\nu-\bar{\nu})^2}{2\sigma_\nu^2}\right]$$

$$\sigma_\nu=\sqrt{\gamma_\nu\Sigma_\nu} \tag{3}$$

The r.m.s. value σ_ν is linked to the inhomogeneous broadening parameter (see Appendix) by

$$\sigma_\nu=\pi\mu_\varepsilon\ ,\quad \mu_\varepsilon=4N\sigma_\varepsilon \tag{4}$$

where σ_ε is the relative energy spread and N is the number of undulator periods. The degree of energy-phase correlation is fixed by the parameter α_ν.

The modifications induced in the FEL dynamics by a distribution of the type (2) are obtained from the FEL coupled pendulum equations, which can be written as 5

$$\frac{d^2\zeta}{d\tau} = |a|\cos(\zeta+\varphi)\ , \qquad \left(\frac{d\zeta}{d\tau} = v\right)$$
$$\frac{d}{d\tau}a = -2\pi g_0 < e^{i\zeta} >_{\zeta,v} \tag{5}$$

Equations (5) holds either in small and strong signal regime. The small signal approximation of eqs (5) is obtained, by expanding all the relevant variables, up to the first order in the dimensionless field amplitude a. By setting

$$\zeta \cong \zeta_0 + v_0\tau + \delta\zeta \tag{6}$$

we can compute the electron phase variation $\delta\zeta$ by integrating the first of eqs (5) which, at the lowest order, yields

$$\delta\zeta \cong \int_0^\tau d\tau'(\tau-\tau')\Re e\left[a(\tau')e^{i(\zeta_0+v_0\tau')}\right] \tag{7}$$

By inserting (7) into the second of (5) and by expanding the exponential up to first order in $\delta\zeta$, we obtain the following integro-differential equation, specifying the unsaturated behavior of the optical field

$$\begin{aligned}\frac{da}{d\tau} &= -2\pi g_0 < e^{-i(\zeta_0+v_0\tau)} > + i\pi g_0 \int_0^\tau d\tau'\,(\tau-\tau')a(\tau')\cdot \\ &\cdot < e^{-iv_0(\tau-\tau')} > + i\pi g_0 \int_0^\tau d\tau'\,(\tau-\tau')a^*(\tau')\cdot \\ &\cdot < e^{-2i\zeta_0 - iv_0(\tau+\tau')} >\end{aligned} \tag{8}$$

By performing the averages on the initial distribution (2), we end up with

$$
\begin{aligned}
\frac{da}{d\tau} &= -2\pi g_0 \exp\left[-\frac{1}{2}\frac{\sigma_\zeta^2}{1+\alpha_\upsilon^2} - \frac{1}{2}\left(\tau - \frac{\alpha_v}{\gamma_v}\right)^2 \sigma_v^2 - i\bar{v}\tau\right] \\
&+ i\pi g_0 \int_0^\tau d\tau' (\tau-\tau')\exp\left[-i\bar{v}(\tau-\tau') - \frac{1}{2}(\tau-\tau')^2\sigma_v^2\right] a(\tau') \\
&+ i\pi g_0 \int_0^\tau d\tau' (\tau-\tau')\exp\left[-2\frac{\sigma_\zeta^2}{1+\alpha_v^2} - i\bar{\upsilon}(\tau+\tau') - \frac{1}{2}\left(\tau+\tau' - \frac{2\alpha_v}{\gamma_v}\right)^2 \sigma_v^2\right] a^*(\tau')
\end{aligned}
\tag{9a}
$$

where

$$
\sigma_\zeta = \sqrt{\beta_v \Sigma_v} \tag{9b}
$$

is the r.m.s. value of the ζ distribution functions.

It is evident therefore that for an unbunched e-beam ($\sigma_\zeta \rightarrow \infty$) eq. (9a) reduces to the usual FEL small signal equation. The enhancement of the bunching effects demands for small σ_ζ values or suitable α_v values as we will discuss in the forthcoming section.

2. COHERENT SPONTANEOUS EMISSION

The first term in eq. (9a) accounts for the coherent spontaneous emission, the second is the usual FEL gain contribution, the third, due to the initial e-beam prebunching, contributes to the stimulated part of the emission.

Let us now consider the spontaneous coherent part, where the first exponential accounts for the bunching contribution and the second for the energy spread inhomogeneous term. It is evident that the energy-phase correlation coefficient αv may be exploited to enhance the coherently generated power.

To better understand this point, we define the coherent dimensionless amplitude

$$
a_c(\tau) = -2\pi g_0 \int_0^\tau \exp\left[-\frac{1}{2}\frac{\sigma_\zeta^2}{1+\alpha_v^2} - \frac{1}{2}\left(\tau' - \frac{\alpha_v}{\gamma_v}\right)^2 \sigma_v^2 - i\bar{v}\tau'\right] d\tau' \tag{10}
$$

and study the dependence of the associated intensity $|a_c|^2$ on the parameter α_v.

We consider two possibilities

a) constant bunching

b) constant energy spread.

In the first case we keep fixed σ_ζ and very α_v. From the physical point of view this assumption is equivalent to a variation of the energy spread, the through variation of γ_v (see eq. (2b). To this aim it is convenient to recast eq. (10) in the form

$$a_c = -2\pi g_0 \int_0^\tau d\tau' \; \exp\left[-\frac{1}{2}\frac{\sigma_\zeta^2}{1+\alpha_v^2} - \frac{1}{2}\left(\tau' - \frac{\alpha_v \beta_v}{1+\alpha_v^2}\right)^2 \frac{1+\alpha_v^2}{\beta_v}\Sigma_v - i\bar{v}\tau' \right] \tag{11}$$

It is evident that an increase of α_v may have two beneficial consequences. It may reduce the effect of the bunching term, we can indeed define an equivalent r.m.s. bunching width

$$\sigma_\zeta^{eq} = \frac{\sigma_\zeta}{\sqrt{1+\alpha_v^2}} \tag{12}$$

and it may counteract the effect of the energy spread. However when α_v becomes too large the energy spread

$$\sigma_v = \sqrt{\frac{1+\alpha_v^2}{\beta_v}\Sigma_v} \tag{13}$$

becomes dominating and the amount of radiated power can be reduced. We can expect therefore an optimum value occurring for positive α_v values.

The situation is summarized in Figs (1,2,3). When $\beta_v > 1$ a significant increase of the coherently generated power occurs for positive α_v values. Along with $|a_c|^2$ we have reported the equivalent bunching width and the energy spread. It is clear that the optimum α_v is a compromise between the "small" values of σ_ζ^{eq} and the "large" values of σ_v. The case with $\beta_v < 1$ is similar, the optimum α_v occurs at smaller values because of the rather fast variation of the energy spread.

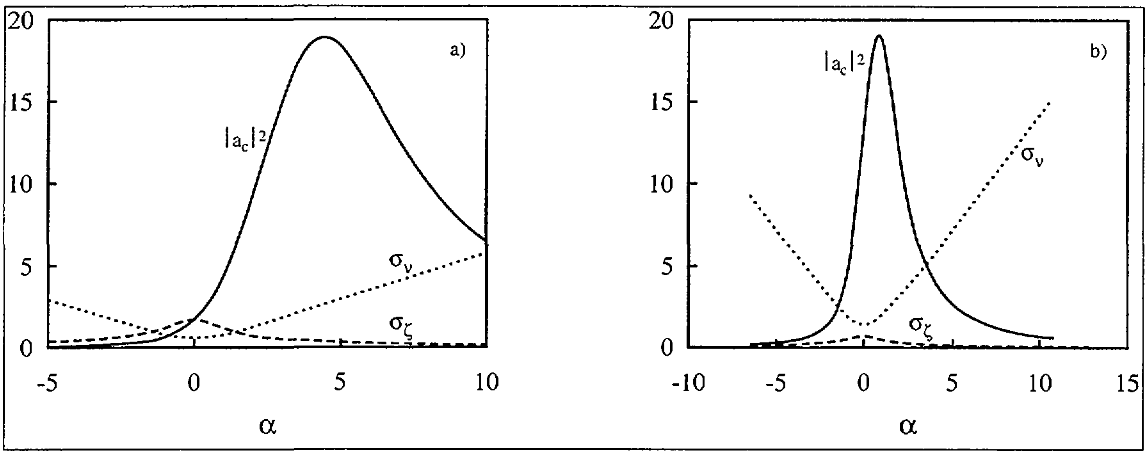

Figure 1 - a) $|a_c|^2$, σ_v *and* σ_ζ *vs* α. $\beta_v=3$, $\Sigma_v=1$, $\bar{v}=0$; *b) Same as* $\beta_v=0.5$

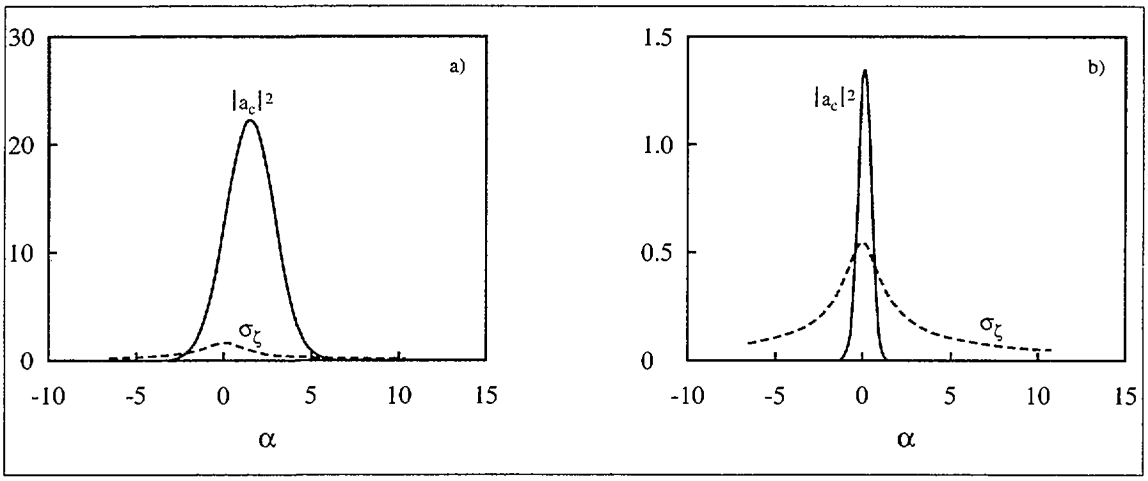

Figure 2 - a) $|a_c|^2$, *and* σ_ζ *vs* α. $\Sigma_v=1$, $\gamma_v=3$, $\bar{v}=0$; *b) Same as (a)* $\gamma_v=0.3$.

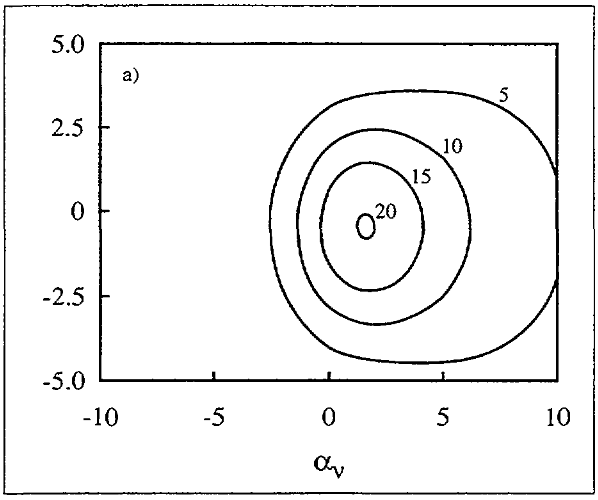

Figure 3 - Contour plots $(\alpha_v \bar{v})$ *of the coherent power (constant bunching* $\Sigma_v=1$, $\beta_v=3$*).*

In the hypothesis of constant energy spread, we keep fixed σ_v and vary α_v. It is therefore convenient to cast Eq. (10) in the form

$$a_c = -2\pi g_0 \exp\left[-\frac{\Sigma_v}{2\gamma_v}\right] \int_0^\tau \exp\left[-\frac{1}{2}\left(\tau' - \frac{\alpha_v}{\gamma_v}\right)^2 \gamma_v \Sigma_v\right] \exp(-i\bar{v}\tau')d\tau' \qquad (14)$$

In this case, since γ_v and Σ_v are fixed, the effect of the α_v variation is that of counteracting the energy spread and we expect that the optimum α_v should occur for $\alpha_v<\gamma_v$ as illustrated in Figs. 2. In this hypothesis we can write the r.m.s. bunching as

$$\sigma_\zeta = \sqrt{\frac{1+\alpha_v^2}{\gamma_v}\Sigma_v} \qquad (15)$$

It is evident that an increase of α_v determines an increase of σ_ζ and thus a dilution of the bunching.

The figures we have shown are all relevant to the case $\bar{v} = 0$, a more complete view is provided by Fig. 3, where we have reported the $(\alpha_v, \bar{v})$ contour plots.

3. COHERENT AND STIMULATED PART

In the previous section we have considered the amount of the coherent spontaneous power emitted in one passage. If the gain coefficient g_0 is large enough, a non negligible contribution to the emission can be provided by the stimulated part, i.e. by those terms in Eq. (9a) explicitly containing the dependence on the field amplitude $a(\tau)$.

The integro-differential nature of eqs. (9a), resembling those of a Volterra type, allows to get naive perturbative solutions. By assuming that the initial field $a(0)$ is vanishing and by limiting ourselves to the first order, we find that the corrections due to the stimulated part and to the coherent contribution can be written as.

$$a_G \cong i\pi g_0 \int_0^{\tau} d\tau_1 \int_0^{\tau_1} d\tau_2 \tau_2 \ \exp\left[-i\bar{v}\tau_2 - \frac{1}{2}\tau_2^2\sigma_v^2\right] a_c(\tau_1 - \tau_2)$$

$$a_{B_s} \cong i\pi g_0 \exp\left[-2\frac{\Sigma_v}{\gamma_v}\right] \int_0^{\tau} d\tau_1 \ e^{-2i\bar{v}\tau_1} \int_0^{\tau_1} d\tau_2 \ \tau_2 \cdot \tag{16}$$

$$\exp\left[i\bar{v}\tau_2 - \frac{1}{2}\left(2\tau_1 - \tau_2 - \frac{2\alpha_v}{\gamma_v}\right)^2 \sigma_v^2\right] \ a_c^*(\tau_1 - \tau_2)$$

The total field will be therefore provided by

$$a_T = a_G + a_{B_s} + a_c \tag{17}$$

The subscript G denotes the party contributing to the small signal FEL gain and B_s the bunching stimulated part.

The importance of these new terms is provided by figs (4,5), where we have plotted $|a_T|^2$ and $|a_G|^2$, $|a_{Bs}|^2$, $|a_c|^2$ vs α in the hypothesis of constant bunching. The emission is always dominated by the coherent contribution, that of the other terms become appreciable for $g_0>1$. For $g_0>4$ at $\bar{v} = 2.6$, G and B_s contribution amount to about 35% of the total (see Figs 4a, 4b). In particular 4b) shows that a constructive interference exhists between the various contributions in eq. (17), in fact $|a_T|^2>|a_G|^2+|a_{Bs}|^2+|a_c|^2$.

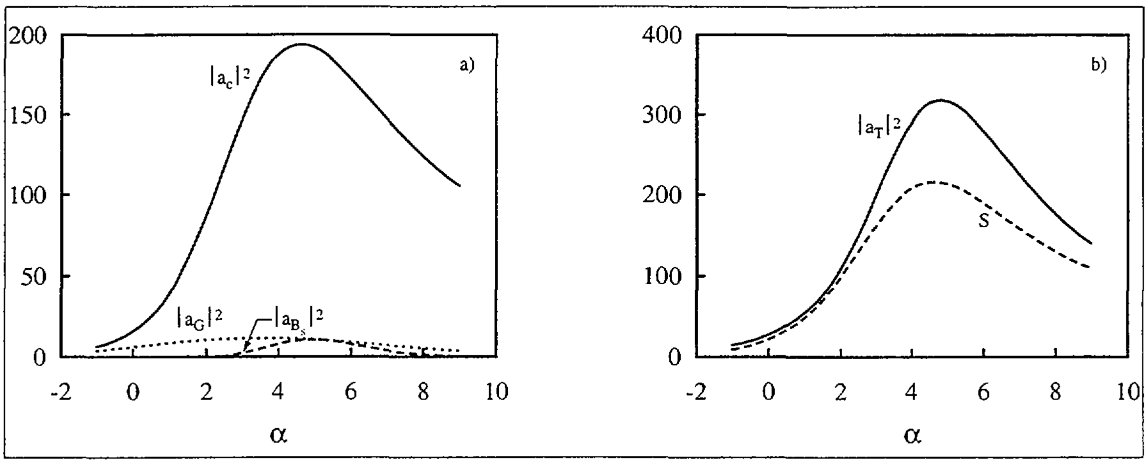

Figure 4 - a) $|a^c|^2$, $|a_G|^2$, $|a_{Bs}|^2$ vs α g_0=4. Same parameters of 1a, $\bar{v} = 2.6$. b) $|a_T|^2$ and s=$|a_c|^2$ +$|a_G|^2$+$|a_{Bs}|^2$vs α. Same parameters of (a).

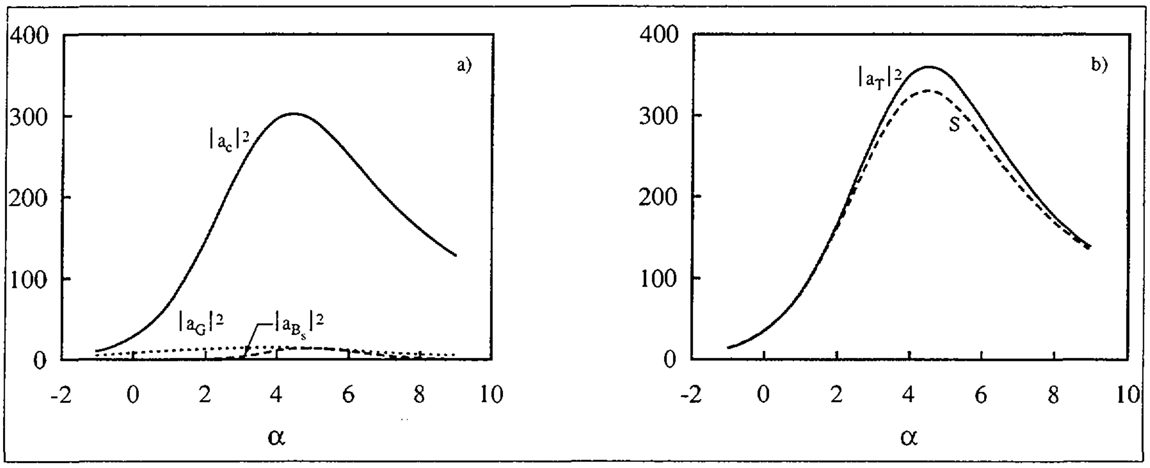

Figure 5 - a) Same as Fig. 4a $\overline{\nu} = 0$. *b) Same as 4b,* $\overline{\nu} = 0$.

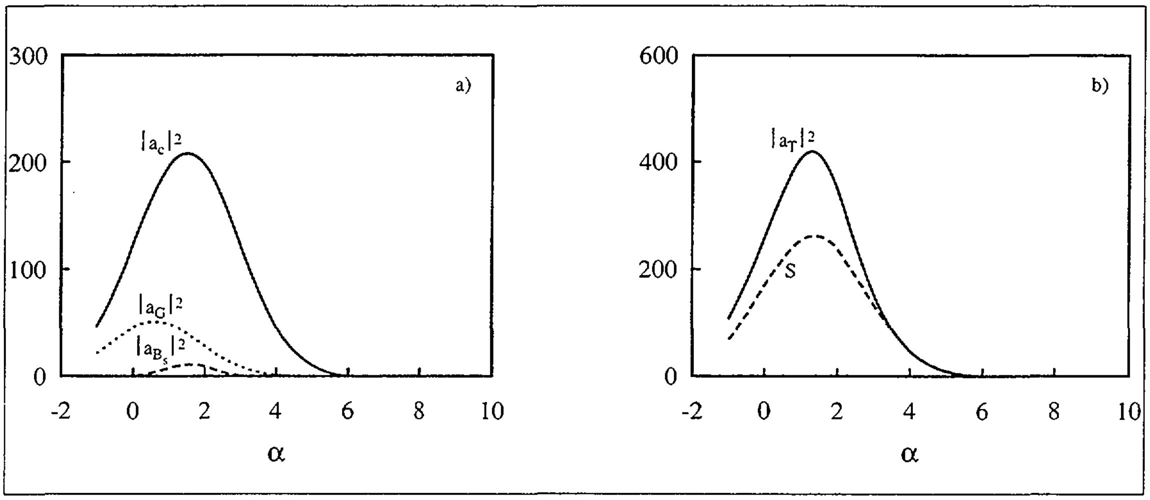

Figure 6 - a) Same as 4a for $\gamma_V=3$, $\overline{\nu} = 2.6$. *b)* $|a_T|^2$ *and s vs* α. *Same as (a).*

The same conclusion holds for $\overline{\nu} = 0$, with the difference that the total power is larger (see figs 5).

The case of constant energy spread is provided by Figs (6,7)

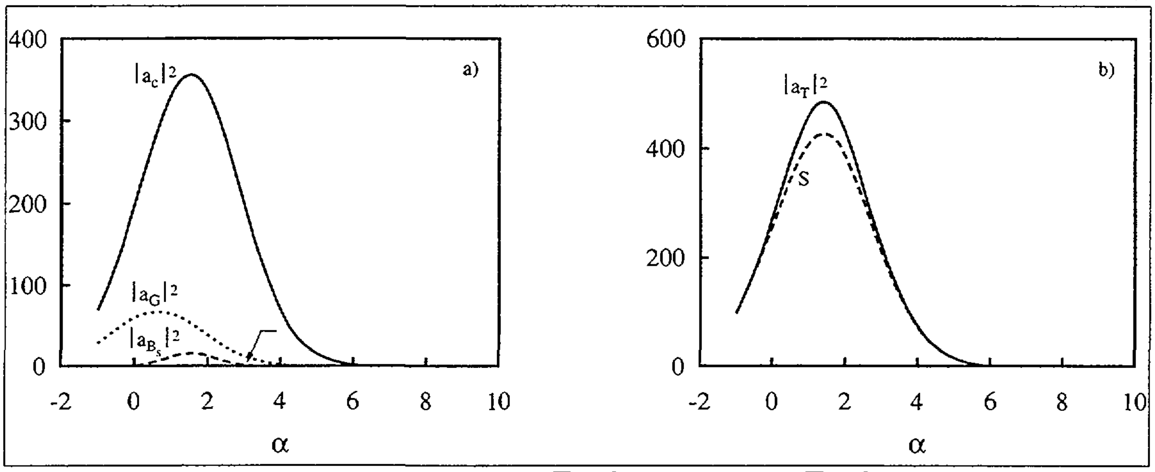

Figure 7 - a) Same as 6a, $\bar{\nu} = 0$. *b) Same as 6b,* $\bar{\nu} = 0$.

4. CONCLUDING REMARKS

In the previous sections we have exploited the concepts and the formalism associated to charged beam transport, to characterize the e-beam longitudinal phase-space distribution. We have used a Courant-Snyder form and we have introduced the quantity Σ_V, which provides the longitudinal phase-space emittance. We have also mentioned the possibility of varying the Twiss parameters by keeping constant the bunching or the energy spread. In the case of constant energy spread, we keep fixed γ_V and vary α_V while the variation of β_V is ensured by eq. (2b) (we are assuming the constancy of Σ_V). It is well known from electron beam optics that such a possibility is ensured by a drift space. In the present case, by replacing the drift length by the dimensionless time τ we find 6

$$\begin{pmatrix} \beta_V \\ \alpha_V \\ \gamma_V \end{pmatrix} = \begin{pmatrix} 1 & -2\tau & \tau^2 \\ 0 & 1 & -\tau \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_V^0 \\ \alpha_V^0 \\ \gamma_V^0 \end{pmatrix} \tag{18}$$

where the superscript "0" denotes initial values. It is therefore evident that, in some conditions, a standard drift section before the undulator can be exploited to optimize the e-beam parameters to enhance the effect of the energy-phase correlation (in this case there is not a substantial difference from the optical-klystron).

As to the case of constant bunching, we keep fixed β_v and vary (α_v,γ_v). This situation is more similar to that of an e-beam crossing a quadrupole, treated in the thin lens approximation. In this case we should have 6

$$\begin{pmatrix}\beta_v\\ \alpha_v\\ \gamma_v\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ -F & 1 & 0\\ F^2 & -2F & 1\end{pmatrix}\begin{pmatrix}\beta_v^0\\ \alpha_v^0\\ \gamma_v^0\end{pmatrix} \tag{19}$$

In conventional e-beam optics F is linked to the quadrupole focal length. In the present context the same role could be provided by an additional radio-frequency (r.f.) which bunches the e-beam before the undulator, F is in this case linked to the peak r.f. value and reads

$$|F|=\frac{e\hat{V}}{E_0}\frac{K_{RF}}{K}\zeta \tag{20}$$

where $e\hat{V}$ is the peak field, E0 the nominal e-beam energy, while KRF and K are the wave vectors associated to the r.f. and FEL waves respectively.

The above considerations indicate two practical possibilities of manipulating the e-beam to modify the parameters of the longitudinal distribution and to study the relevant effect.

We must however underline that this type of beam handling is reasonable for longer wave length 4 (mm,FIR) while, for devices operating at shorter wavelengths, the most convenient tool to prebunch the e-beam and provide an energy-phase modulation is the FEL itself or an external FEL, as discussed in Ref. [2]. Limiting ourselves to the small signal low gain case, the FEL induces an evolution similar to the constant bunching case. This indeed is justified by the fact that the FEL Hamiltonian can be cast in the form

$$H=1/2v^2-k\cos\zeta \tag{21}$$

and thus for small ζ values the effect is that of a quadratic Hamiltonian. We must however underline that in this case we cannot fully apply the Courant-Snyder method since the FEL Hamiltonian is not quadratic in the phase-space coordinates.

Before concluding this paper we should touch other two points

5. the link with previous work

6. the reliability of the present results

The analysis of the emission from a prebunched e-beam, with the inclusion or not of energy correlation effects, is usually accomplished by expanding the beam distribution in terms of the bunching coefficients b_n, namely 2

$$f(\nu,\zeta) = \sum_{n=-\infty}^{+\infty} b_n(\nu) e^{in\zeta} \tag{22}$$

The b_n contains the dependence on ν and are in term linked to the distribution by the relation

$$b_n(\nu) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} f(\nu,\zeta) e^{-in\zeta} d\zeta \tag{23}$$

The energy averaged values of the b_n usually decreases with increasing n, reasonable values for the first coefficients $b_{1,2}$ are around values of percent. By assuming that σ_ζ is small enough that the interval $(-\pi,\pi)$ can be replaced by $(-\infty,\infty)$ we find for the energy average coefficients

$$\bar{b}_n = \frac{1}{2\pi} e^{-\frac{1}{2}n^2\sigma_\zeta^2} \tag{24}$$

As to the reliability of the present analysis, let us stress that we have checked our analytical formulae with the results from a more accurate and complete numerical code; the results of the comparison are those given in Fig. 8, and the agreement can be considered satisfactory.

For the evolution of the intensity values involved in, we recall that the Colson's dimensionless amplitude is linked to the optical power density I by

$$|a|^2 = 0.8\pi^4 \frac{I}{I_s} \tag{25}$$

where I_s is the saturation intensity, while in practical units reads

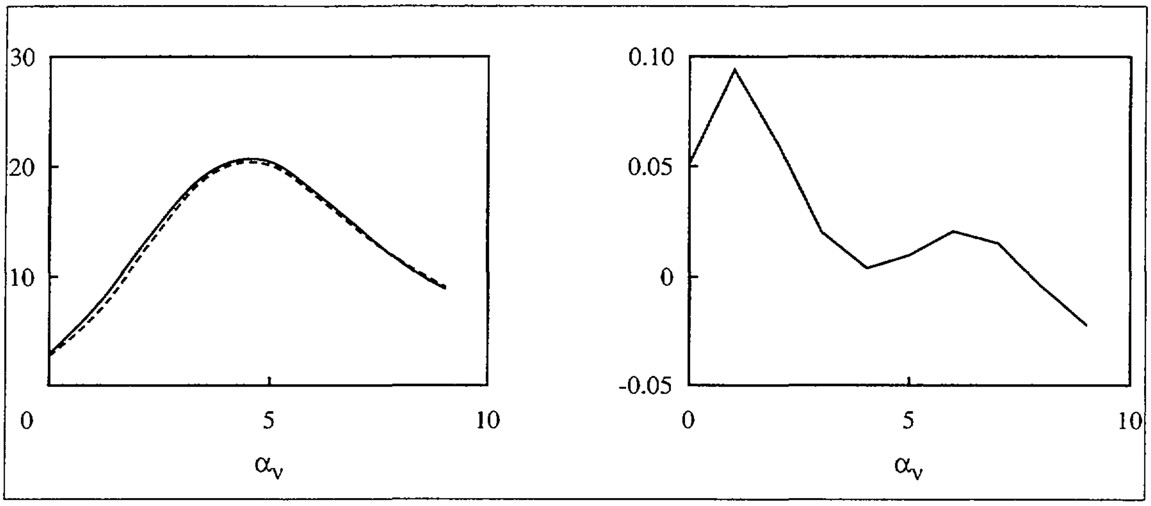

Figure 8 - a) Comparison between analytical and numerical results (——). Same parameters of fig. 1a. b) Relative error.

$$I_s\left[\frac{MW}{cm^2}\right] \cong 6.9\times10^2\left(\frac{\gamma}{N}\right)^4 \frac{1}{[\lambda_u[cm]kf_b]^2}$$

$$f_b = J_0(\xi) - J_1(\xi), \tag{26}$$

$$\xi = \frac{1}{4}\frac{k^2}{1+k^2/2}$$

with γ being the electron relativistic factor, N, λ_u and k the undulator number of periods, period length and strength respectively.

By assuming γ=50, N=40, $k=\sqrt{2}$, λ_u=3 cm we obtain for g_0=1 a maximum output power of about $30\frac{MW}{cm^2}$ while for g_0=4 (including the contribution from the stimulated parts) about $450\ \frac{MW}{cm^2}$.

APPENDIX

Denoting by $\varepsilon = \frac{\gamma - \gamma_0}{\gamma_0}$ the relative electron energy, we can write the longitudinal phase space distribution in the form

$$f_\varepsilon(\varepsilon,\zeta) = \frac{1}{2\pi\bar{\varepsilon}} \exp\left\{-\frac{1}{2\bar{\varepsilon}}\left(\beta_\varepsilon \varepsilon^2 + 2\alpha_\varepsilon \varepsilon\zeta + \gamma_\varepsilon \zeta^2\right)\right\} \tag{A.1}$$

where $\sigma_\varepsilon = \sqrt{\gamma_\varepsilon \bar{\varepsilon}}$ is the relative energy spread and $\sigma_\zeta = \sqrt{\beta_\varepsilon \bar{\varepsilon}}$ is the r.m.s. bunching width. If we choose $\bar{\varepsilon} \cong 10^{-3}$, $\gamma_\varepsilon \cong 10^{-3}$, $\beta_\varepsilon \cong 10^{+3}$ we get $\sigma_\varepsilon \cong 10^{-3}$, $\sigma_\zeta \cong 1$.

The same distribution in the (ν,ζ) space is provided by eq. (2a) where

$$\Sigma_\nu = 4\pi N\bar{\varepsilon},\ \ \gamma_\nu = 4\pi N\gamma_\varepsilon,\ \ \beta_\nu = \frac{1}{4\pi N}\beta_\varepsilon,\ \ \alpha_\nu = \alpha_\varepsilon \tag{A.2}$$

REFERENCES

1 See e.g. the Proc. of the 18th Int. FEL Conf., ed. by G. Dattoli and A. Renieri, Sec. V "Beam Prebunching and Superradiance", Nucl. Instrum. & Meth. **A393** (1997)

2 L.H.Yu, Phys. Rev. **A44**, 5178 (1991)

3 G. Dattoli, L. Giannessi, P.L. Ottaviani and A. Segreto, Nucl. Instrum. & Meth. **A393**, 339 (1997)

4 A. Doria, G.P. Gallerano, E. Giovenale, G. Messina and C. Ronsivalle, Phys. Rev. Lett. to be published

5 W.B. Colson, G. Pellegrini and A. Renieri (Eds), Laser Handbook Vol. VI, North Holland Amsterdam (1990)

6 See e.g. G. Dattoli, A. Renieri and A. Torre "Lectures on Free Electron Laser Theory and Related Topics", World-Scientific Singapore (1993)

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