One of the most important problems in the HTGR development is the creation of the fuel element gas-tight for the fission products. This problem is being solved, using fuel elements of dispersion type [1,2] representing ensemble of coated fuel particles dispersed in the graphite matrix. Gastightness of such fuel elements is reached at the expense of depositing a protective coating on the fuel particles, which is composed of some layers serving as diffusion barriers for fission products. It is apparent that the rate of fission products diffusion from coated fuel particles is determined by the strength and temperature of the protective coating. In this connection arise the problems of the optimum designing of the protective coating of the maximum strength and of optimum fuel distribution in the fuel element ensuring the maximum reduction of the temperature difference in the fuel element.

I. Let us consider at first the problem of the optimum fuel distribution in the fuel element, supposing the achievement of the minimum temperature drop between the fuel element centre and its surface. Look at this problem from the mathematical point of view. Let it be a spherical fuel element, in which fuel is mixed with
the matrix material. Then for the mixture the thermal conductivi-
ity coefficient is the function of fuel concentration $\lambda_r$ and
matrix $\lambda_m$. In this case one-dimensional temperature field is
described by the solution of the equation
\[
\frac{d^2 T}{dz^2} + \frac{2}{z} \frac{dT}{dz} + \frac{1}{\lambda(z)} \frac{dT}{dz} + \frac{Q(z)}{\lambda(z)} = 0, \tag{1.1}
\]
satisfying the boundary conditions
\[
T(0) < M < \infty, \quad T(R) = T_0. \tag{1.2}
\]
It is necessary to determine the fuel distribution $C(z)$, at which
the minimum temperature drop between centre and fuel element surface
is realized. In this case the mean fuel concentration in the fuel
element
\[
\bar{C} = \frac{3}{R^3} \int_0^R z^2 C(z) \, dz = \frac{1}{R^2} \int_0^R C(\rho) \rho^2 d\rho, \quad \rho = \frac{z}{R}, \tag{1.3}
\]
and, hence, the thermal power of the fuel element are constant
values.

Introducing a simplifying assumption about the linear de-
pendence of the thermal conductivity coefficient $\lambda$ and internal
power output on fuel concentration
\[
\lambda = \lambda_m - (\lambda_m - \lambda_r) C(z), \quad Q = A C(z), \quad A = \text{const}
\]
after the integration of Eq. (1.1) and the boundary conditions
(1.2) satisfaction we obtain the following expression for the
temperature drop in the fuel element
\[
\Theta = \frac{\lambda_m A \Delta T}{\lambda_m R^2} = \int_0^1 \frac{C(\rho') \rho^4 d\rho}{[1 - 8 C(\rho)] \rho^2 R^2} \, d\rho, \quad \beta = 1 - \frac{\lambda_r}{\lambda_m} \tag{1.4}
\]
Now the problem of the optimum solution is formulated as follows:
to find such a fuel distribution $C(\rho)$ along the fuel element ra-
dius at which the functional (1.4) has the least value at the
given mean fuel concentration $\bar{C}$. Then $0 \leq C \leq 1$, $\beta < 1$. 67
Let us introduce the designations
\[ \mathcal{X}(\rho) = \int_0^\rho \frac{x(t)dt}{[1 - B u(t)]^{1/2}}, \quad x'(\rho) = \int_0^\rho u(t)\frac{dx}{dt} dt, \quad u(\rho) = c(\rho) \]
and instead of the functional (1.4) consider the system of the differential equations
\[
\frac{d\mathcal{X}(\rho)}{d\rho} = \frac{x'(\rho)}{[1 - B u(\rho)]^{1/2}}, \quad \frac{dx'(\rho)}{d\rho} = \rho \frac{dx}{dt} u(\rho).
\]
with the boundary conditions
\[ x(0) = 0, \quad x'(0) = \theta_{\text{min}}, \quad x(1) = 0, \quad x'(1) = \frac{c}{3}. \]
Thus the optimization problem is reduced to the determination of \( u(\rho) \) and the corresponding trajectory in the phase \((x, x')\), which would give the minimum value \( x(1) \).

To solve the given problem we shall use Pontryagin maximum principle \([3]\). Having made appropriate calculations according to the scheme proposed in Ref. \([4]\), we shall obtain the following result:

1) in the fuel element centre there always is a radius \( \rho_0 \) zone where fuel is absent and
\[ u(\rho) = \begin{cases} \frac{1}{2B}, & \rho > 0.5 \\ 1, & \rho < 0.5 \end{cases} \]

2) if the parameter \( B \) satisfies the inequality \( 0 < B < 0.5 \), then the extremum problem solution has a form
\[ u(\rho) = \begin{cases} 0, & 0 \leq \rho \leq \rho_0 \\ 1, & \rho_0 < \rho \leq 1 \end{cases} \]
where \( \rho_0 = \sqrt{1 - e} \).
3) in the case of \( 0.5 \leq B \leq 1 \) the extremum problem solution may be obtained using the method of the successive approximations from the following system

\[
\begin{align*}
\chi^1(\rho) &= \int_0^\rho \mathcal{U}(\xi) \xi^4 d\xi, \\
\psi_1(\rho) &= \int_{\rho_0}^\rho \frac{d\xi}{[1-B \mathcal{U}(\xi)] \xi^2}, \\
\mathcal{U}(\rho) &= \frac{1}{B} \left[ 1 - \frac{1}{\rho^2} \sqrt{\frac{B \chi^1(\rho)}{\psi_1(\rho)}} \right] \tag{1.9}
\end{align*}
\]

As a zero approximation the value \( \mathcal{U}_0(\rho) = \frac{1}{2} B \) is chosen and then from the first two equations the functions of \( \chi^1(\rho), \psi_1(\rho) \) are determined. The substitution of these functions in the third equation of the system (1.9) gives the first approximation with the help of which the calculation cycle is repeated once more etc.

Fig.1 presents as an example the optimum fuel distributions along sphere radius for the values of the mean concentration \( C=0.4 \) and various ratios of thermal conductivity coefficients of fuel matrix materials. The character of the optimum distribution is explained as follows. When the thermal conductivity coefficients of fuel and matrix are close, it is advantageous to distribute the fuel near the surface of a sphere. When the parameter \( B \) (\( B \in [0.5; 1] \)) increases and \( \lambda_f \) becomes substantially lesser than \( \lambda_m \), the fuel on the surface prevents heat transfer from the internal layers and must be diluted with the matrix material.

The suggested solution of the extremum problem presents
the shape of the optimum fuel distribution in the fuel element and its physical nature. However, the practical realization of the fuel profiling in the fuel element is difficult. Besides, as a rule, a fuel element has the outer shell ensuring its operability during the specified service life. Taking into account these considerations the problem of the optimum designing of the HTGR spherical fuel element can be formulated as follows: let a sphere with the radius $R$ be given having the outer layer of a thickness $h$ with no power output. It is necessary to determine such fuel concentration distribution which would provide the minimum temperature drop between the sphere centre and its surface. It is supposed a priori that the solution is of a piece-constant function type.

The given problem is easily solved using the common methods of the mathematical analysis of the extremum functions estimation. After appropriate calculations, we came to the conclusion that the thickness of the energy generating layer is defined by the formula

$$ \rho_0 = \frac{\xi}{2} \left( 1 + \sqrt{9 - 8 \beta \xi} \right), $$

and the fuel concentration in the layer and the temperature drop between the fuel element centre and its surface are

$$ C = \frac{\xi^3 \rho}{\xi^3 - \rho^3}, \quad \Delta T = \frac{A R^2 \xi^2}{\lambda_M} \frac{\xi^3 + 3 \rho_0^3 - 3 \xi \rho^2_0}{6(\xi^3 - \rho^3 - \beta \xi)} \left\{ \frac{\xi^3 + 3 \rho_0^3 - 3 \xi \rho^2_0}{6(\xi^3 - \rho^3 - \beta \xi)} + 1 - \frac{\xi}{3} \right\}, $$

where

$$ \xi = 1 - \frac{h}{R}, \quad \rho_0 = \frac{\rho}{R}. $$

Fig. 2 shows the dependences of the temperature drop and the level of the maximum stresses in the fuel element on the power generating layer thickness for VG-400 reactor [5]. It follows from the graphs that by profiling the fuel distribution a substan-
tial decrease in the temperature drop in the fuel element and some decrease in the tensile stress level can be obtained.

The comparative analysis of the stressed state in fuel elements of various constructions was carried out as applied to the conditions of the VG-400 operation. In [2] the diameter of the two-zoned fuel element was set to be 60 mm, and fuel core 50 mm. The three-zoned fuel element was of the same size but unlike the two-zoned one contained the central spherical fuelless region, whose diameter was 35 mm (the fuel zone thickness was 7.5 mm). The fuel loading ratio $k_v$ (ratio of the total volume of the coated particles to the volume of the fuel core) is 7.6% for the two-zoned fuel element and 11.6% for the three-zoned one, which is quite acceptable from the fuel element fabrication technology point of view. The calculations were carried out using the following data: full element thermal power - 4250 W, $E = 7000$ MPa, $\lambda = 20$ W/mK, $\lambda = 5.8 \times 10^{-6} K^{-1}$, $\nu = 0.2$. The graphs of the radial temperature and stress distributions in the two-zoned and three-zoned fuel elements show that the transition from the two-zoned fuel element to the three-zoned one results in the decrease in the maximum temperature of the fuel element by more than 200 K in tensile stress level by about 20% (Figs. 3, 4). It must be noted that in the three-zoned fuel element a considerable part of the coated fuel particles is in the region of lower temperatures in comparison with the two-zoned fuel element, which undoubtedly enables the decrease in the rate of the fission products release from the coated fuel particles.

2. The protective coating of the coated fuel particle is a multilayer construction in which every layer has a definite function and the destruction of even one of the layers (except for
the buffer layer) may deteriorate the tightness of the coating as a whole. Therefore it is advisable to solve the problem of the optimum protective coating construction designing proceeding from the conditions of the maximum construction strength and the criterion of the best construction choice in the given case will be

\[
\min \left\{ \max_i \frac{\sigma_i^{\text{max}}}{\sigma_o} \right\},
\]

where \(\sigma_o\) is the maximum permissible stress value for the material of the i-th coating layer.

Thus formulated extremum problem leads to the search for the global extremum of a certain function depending on the discrete parameters. When the multiparametered function has a complicated form, the combined method of the global extremum search [6] is effective. The essence of the method is repeated search for the local extrema but each time for the initial conditions chosen arbitrarily. Local minimum the result of each step, is stored and compared with local minima of the following steps from which the least minimum is chosen as a global minimum. The initial conditions chosen on such a basis permit the global extremum to be determined with greater probability. Gauss-Zeidal method was used for the determination of local extrema[7].

The construction of coated fuel elements protective coating was optimized for VG-400 reactor operation conditions. A coated fuel element contains a 500 mkm dia spherical fuel kern of UO₂ and a fourlayer protective coating 250 mkm in thickness. The first layer of the protective coating serving as a reservoir for gas fission products is made of the low density pyrocarbon, the rest of the layers are diffusion barriers: the second and the fourth
layers are fabricated of high density pyrocarbon, the third layer is of silicon carbide. The construction of the protective coating was considered to be optimal for which condition (2.1) is satisfied. The radii of the kern and coated fuel particles are constant, thickness of the coating layers are restricted from the bottom

\[ R_{i+1} - R_i \geq h_i, \tag{2.2} \]

where \( i \) is the number of the coating layer and the values of \( h_i \) are determined by technological possibilities.

The calculations were carried out when estimating the stressed-strained state of the protective coating in the elastic approximation, id est, the effects due to radiation creep, stress relaxation etc. were not taken into account. The calculations permit to make the conclusion that when \( h_i \geq 15 \text{ mkm} \) (it is hardly advisable to consider lower values) the optimum construction of the protective coating is characterized by the increase of the thicknesses of the buffer layer and the silicon carbide layer. These features of the optimum construction are explained as follows: the buffer layer increase reduces pressure of gas fission products and decreases stress in the protective coating; the increase of carbide layer thickness decreases radial movement of this layer under the influence of gas fission products pressure and, as a consequence, the radial movement of all the rest protective coating layers is decreased because it is the carbide layer that is the main force layer in the coating. Therefore the increase in the SiC layer thickness decreases stresses in all the rest layers. Safety factor in the optimal construction increases by two or three times in comparison with the constructions.
where thickness of high density pyrocarbon and silicon carbide are approximately the same.

The above optimization of the protective coating construction takes into account only a strength aspect of coated fuel elements operability. However the use of pyrocarbon layers permits to decrease carbide layer corrosion to reduce the destructive action of amoeba effect on the coating and solve some other problems aimed at the increase of coated fuel particle gas tightness. The experience gained showed that for these problems to be solved and also for providing sufficiently low contamination level of uranium coating when high density PyC layers from methane mixtures having a density of 1.8 g/cm$^3$ and higher are formed, it is reasonable to use a five-layer construction where the first buffer layer is of porous pyrocarbon PyC with a density of about 1.0 g/cm$^3$, the second layer is of PyC with an intermediate density (\(\rho \sim 1.4-1.6\) g/cm$^3$) and a thickness of 30 mkm, the third layer is of dense PyC with a density of 30 mkm and \(\rho \sim 1.8-1.9\) g/cm$^3$. The thickness of outer dense PyC layer providing integrity of the brittle carbide layer usually is about 50 mkm. In this case the problem of the optimal designing is to determine the thicknesses of the buffer and carbide layers. If we fix the diameter of a fuel particle at the level 0.5 mm and the diameter of a coated fuel particle at the level 1 mm, then the optimal construction of the protective coating will be characterized by the following parameters: porous PyC buffer layer - 90 mkm, the second layer of intermediate density PyC - 30 mkm, the third layer (dense PyC) - 30 mkm, the fourth layer (SiC) - 50 mkm, the fifth layer (dense PyC) - 50 mkm.

Optimization calculations were carried out in terms of the theory of elasticity. Only temperature loadings and gas fission
products pressure were taken as stresses sources. Therefore the data obtained need an experimental verification or additional analysis of stress-strain state with allowance for radiation creep and PyC layers dimension changes.

Coated fuel particles of protective coating construction close to the calculation one were in-pile tested. Temperature of test was 1470 K, kinetics of gas fission products output from the burnup fraction is shown in Fig.5. Relative output (R/B) of gas fission products at the end of the experiment (fima ~ 10%) did not exceed ~ 10^-6. This is indicative of high gastightness of coated fuel particles and confirms the correctness of the calculated recomendations. In [2] the stresses that originated in the coated fuel particles of the discussed construction during in-pile tests were, analyzed taking into account radiation creep and PyC shrinkage. It was shown that the values of stresses were within permissible limits.

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Fig. 1. Optimal distribution of relative fuel concentration in a fuel element at a mean concentration $\bar{c} = 0.4$. 
Fig. 2. Dependences of temperature drop and maximum stresses in a fuel element upon radius of the internal zone free from fuel. 1. T drop along fuel element; 2. Radius of fuelless zone; 3. Stress.

Fig. 3. Radial temperature distribution in two-zoned (1) and three-zoned (2) fuel elements.
Fig. 4. Radial distribution of circumferential stresses in two-zoned (1) and three-zoned (2) fuel elements.

1. Stress.

Fig. 5. Dependence of $^{85m}$Kr nuclide output on fuel burnup during irradiation of fuel elements with thermal neutron flux $(7-8) \times 10^{13}$ n/cm$^2$/s, generating heat rating (2.1-2.3) kW, irradiation temperature 1470 K.

1. Relative output $^{85m}$Kr; 2. Burnup, % fima.