THE REACTIVITY METER AND CORE REACTIVITY

P. Siltanen
Fortum Engineering Ltd, Vantaa, Finland

ABSTRACT

This paper discusses in depth the point kinetic equations and the characteristics of the point kinetic reactivity meter, particularly for large negative reactivities. From a given input signal representing the neutron flux seen by a detector, the meter computes a value of reactivity in dollars (ρ/β), based on inverse point kinetics. The prompt jump point of view is emphasised.

A simple point model of the reactor and a local flux distortion factor are used to generate input signals into a simulated reactivity meter. The obtained results show how the reading of the reactivity meter will approach the reactivity of the core model, if the reactivity is lower than −1 dollars. However, for reactivity values higher than −1 dollars, the influence of the flux distortion on the reading of the reactivity meter persists. The influence of using different kinetic parameters in the meter than in the core is also investigated. A typical example shows that using data by Keepin in the reactivity meter can produce differences up to ten percent of the reading value.

Reactivity meter measurements taken during typical rod drop experiments in VVER-440 reactors do not produce accurate indications of the (static) core reactivity. However, such measurements are useful as integral neutronic validation tests. The measurement results should be predicted by comprehensive 3-dimensional kinetic models for the core, by neutron transport from the core to the ionisation chambers outside the core, and finally by simulating the characteristics of the reactivity meter used in the experiment. The reactivity meter is a tool that performs a transformation of the strongly time dependent flux signal to a nearly constant output quantity, that is easily compared to a calculated prediction.

1. INTRODUCTION

In VVER reactors rod drop measurements are typically performed for full reactor scram and for reactor scram with a stuck control rod. Negative reactivity up to −15 dollars can be involved. In addition, the control rods introduce strong distortions into the neutron flux distribution. This situation sets strict requirements on the correct interpretation of the measurement results. Over several years there has been increased interest in the interpretation of rod drop measurements in organisations involved with VVER reactors.

Early investigations on the problem were performed in Finland about ten years ago [1, 2, 3]. These investigations considered the performance of the reactivity meter and the simulation of core behaviour by two-dimensional calculations in the prompt jump approximation. Since that time, many papers have been published within AER on the topic of simulating core behaviour by 3-dimensional neutron kinetic codes during the drop of control rods and simulating the response of a reactivity meter in some approximation. A special seminar on "The problems of reactivity (rod drop) measurements" was organised in Moscow, May 27 – 28, 1999. Some of the material in this paper was presented at the seminar. New material has also been added, inspired by presentations and discussions at the seminar. All this material is acknowledged without specific reference to it.

2. POINT KINETICS AND THE REACTIVITY METER

An extensive review of the point kinetics equations and their application to a reactivity meter is given in Appendix A. The equations are considered from the point of view of large negative reactivity. Emphasis is given to the correct interpretation of the kinetics parameters and to the behaviour of the solution. Appendix A is an integral part of this paper and contains all the equations and formulas, giving their derivation as well. For simplicity of notation, a one-group interpretation is used. However, this is not a severe limitation when all quantities involved are allowed to change with time, including the neutron velocity.

The prompt response or prompt jump of the neutrons to reactivity changes is an important characteristic of the point kinetics equations. The first lesson from the derivation of the prompt jump is that it is necessary to distinguish between the prompt neutron lifetime and the mean neutron generation time. This removes all virtual conflicts between different forms of the kinetics equations.

Another lesson is that the magnitude of the jump depends on the specific quantity that is observed: neutron density (or population), neutron flux density, or production rate of fission neutrons. The jumps of the first two quantities are not determined by the reactivity (change) alone. They are also influenced directly by changes in the cross sections and in neutron velocity. However, the prompt jump of the fission neutron production rate depends only on the reactivity (Eq. 16). This indicates that fission neutron production is the natural amplitude function of neutron kinetics.

In a standard reactivity meter with constant kinetics parameters, the prompt jump has the correct form for the fission neutron production rate. In this sense it is well adapted to measuring reactivity. However, all effects that influence the proportionality between the detector signal and the rate of fission neutron production will also distort the reactivity reading of the meter. A typical cause is a change in the spatial distribution and hence a change in the detector efficiency relative to the total rate of fission neutron production.

Once a deviation is introduced into the meter reading, it persists for a long time for large negative reactivity. Even for reactivities smaller than 1 dollar the recovery of the meter towards the correct reading is slow from the practical point of view. This is illustrated in Figures 1 to 4. In these simulations, the core is simulated by a point model representing the average fission neutron source in the core. The detector signal is derived from the core aver-

age by applying a local flux distortion factor K in parallel with the reactivity change. Another way of interpreting K is a change in the detector efficiency: $K = \varepsilon_1/\varepsilon_0$. The point kinetics parameters in the core and in the reactivity meter are the same. This is why the simulated meter reproduces faithfully the input reactivity to the simulated core, when K = 1.

The reason that the meter is unable to recover from the local distortion is the following. A reactivity meter derives the reactivity essentially based on the prompt jump that is sees. After the reactivity change is over, the asymptotic behaviour of the core and the detector signal contains very little information on reactivity. The behaviour is practically the same for different values of negative reactivity in a wide band. Note also that in a real rod drop case the spatial distortion tends to increase somewhat with time.

Another test illustrating the influence of differences in neutron kinetics parameters is shown in Figure 5. In this simulation, the data in the meter is always that given by Keepin for the thermal fission of U-235. This is the data nominally used in the PIR meters employed during physical start-up tests of Loviisa-1&2. The data simulating the core is varied from Keepin data to data produced by the assembly code CASMO-4 for different fuel types in the initial core and by HEXTRAN for the core as a whole in its initial state. Both delayed neutron fractions and decay constants are different. The data is shown in Table 1. The reactivity insertion into the core was always –15 dollars. This eliminates the influence of differences in total β_{eff}. When the data in the core is entirely based on CASMO, the output of the "Keepin reactivity meter" at 60 ... 100 s is –16.75 dollars or almost 12 % more in absolute value. This is a significant effect and needs to be taken into account when interpreting experimental data.

It is also shown in Appendix A that the asymptotic time behaviour of the fission rate depends only weakly on large negative reactivity values. This implies that the logarithmic derivative is not a good indicator of the reactivity. Another expression of this fact is that the reactivity meter is not able to find the correct reactivity in reasonable time, although it works with the complete set of kinetics equations.

Appendix A contains further an interpretation for the concept of dynamic reactivity as distinct from static reactivity. According to this interpretation, dynamic reactivity is needed to reproduce the correct magnitude of the prompt jump, when the source of delayed neutrons has a distribution that differs from the distribution of the fission neutron production in the subcritical core. Hence, dynamic reactivity is not an invariant property of the subcritical core. Rather, it depends on the distribution of the driving source. If the source is concentrated in regions of high absorption or other loss, then the dynamic reactivity is large compared to the static reactivity. Conversely, if negative reactivity is reduced, the significance of the original source is diminished as a new delayed neutron source builds up. Dynamic reactivity increases faster than static reactivity. When criticality is reached, both reactivities reach zero.

3. PROCEDURE FOR INTERPRETING ROD DROP MEASUREMENTS

In a purely 3-dimensional change in the core, such as that produced by the drop of control rods, the reading of a reactivity meter can be influenced by several disturbances from the ideal point reactor. The common feature of these disturbances is that, for large negative reactivities, the meter cannot distinguish them from changes in the signal due to a real reactivity

change. Also, the kinetic parameters of the meter can differ from the true ones governing core behaviour.

All the complication can be taken into account by simulating the entire dynamic process from core behaviour to reactivity meter output. The main components in this chain are:

- Simulation of the core by a 3-dimensional kinetic code.
- Calculation of neutron transport from the core to the neutron detectors in the reflector, accounting for the distribution of the neutron source in the core.
- Simulation of the reactivity meter with the kinetic parameters actually used during the experiment at the plant.

The predicted output of the reactivity meter can be directly compared with the measurement. In this approach, no specific meaning is attached to the absolute value of the meter reading. Essentially, the reactivity meter is an instrument of observation that performs a useful mathematical transformation of the rapidly changing local neutron flux (detector current) to a much more constant function of time after the drop of control rods. The simulation of the reactivity meter is the least demanding of the whole simulation task and the data involved is perhaps best known.

The prime purpose of rod drop tests is to "measure" the reactivity worth of control rods. However, in such strong changes to the core, simple interpretations fail for reactivity meter readings based on ex-core detectors. The experiment becomes an integral test for the ability of the calculation system to predict a large neutronic transient. Both static and kinetic models and data are put to a severe test. Hence, the simulation of the experiment is an exercise in code system validation.

If there is a discrepancy between prediction and experiment, this cannot be simply interpreted as an error of control rod modelling. All models and data need to be examined carefully.

In the end, when other sources of error have been exhausted, the control rod model can be adjusted to achieve a better agreement with experimental data. The best static reactivity worth of the control rods is then the one calculated by the same model for the core.

Work to the above effect is being performed at VTT Energy in Finland. A summary of the current results is given in ref. [4].

REFERENCES

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[4] Kaloinen E, Kyrki-Rajamäki R, Wasastjerna F: *Simulation of rod drop experiments in the initial cores of Loviisa and Mochovce.* 9th AER Symposium, Demenovska Dolina, Slovakia, Oct. 4 – 8, 1999.

Table 1. Neutron kinetics parameters applied in simulations of the core and the reactivity meter.

i	KEEPIN		CASMO	
	λ_i (1/s)	β_i/β	λ_i (1/s)	β_i/β
1	0.0124	0.033	0.0128	0.034
2	0.0305	0.219	0.0318	0.201
3	0.1110	0.196	0.1190	0.184
4	0.3010	0.395	0.3180	0.404
5	1.1400	0.115	1.4020	0.143
6	3.0100	0.042	3.9250	0.034
β	0.00650		0.00716	
Λ (μs)	25		25	

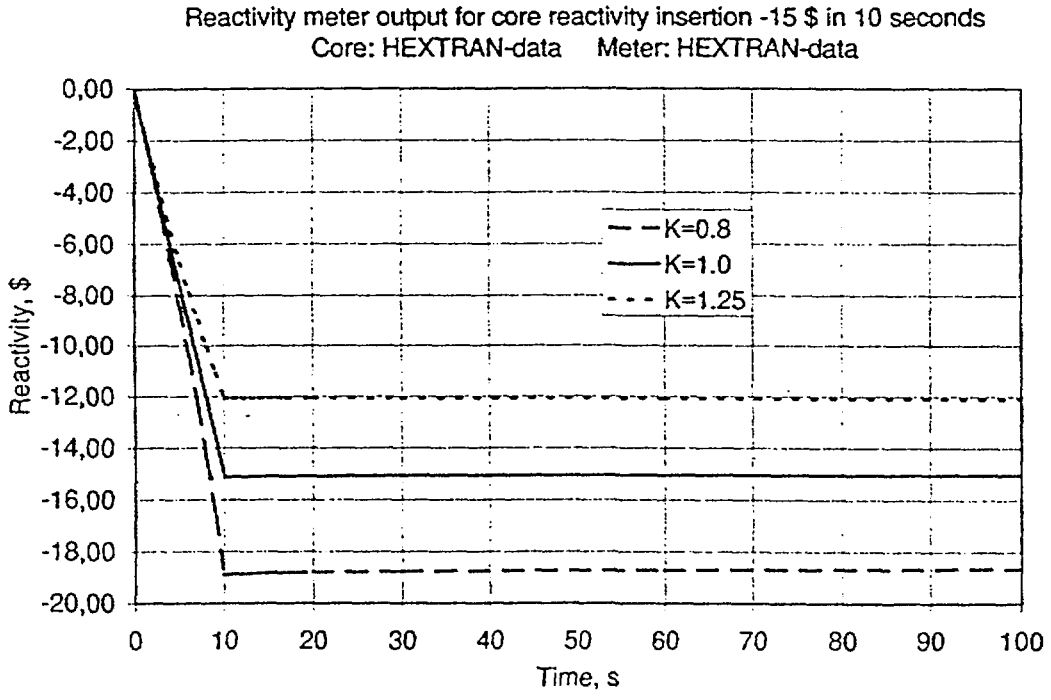

Figure 1. Simulation of reactivity meter performance for different values of the local flux distortion factor K. Core reactivity insertion –15 dollars.

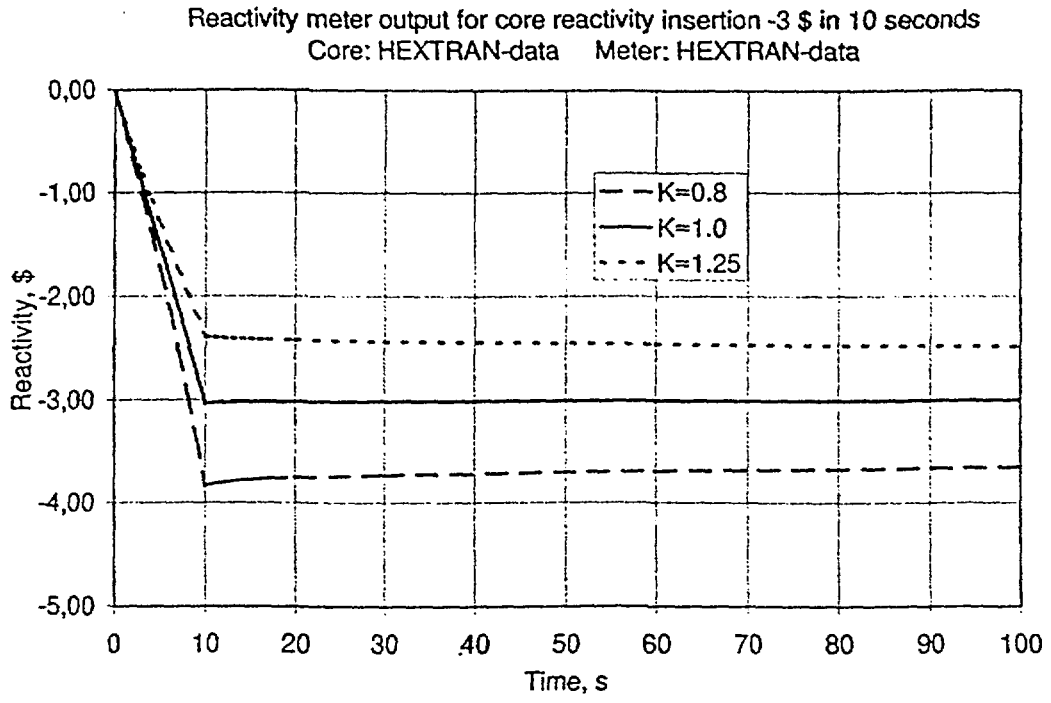

Figure 2. Simulation of reactivity meter performance for different values of the local flux distortion factor K. Core reactivity insertion –3 dollars.

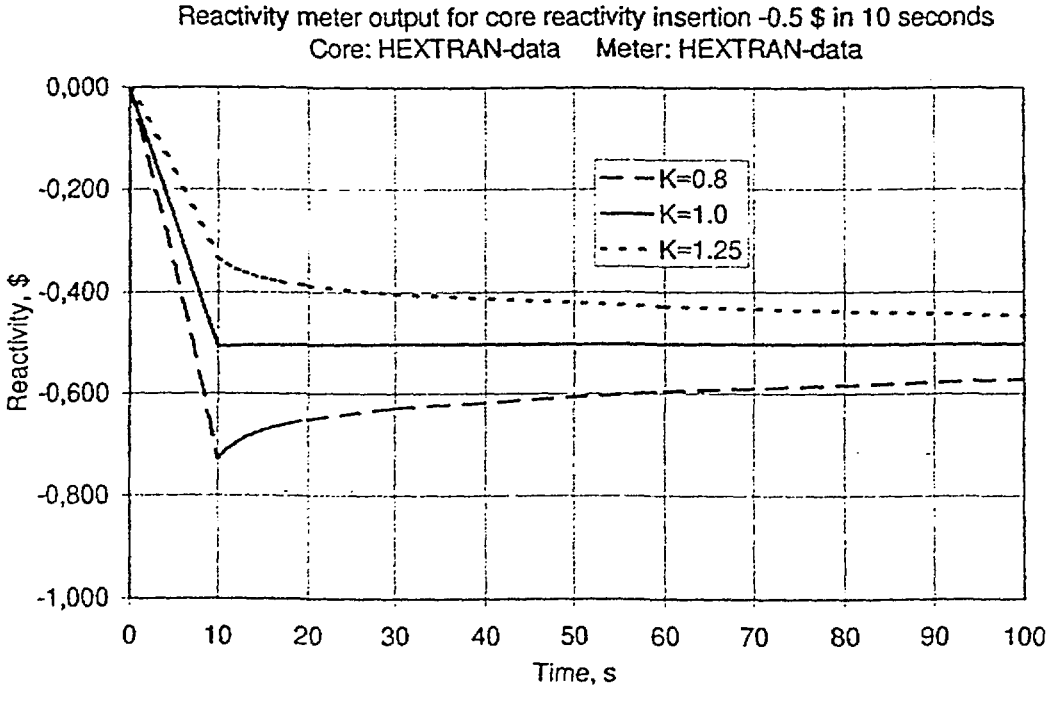

Figure 3. Simulation of reactivity meter performance for different values of the local flux distortion factor K. Core reactivity insertion –0.5 dollars.

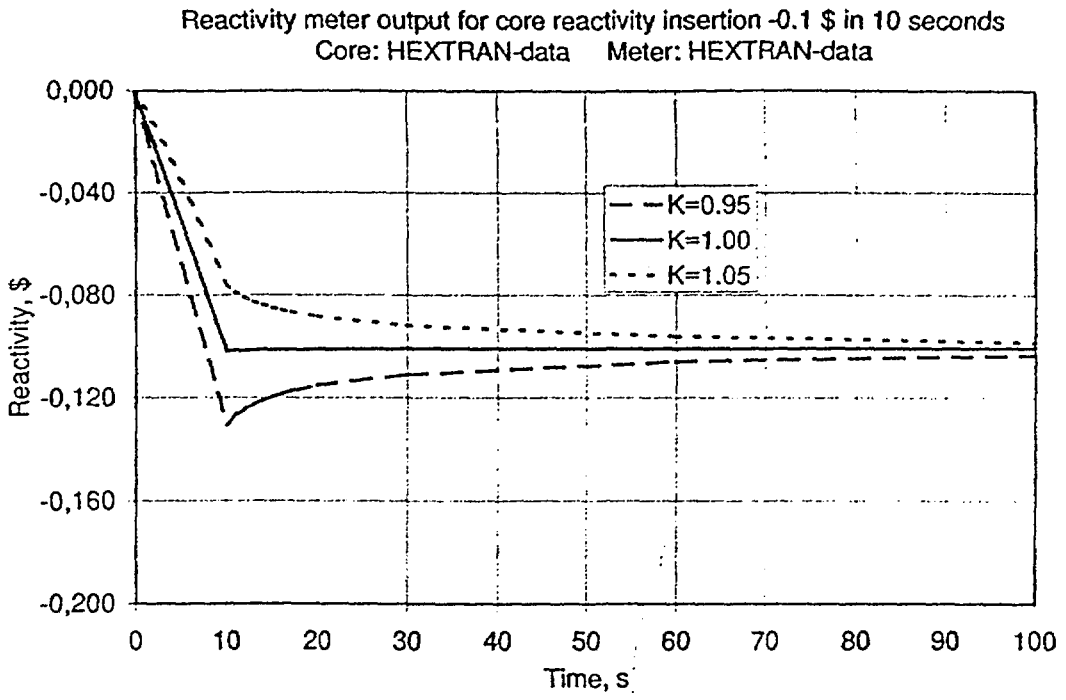

Figure 4. Simulation of reactivity meter performance for different values of the local flux distortion factor K. Core reactivity insertion –0.1 dollars.

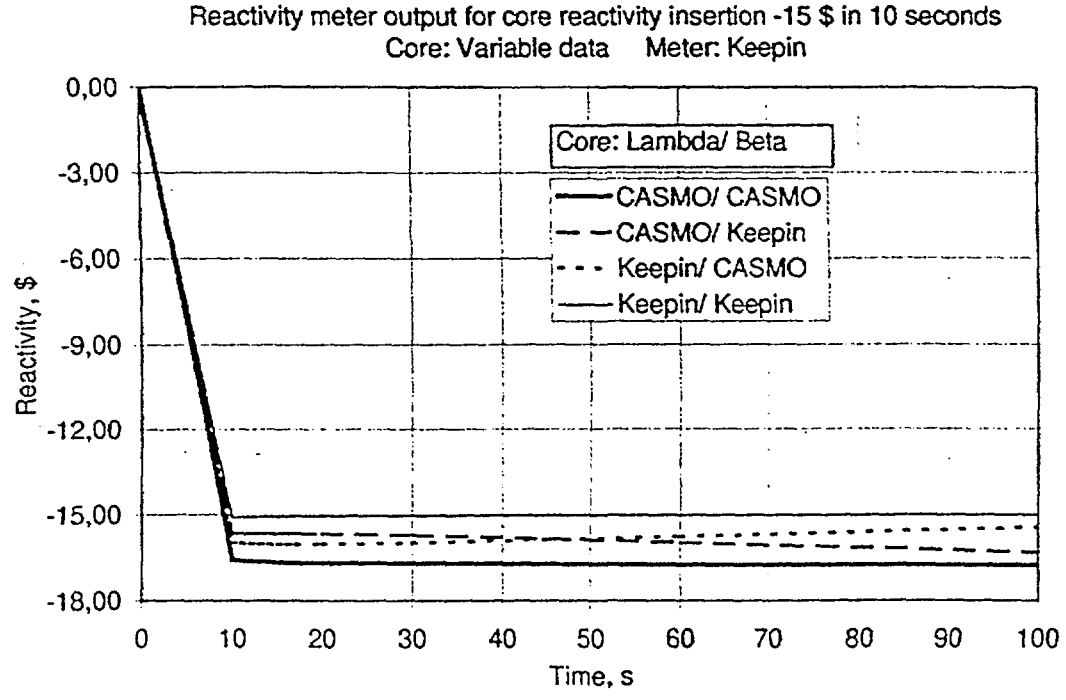

Figure 5. Simulation of the "Keepin reactivity meter" performance for different sets of neutron kinetics parameters in the core. Core reactivity insertion –15 dollars.

APPENDIX A: THE POINT KINETICS EQUATIONS

The purpose of this appendix is to review the point kinetics equations from the point of view of large negative reactivities. Emphasis is given to the correct interpretation of the kinetics parameters and to the behaviour of the solution.

A1. Basic equations

Let us assume a point reactor or a uniform finite reactor with leakage included. In order to keep this presentation simple, we assume a one-velocity or one-group model of the neutron flux. We also assume that diffusion theory is valid for describing neutron leakage. These assumptions are not very restrictive, particularly when we assume that all parameters can be time dependent. To keep the notation simple, this time dependence is not explicitly shown. Further, we make a clear distinction between the neutron density (or population) n and neutron flux density $\varphi = nv$, where v is the neutron velocity.

Using conventional notation, the time dependent diffusion equation and the equations for concentrations of delayed neutron precursors can be written in the following form:

$$\begin{aligned} \frac{dn}{dt} &= (1-\beta)v\Sigma_f nv - \Sigma_a nv - DB^2 nv + \sum_i \lambda_i C_i + Q \\ \frac{dC_i}{dt} &= \beta_i v\Sigma_f nv - \lambda_i C_i \quad ; \quad i = 1,\ldots,6 . \end{aligned} \tag{1a, 1b}$$

We then define the quantities

$$\begin{aligned} k &= \frac{v\Sigma_f}{\Sigma_a + DB^2} = \frac{k_\infty}{1 + B^2 M^2} \\ \ell &= \frac{1}{(\Sigma_a + DB^2)v} = \frac{1}{\Sigma_a v(1 + B^2 M^2)} . \end{aligned} \tag{2a, 2b}$$

Quantity k is the effective multiplication factor of the system and ℓ is the prompt neutron lifetime. The latter forms are given for information only and they employ the auxiliary definitions

$$k_\infty = v\Sigma_f / \Sigma_a \quad \text{and} \quad M^2 = D/\Sigma_a \ .$$

Using the first forms given in definitions (2) in equations (1) we obtain the basic form of the point kinetics equations:

$$\begin{aligned} \frac{dn}{dt} &= \frac{(1-\beta)k - 1}{\ell} n + \sum_i \lambda_i C_i + Q \\ \frac{dC_i}{dt} &= \frac{\beta_i k}{\ell} n - \lambda_i C_i \quad ; \quad i = 1,\ldots,6 . \end{aligned} \tag{3a, 3b}$$

We further define reactivity ρ as

$$\rho = \frac{k-1}{k} \quad \Rightarrow \quad k = \frac{1}{1-\rho} \; . \tag{4a, 4b}$$

k could now be completely substituted by ρ in equations (3), but there is also an alternate expression that can be used in equation (3a):

$$(1-\beta)k-1 = \frac{\rho-\beta}{1-\rho} = (\rho-\beta)k \; .$$

By defining the neutron generation time Λ as the quantity

$$\Lambda = \frac{\ell}{k} = \frac{1}{\nu\Sigma_f v} \tag{5}$$

equations (3) can be written in the standard form of the point kinetics equations

$$\begin{aligned} \frac{dn}{dt} &= \frac{\rho-\beta}{\Lambda} n + \sum_i \lambda_i C_i + Q \\ \frac{dC_i}{dt} &= \frac{\beta_i}{\Lambda} n - \lambda_i C_i \quad ; \quad i = 1,\ldots,6 \; . \end{aligned} \tag{6a, 6b}$$

Up to this point, no approximations are made regarding e.g. the value of k. The kinetic equation forms (3) and (6) are identical. The equations suggest that the kinetic behaviour of the neutron density is influenced by changes in cross sections and in buckling (leakage) as they are reflected in both reactivity ρ and in the lifetime ℓ or Λ.

It is important to distinguish between the two quantities of lifetime, which differ by a factor of k from each other. They can be written in the following informative forms by expanding the numerator and denominator by n in definitions (2b) and (5):

$$\ell = \frac{n}{(\Sigma_a + DB^2)\varphi} \quad ; \quad \Lambda = \frac{n}{\nu\Sigma_f\varphi} \; .$$

The prompt neutron lifetime ℓ is the virtual time required for removal of the existing neutron population at the current rate of neutron absorption and leakage. The neutron generation time Λ is the virtual time required for generation of the existing neutron population at the current rate of neutron production (including the associated delayed neutrons).

A2. Inverse point kinetics and the reactivity meter

Inverse point kinetics basically involves the solution of equations (6) for a given time dependence $n(t)$. Equation (6a) is solved algebraicly for reactivity and equations (6b) are used as such to follow the concentrations of delayed neutron precursors. Modern digital reactivity meters can easily work directly with equations (6).

In older types of reactivity meters, particularly in analogue devices, equations (6) are transformed into a more convenient form. Such transformations are also useful to gain insight into the equations and into the operation of a reactivity meter. A change of variables is performed for the concentrations of delayed neutron precursors by defining

$$C'_i = \frac{\lambda_i \Lambda}{\beta_i} C_i \quad \Rightarrow \quad C_i = \frac{\beta_i}{\lambda_i \Lambda} C'_i \ . \tag{7}$$

Substitution into equations (6) and multiplication of equation (6a) by Λ/β and equations (6b) by $\lambda_i \Lambda/\beta$ we obtain the following set of equations:

$$\begin{aligned} \frac{\Lambda}{\beta}\frac{dn}{dt} &= (\rho/\beta - 1)n + \sum_i \frac{\beta_i}{\beta} C'_i + \frac{\Lambda}{\beta} Q \\ \frac{dC'_i}{dt} &= \lambda_i (n_i - C'_i) \ ; \ i = 1,\ldots,6 . \end{aligned} \tag{8a, 8b}$$

The only assumption in the derivation is that the multiplier in transformation (7) is a constant that can be extracted from the time derivatives of C_i. In practice, this implies that the parameters λ_i , Λ , β_i , and hence also β are all constants. This is usually the case for all reactivity meters, except perhaps those that are embedded into a 3-D kinetic core model. By a further transformation of the precursor concentrations

$$C''_i = \nu C'_i = \frac{\lambda_i \Lambda \nu}{\beta_i} C_i \tag{9}$$

and upon multiplication of equations (8) by ν we obtain the following rather similar set of equations:

$$\begin{aligned} \frac{\Lambda}{\beta}\frac{d\varphi}{dt} &= (\rho/\beta - 1)\varphi + \sum_i \frac{\beta_i}{\beta} C''_i + \frac{\Lambda \nu}{\beta} Q \\ \frac{dC''_i}{dt} &= \lambda_i (\varphi_i - C''_i) \ ; \ i = 1,\ldots,6 . \end{aligned} \tag{10a, 10b}$$

Note the change of variable from neutron density n to neutron flux density φ in these equations. The assumption in the derivation is that also the neutron velocity ν is now a constant.

From equations (10) we can further obtain a standard prompt jump approximation as $\Lambda \rightarrow 0$. The time derivative in (10a) disappears and all other terms and definitions of variables remain finite, since by definition

$$\Lambda \nu = 1/\nu\Sigma_f \ .$$

This demonstrates the mathematical soundness of the zero lifetime approximation, in which the neutron flux is assumed to respond instantly to changes in reactivity. This approximation also implies that the specific value of Λ is not very important, as long as it remains constant as assumed and the real relaxation time of the prompt neutron response is indeed fast enough.

The inverse equations for solving reactivity obtained from equations (10) are

$$\frac{\rho}{\beta} = 1+\frac{\Lambda}{\beta}\frac{d\varphi/dt}{\varphi}-\frac{1}{\varphi}\left[\sum_i \frac{\beta_i}{\beta}C''_i+\frac{\Lambda\nu}{\beta}Q\right]$$
$$\frac{dC''_i}{dt} = \lambda_i(\varphi_i - C''_i) \quad ; \quad i=1,\ldots,6. \qquad (11a, 11b)$$

If we set Q=0 and $\Lambda\rightarrow 0$, then equation (11a) can further be simplified into the form

$$\frac{\rho}{\beta} = 1-\frac{\varphi_d}{\varphi} \quad \text{,where} \quad \varphi_d = \sum_i \frac{\beta_i}{\beta}C''_i . \qquad (12)$$

Equations (12) and (11b) have the following interpretation. The quantities C''_i are filtered values of the neutron flux, obtained by simple filtering of the actual neutron flux with different time constants λ_i according to equations (11b). The quantity φ_d is a delayed neutron flux value obtained as the weighted average value of the filtered neutron fluxes. The reactivity in dollars at any particular time is determined by the ratio of the delayed and instantaneous neutron flux values. A more physical interpretation is that the reactivity at any time is determined by the ratio of the magnitude of the delayed neutron source and the actual neutron flux level that is maintained by this source.

A3. The prompt response

The initial response of a point kinetic reactor can be characterised by the prompt response to a step change in reactivity. Let us take a closer look at the magnitude of this prompt response from an initial critical condition. In the initial state "0" reactivity is zero and $dn/dt = 0$. In the final state "1" reactivity assumes a (negative) value ρ and after the rapid transient $dn/dt = 0$ again. During the whole transient the source of delayed neutrons remains at a constant initial level. For simplicity, we also assume β to be constant.

Let us examine the prompt response obtained from both equations (3a) and (6a). With Q= 0 both equations have the form

$$\frac{dn}{dt} = \frac{\mathrm{f}(\rho)}{T}n+\sum_i \lambda_i C_i , \qquad (13)$$

where

$$\mathrm{f}(\rho) = \frac{\rho-\beta}{1-\rho} \quad ; \quad T=\ell \quad \text{in Eq. (3a)}$$
$$\mathrm{f}(\rho) = \rho-\beta \quad ; \quad T=\Lambda \quad \text{in Eq. (6a)} .$$

Substituting into equation (13) values in the initial and final states and eliminating the constant delayed neutron source we obtain the following formula for the prompt jump:

$$\frac{n_1}{n_0} = \frac{\mathrm{f}(0)}{\mathrm{f}(\rho)}\frac{T_1}{T_0} . \qquad (14)$$

The following two alternate formulas for the prompt jump of the neutron density are obtained:

$$\frac{n_1}{n_0} = \frac{1-\rho}{1-\rho/\beta}\frac{\ell_1}{\ell_0}, \text{ where } \frac{\ell_1}{\ell_0} = \frac{(\Sigma_a + DB^2)_0 v_0}{(\Sigma_a + DB^2)_1 v_1}$$

$$\frac{n_1}{n_0} = \frac{1}{1-\rho/\beta}\frac{\Lambda_1}{\Lambda_0}, \text{ where } \frac{\Lambda_1}{\Lambda_0} = \frac{(\nu\Sigma_f)_0 v_0}{(\nu\Sigma_f)_1 v_1}. \qquad (15a, 15b)$$

It can be shown that these two formulas are equivalent in spite of the apparent differences. If we assume ν is a constant, then there are two special cases. If the cross section for absorption and leakage does not change, then $\ell_1/\ell_0 = 1$ in equation (15a). If the neutron production cross section does not change, then $\Lambda_1/\Lambda_0 = 1$ in equation (15b). Note that the dependence of the jump on reactivity is different in these two cases. If ρ is of the order –0.10 (or –10 %), the difference is significant. Both cases can be found in the literature.

Equation (15b) can be written in the following simple form:

$$\frac{P_1}{P_0} \equiv \frac{\nu\Sigma_{f1} n_1 v_1}{\nu\Sigma_{f0} n_0 v_0} = \frac{1}{1-\rho/\beta}. \qquad (16)$$

This form tells that the prompt jump of the fission neutron production P depends only on the reactivity and not on the cross sections or neutron velocity. In a similar manner it can be seen from equations (15a) and (15b) that the prompt jump of the neutron flux density nv depends on cross section changes, but not on changes in neutron velocity.

A reactivity meter does not react directly to any of the neutron physical quantities discussed above. It reacts to the input current signal I. In a reactivity meter based on equations (6), (8) or (10) and using constant kinetics parameters the prompt jump always assumes the form

$$\frac{\rho}{\beta} = 1 - \frac{I_0}{I_1} \quad \Rightarrow \quad \frac{I_1}{I_0} = \frac{1}{1-\rho/\beta}. \qquad (17)$$

Therefore, such a reactivity meter will indicate the correct reactivity, if the measured signal is directly proportional to the fission neutron production in the reactor. If the efficiency ε of the measurement changes during the transient for some reason, e.g. due to distortion in the distribution of neutron production, then we can write

$$I_0 = \varepsilon_0 P_0 \quad \text{and} \quad I_1 = \varepsilon_1 P_1.$$

Using these definitions in equation (16), we obtain the following formula for the real reactivity of the reactor in terms of the measured currents:

$$\frac{\rho}{\beta} = 1 - \frac{I_0}{I_1}\frac{\varepsilon_1}{\varepsilon_0}. \qquad (18)$$

This can be compared with formula (17). By interpreting the measured reactivity value $(\rho/\beta)_m$ back to a detector current ratio in line with formula (17), the following expression is obtained between the real and measured reactivities:

$$[1-\rho/\beta] \;=\; [1-(\rho/\beta)_m]\frac{\varepsilon_1}{\varepsilon_0}. \qquad (19)$$

For large negative reactivity values, changes in detector efficiency, due e.g. to redistribution of the fission power, are reflected in the measured reactivity nearly in inverse proportion to the change in efficiency.

Finally, let us consider another effect resulting from a distortion in the distribution of neutron production during a step change in reactivity. The source of delayed neutrons has the initial distribution. After the change in the core, these neutrons can have a lesser importance, if they are born in regions of high absorption or otherwise low neutron flux. Based directly on equations (1b) we can write the initial delayed neutron source as

$$\sum_i \lambda_i C_{i0} \;=\; \sum_i \beta_i (\nu\Sigma_f n v)_0 \;=\; \beta_0 (\nu\Sigma_f n v)_0 .$$

After the reactivity change, the importance of this source is assumed to change, so that the source is effectively reduced by the factor η, as if the effective number of delayed neutrons per fission were only $\eta\beta_0$. A new value β_1 for new delayed neutron precursors may also be appropriate after the change. Proceeding in a manner similar to that given in equations (13) and (14), we obtain the prompt jump of the neutron density in the general form

$$\frac{n_1}{n_0} \;=\; \frac{\mathrm{f}(0,\beta_0)}{\mathrm{f}(\rho,\beta_1)}\frac{T_1}{T_0}\eta . \qquad (20)$$

Substituting in the values $\mathrm{f} = \rho - \beta$ and $T = \Lambda$, we can write in analogy with equation (16)

$$\frac{P_1}{P_0} \;\equiv\; \frac{\nu\Sigma_{f1} n_1 v_1}{\nu\Sigma_{f0} n_0 v_0} \;=\; \frac{1}{1-\rho/\beta_1}\frac{\eta\beta_0}{\beta_1}. \qquad (21)$$

This result indicates that the true measure of reactivity in dollars is β_1, which is effective after the change in the core. The additional complications of changes in distributions are reflected in the second factor on the right hand side. If $\eta < 1$, then the prompt jump in neutron production will be "greater" than that given by formula (16). Note that the definition of reactivity ρ is still unchanged in equation (21) and corresponds to the concept of static reactivity ρ_S.

However, if the prompt jump is described without assuming any jump in the effective delayed neutron source, then an equal jump can be obtained by assuming a different value of "dynamic reactivity" ρ_D in formula (21) with $\eta = 1$. Setting the two forms of the prompt jump to be equal yields the following relationship:

$$1-\rho_D/\beta_1 \;=\; \frac{1-\rho_S/\beta_1}{\eta}. \qquad (22)$$

This relationship gives one interpretation for the concept of dynamic reactivity and explains why big differences from static reactivity can be observed particularly during reactor scram.

A4. Time dependent solutions

In the following, we again assume that the parameters of the kinetics equations are constants. This assumption enables to find time dependent analytical solutions. We may as well work with equations (6), assuming $Q = 0$.

In closer view, the prompt response of the neutrons to a step change in reactivity is a fast transient. Solving from equations (6b) the concentrations of the delayed neutron precursors in the initial steady state and substituting into equation (6a) we obtain the following equation, valid after a step change in reactivity:

$$\frac{dn}{dt} = \frac{\rho-\beta}{\Lambda} n + \frac{\beta}{\Lambda} n_0 . \tag{23}$$

The solution to this equation with constant coefficients is

$$\frac{n(t)}{n_0} = \frac{P(t)}{P_0} = \frac{1}{1-\rho/\beta}\left[1 - \frac{\rho}{\beta}\exp\left(\frac{\rho-\beta}{\Lambda} t\right)\right]. \tag{24}$$

Note that the prompt jump given by the first factor is generally correct for the fission neutron production P. If the exponent term is denoted by αt, we have

$$\alpha = \frac{\rho-\beta}{\Lambda} = -\frac{\beta}{\Lambda}(1-\rho/\beta). \tag{25}$$

For typical VVER values $\beta = 0{,}006$ and $\Lambda = 25$ μs this time constant has the following values

ρ/β	α (1/s)	$T = -1/\alpha$ (ms)
0	-240	4.2
-1	-480	2.1
-5	-1440	0.7
-20	-5040	0.2

The prompt response is indeed fast and becomes increasingly faster for large negative reactivities. This demonstrates the validity of the prompt jump approximation. If the reactivity changes linearly, the neutrons will follow the prompt jump approximation with a delay of T.

The same exponential term represents the solution for the decay of neutrons in a pulsed experiment in a subcritical system. In this case, after removal of the external source, only the homogeneous part of equation (23) remains. From equation (25) we obtain the reactivity in terms of the measured value of α as

$$\rho/\beta = 1 + \alpha(\Lambda/\beta). \tag{26}$$

It is particularly evident in this case that the parameters β and Λ belong to the subcritical system. The same is true for the step change in reactivity from an initially critical state.

The general time-dependent solution of equations (6) with constant coefficients can be searched in the following form, replacing n with P:

$$P(t) = \sum_{k=1}^{7} P_k \exp(\omega_k t). \tag{27}$$

Similar expressions apply to the concentrations of delayed neutrons. The values of ω_k are determined as solutions of the reactivity equation or inhour equation [see e.g. Bell & Glasstone, p. 478]

$$\rho = \Lambda\omega_k + \sum_i \frac{\beta_i \omega_k}{\omega_k + \lambda_i}. \tag{28}$$

There are seven roots to this equation. It is known from textbooks that for large negative values of reactivity the roots approach the values $-\lambda_i$ (i = 1,...,6) and $-1/\Lambda$. By searching for the first six roots in the form

$$\omega_k = -\lambda_k + \varepsilon_k \tag{29}$$

we find by truncating small terms that

$$\varepsilon_k \cong \frac{-\beta_k \lambda_k}{\rho - \beta} \quad \Rightarrow \quad \frac{\varepsilon_k}{\lambda_k} \cong -\frac{\beta_k}{\rho - \beta} = \frac{\beta_k/\beta}{1 - \rho/\beta}. \tag{30}$$

The relative deviations of the roots from their asymptotic values (i.e. ε/λ) are at most a few percent when the reactivity exceeds –10 dollars. When the reactivity varies by 1 dollar, the variation in ω_k is less than 0.4 percent for all delayed neutron groups; for the first group with the longest decay time it is only 0.03 percent. Hence, there is a very weak dependence of the six time constants on reactivity. For the fastest time constant we obtain by truncating small terms

$$\omega_7 \cong \frac{\rho - \beta}{\Lambda}. \tag{31}$$

This is the same value obtained in formula (25) for α. The coefficients P_k are given by the following formula [see e.g. Bell & Glasstone, p. 478]:

$$\frac{P_k}{P_0} = \frac{\Lambda + \sum_i \frac{\beta_i}{\omega_k + \lambda_i}}{\Lambda + \sum_i \frac{\lambda_i \beta_i}{(\omega_k + \lambda_i)^2}}. \tag{32}$$

By substituting the values of ω_k into (32) and by truncating small terms we obtain the following approximate values for the first six coefficients:

$$\frac{P_k}{P_0} \cong \frac{\varepsilon_k}{\lambda_k} = \frac{\beta_k/\beta}{1-\rho/\beta}. \tag{33}$$

The seventh coefficient is given by

$$\frac{P_7}{P_0} \cong \frac{\rho}{\rho-\beta} = \frac{-\rho/\beta}{1-\rho/\beta}. \tag{34}$$

The sum of the seven coefficients is unity, which indicates that any small errors compensate each other in the approximations. The entire solution can be written as

$$\frac{P(t)}{P_0} \cong \frac{1}{1-\rho/\beta}\left[\sum_i \frac{\beta_i}{\beta}\exp(\omega_i t) - \frac{\rho}{\beta}\exp(\alpha t)\right], \tag{35a}$$

where

$$\omega_i \cong -\lambda_i\left(1-\frac{\beta_i/\beta}{1-\rho/\beta}\right); \quad \alpha = -\frac{\beta}{\Lambda}(1-\rho/\beta). \tag{35b, 35c}$$

A general implication of formula (35) is that information on large negative values of reactivity is present in the prompt jump and in the time constant of the fast response. There is rather weak information on reactivity in the asymptotic behaviour after the prompt response, i.e. after the reactivity change is over.

After the prompt response, also the logarithmic derivative of P is rather insensitive to reactivity. The explicit expression for the logarithmic derivative is

$$\frac{1}{P}\frac{dP}{dt} = \frac{\sum_i \omega_i(\beta_i/\beta)\exp(\omega_i t)}{\sum_i(\beta_i/\beta)\exp(\omega_i t)} \cong \frac{\sum_i \omega_i(\beta_i/\beta)\exp(-\lambda_i t)}{\sum_i(\beta_i/\beta)\exp(-\lambda_i t)}. \tag{36}$$

The derivative is a weighted average of the time constants ω_i. In principle, there is a small shift in the values when the reactivity is varied, but this shift is typically only fractions of one percent. Such differences are difficult to observe. Even if the observation is highly accurate, the values of λ_i are not defined to comparable accuracy to determine the difference. According to formula (35b), it is this difference, which depends on reactivity.

The physical explanation is that the asymptotic time behaviour is dominated by the decay of the initial delayed neutron source. The contribution of new delayed neutron precursors has only a weak effect.