The Investigation of Added Masses and Damping Factors for Vibrations of Tube and Tube Bundles in Fluid by V. F. Sinyavskii, V. S. Fedotovskii and A. B. Kukhtin, USSR.

The vibrations of single cylinders in fluid being surrounded by the solid walls of different form as well as the bundles of cylindric rods have been considered in this report.

It has been proposed a model of the hydrodynamic damping of vibrations and the analytic solution of a problem concerning the damping of cylinder vibrations in fluid surrounded by a concentric shell. It has been shown the influence of fluid viscosity and vibration frequency on a value of the fluid added mass and the damping factor of vibrations.

For the cylinder being vibrated near the boundaries of a special type (for example, a flat wall, a plane gap) and for the cophased vibrations of some triangular and square packed rod bundles there have been carried out the calculations for the added masses and the damping factors by an electrodynamic analogy method.

The calculational results are compared with the available experimental data. A satisfactory agreement between the experimental data and the calculational results indicates the possibility for practical using the suggested procedure and the calculational formulae for determining the natural frequencies and the damping factors of vibrations for elastic cylindrical elements in different fluids.

For calculating the natural frequencies and the amplitudes of elastic element vibrations in fluid, for example, the cylindric fule elements in a nuclear reactor or the tubes of a heat exchanger, it is necessary to know a value of the fluid added mass and the damping force.

In a number of the works [1,2] it has been shown that the vibration frequiences of constructive elements within the coolant flows in case of the practicable ranges of velocities are close to the natural frequencies of vibrations in stationary fluid and are not dependent upon the velocity of fluid flow. This fact allows to determine the added mass values for evaluating the frequency characteristics of vibration in some flows obtained in the experiments in stationary fluid.

The damping of vibrations in turbulent flow depends upon the fluid velocity. Its value, as some experiments show, increases with the growth of a velocity [2]. The damping forces for stationary fluid have minimum quantities. However, the damping being even existed in stationary fluid can considerably exceed the constructive damping [3].

As it has been shown by the experiments [3] the fluid viscosity has a considerable influence upon the added mass quantity and the vibration damping. Moreover, these characteristics are strongly dependent upon the positioning of the fixed boundaries and the adjacent vibratory elements that surround the cylinder.

We consider the vibrations of a cylinder of radius a within the cavity filled up of incompressible fluid and being confined by a concentric shell of radius b. If the amplitude of cylinder vibrations is considerably less than its radius and a magnitude of the gap between the cylinder and the shell, then the occuring movement of fluid will possess a nonstalling character.

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When the Reynolds number $Re = U_0 a/\nu >> 1$, then the thin vibrational boundary layers are formed on the surface of vibrating cylinder and on the shell that surrounds this one.

It is believed that the influence of fluid viscosity will appear only in the boundary layers and beyond these layers the fluid movement will be potential.

At the small thickness of boundary layers the fluid movement in those ones one may consider as a plane one and the tangential velocity field may be taken the same as for the boundary layer originating at the vibrating flat plate. It has been known that at vibrating the infinite flat plate according to the harmonic law in a direction of its plane the velocity field in the coordinate system associated with this plate takes the form (4)

$$U(y,t) = U_0\left[\cos\omega t - \exp\left(-\frac{y}{\delta}\right)\cos\left(\frac{y}{\delta} - \omega t\right)\right], \qquad (1)$$

where $\delta = \sqrt{2\nu/\omega}$ is the characteristic thickness of a vibrating boundary layer, ω is the cyclic frequency; y is the distance up to a plate.

Taking into consideration that the added mass is determined by a fluid kinetic energy one can obtain an average value of the energy loss thickness T per one vibration period Δ from the condition:

$$\Delta \frac{1}{T_0}\int_0^T U_0^2\cos^2\omega t\,dt = \frac{1}{T}\int_0^T\int_0^\infty U_0^2\exp\left(-\frac{2y}{\delta}\right)\cos^2\left(\frac{y}{\delta} - \omega t\right)dy\,dt \qquad (2)$$

Integrating (2), we obtain:

when $$\Delta = \frac{1}{2}\delta. \qquad (3)$$

vibrating the cylinder surrounded by a fixed concentric shell we shall regard that some fluid within the boundary layers is braked and it does not take part in vibrational movement on the fixed shell. On the surfaces of vibrating cylinder fluid in the Δ layer moves as a solid body having a kinetic energy of $E = \pi\rho a\Delta u^2(t)$.

Thus for oalculating the added mass it should be computed a fluid energy in the $b-\Delta > r > a+\Delta$ gap and to it must be added the fluid energy in boundary layer on the surface of vibrating cylinder

$$m = \frac{2E(t)}{u^2(t)} = \frac{2\rho\int_0^\pi\int_{a+\Delta}^{b-\Delta}\left[U_r^2(r,\theta,t) + U_\theta^2(r,\theta,t)\right]r\,dr\,d\theta + \pi\rho a\Delta u^2(t)}{U^2\cos^2\omega t}; \qquad (4)$$

here

$$U_r(r,\theta,t) = -U_0\frac{(a+\Delta)^2}{(b-\Delta)^2-(a+\Delta)^2}\left[1 - \frac{(b-\Delta)^2}{r^2}\right]\cos\theta\cos\omega t, \qquad (5)$$

$$U_\theta(r,\theta,t) = U_0\frac{(a+\Delta)^2}{(b-\Delta)^2-(a+\Delta)^2}\left[1 - \frac{(b-\Delta)^2}{r^2}\right]\sin\theta\cos\omega t \qquad (6)$$

are the radial and tangential components of the velocity of a potential flow in the gap.

Performing the integration, we obtain

$$m = \pi\rho(a+\Delta)^2\frac{(a+\Delta)^2+(b-\Delta)^2}{(b-\Delta)^2-(a+\Delta)^2} + \pi\rho a\Delta \qquad (7)$$

At $\Delta \to 0$ the formula (7) transforms into the Stokes' formula (5):

$$m = m_0\frac{a^2+b^2}{b^2-a^2} \qquad (8)$$

where $m_0 = \pi\rho a^2$ is the cylinder added mass in the infinite volume of ideal fluid.

In case of the cylinder vibrating in the infinite volume of viscous fluid $(b\to\infty)$ the expression for an added mass will be of the following form:

$$m_\infty = m_0\left[1 + 2\frac{\delta}{a} + \frac{1}{4}\left(\frac{\delta}{a}\right)^2\right] \qquad (9)$$

Now we consider the problem on a hydrodynamic damping of the cylinder vibrations. The hydrodynamic damping factor is conveniently defined in terms of the velocity of energy dissipation in fluid. The mean velocity of energy dissipation per a vibration period is equal:

$$\frac{dE}{dt} = \frac{1}{T}\int_0^T F \cdot u\,dt = \frac{1}{T}\int_0^T \xi u^2 dt, \qquad (10)$$

from this it follows that

$$\xi = \frac{2(dE/dt)}{u_0^2} \qquad (11)$$

The velocity of energy dissipation in viscous fluid is expressed by the formula (6)

$$-\frac{dE}{dt} = \mu\int[\mathrm{rot}\,\vec{u}]^2 dV + \mu\int\frac{d\vec{u}}{dn}ds + 2\mu\int[\vec{u}\cdot\mathrm{rot}\,\vec{u}]ds \qquad (12)$$

where $\vec{u}$ is the velocity field in fluid occuring at the cylinder vibrations; n is the direction of the outer normal to the streamlined surface. The first integral in Eq.(12) gives the velocity of energy dissipation in the boundary layer region and the second one gives it in the potential flow-around region. The third integral in virtue of the condition of fluid adhesion on a solid surface is equal to zero. The calculations show that in the thin boundary layers the velocity of energy dissipations in the potential flow-around region is negligible as compared to the energy dissipation in the boundary layer.

From the first integral it follows that with due regard for Eq(1) the mean velocity of energy dissipation per unit surface for the plane boundary layer is equal to:

$$\frac{dE}{dt} = \frac{\mu u_*^2}{2}\sqrt{\frac{\omega}{2\nu}}\,, \qquad (13)$$

where

u_* is the local amplitude value of fluid velocity beyond the boundary layer in a system connected with the fixed surface.

For the concentric shell surrounding the cylinder being vibrated, the amplitude value of relative velocity is approximately equal to the amplitude value of a tangential velocity component (6) at $r=b$ for the potential fluid movement at $\Delta = 0$.

$$u_\theta(b,\theta,0) = u_0\frac{2a^2}{b^2-a^2}\sin\theta \qquad (14)$$

For calculating the velocity of energy dissipation in the boundary layer over a cylinder being vibrated it should be taken the velocity in a coordinate system associated with a cylinder as the amplitude velocity.

$$u_\theta(a,\theta,0) = \left[u_0 + u_0\left(\frac{b^2+a^2}{b^2-a^2}\right)\right]\sin\theta = u_0\frac{2b^2}{b^2-a^2}\sin\theta \qquad (15)$$

Substituting Eqs.(14), (15) in Eq. (13) and performing the integration over the surfaces of a cylinder and a shell, we obtain:

$$-\frac{dE}{dt} = \frac{2\pi\mu a u_0^2}{\delta}\left[\frac{b^4+a^3b}{(b^2-a^2)^2}\right] \qquad (16)$$

So, from Eq. (11) we have:

$$\xi = \frac{4\pi\mu a}{\delta}\left[\frac{b^4+a^3b}{(b^2-a^2)^2}\right] \qquad (17)$$

In case of the cylinder vibrating in the infinite volume of fluid the damping factor, ξ_∞, became equal to $4\pi\mu a/\delta$, this coincides with the result in (7). Thus the multiplier enclosed in the square brackets in Eq. (17) characterizes the influence of gap magnitude on the damping factor of vibrations

$$\frac{\xi}{\xi_\infty} = \left[\frac{b^4+a^3b}{(b^2-a^2)^2}\right] \qquad (18)$$

For a number of the practical problems the determination of dynamic characteristics of the elastic cylindric elements surrounded by the walls of arbitrary shape or the bundles of similar elements at different relative intervals between them is of some interest.

Taking into consideration some serious mathematical difficulties of an analytical solution of such problems in this case for determination of the added masses and the damping factors it can be used an electrohydrodynamic analogy method.(EHAM).

The added mass in ideal fluid approximation is calculated from the following formula:

$$m = \frac{\rho \int_s \varphi \frac{\partial \varphi}{\partial n}}{u^2}, \tag{19}$$

where

φ is the hydrodynamic potential on a surface of the cylinder moving with the velocity u ; n is the normal to its surface.

When operating on the electroconductive paper it is convenient to use the EHAM method for a transformed problem, i.e., the problem of a flow - around of the cylinder or the bundle of cylinders by the potential flow. In this case φ corresponds to a potential conditioned by the movement of a cylinder or some cylinders in stationary fluid.

Using the analogy

$$\varphi_{hydro} \to \varphi_{electric}$$

and having written the boundary conditions on the cylinder surface in the form

$$\frac{\partial \varphi}{\partial n} = -u_o \cos\theta,$$

we obtain from Eq. (19) a ratio of the added mass of the cylinder moving in the flat gap or one of the cylinders in the bundle to the added mass of the cylinder moving in the infinite volume of ideal fluid.

$$\frac{m}{m_o} = \frac{2h}{I6\pi a}\int_0^{2\pi} \varphi(a,\theta)\cos\theta \, d\theta - 1, \tag{20}$$

where $I = 2h \cdot j$ is the current; h is the half-width of an electroconductive paper sheet; j is the current density; σ is the specific electric resistance of a paper; a is theradius of a cutted circle.

For calculating a damping factor it has been used the ratio (13) for the mean velocity of energy dissipation in the boundary layer, and as u_*, it has been used the local velocity on the surfaces surrounding the cylinder.

$$u_* = \frac{\partial \varphi}{\partial l} \tag{21}$$

where

l is the direction along the fixed surfaces.

The tangential velocity with respect to an element of the moving cylinder surface is equal:

$$u_* = -\frac{1}{a}\frac{\partial \varphi}{\partial \theta} + u_o \sin\theta \tag{22}$$

Substituting Eqs.(21) and (22) into Eq. (13) and integrating with respect to surfaces of the moving cylinder and surrounding surfaces, we obtain from Eq.(11) the formula for calculating a damping factor for the cylinder vibrating in a flat gap.

$$\frac{\varepsilon}{\varepsilon_\infty} = \frac{h^2}{\pi a^2 I^2 \sigma^2}\left\{\left[\frac{1}{a}\int_0^{2\pi}\left(\frac{\partial \varphi}{\partial \theta}\right)^2 d\theta\right] + 2\int_{-\infty}^{\infty}\left[-\frac{\partial \varphi}{\partial x} - \frac{I\sigma}{h}\right]^2 dx\right\} \tag{23}$$

In case of the cylinders vibrating in the neighbourhood of the wall in Eq.(23) a multiplier interposed before the second integral is equal to unity.

The experimental investigations of the added masses and the vibration damping factors were accomplished on the single cylindric tubes and the rods executing small bending vibrations. In these experiments there have been measured the frequencies of the natural vibrations and the logarithmic decrements in the process of cylinder vibrations in air, water, acetone, mercury and the aqueous solutions of glycerine so that to cover the complete range of coolants being applied in practice with respect to viscosity and density of fluid. For conducting these tests the three experimental sections of round cross-section and the one experimental section of rectangular cross-section with a variable distance between the two walls have been used.

In the first three sections the tubes and the rods made the bending vibrations when employing the end gripping of two kinds (a console and a fixed-hinged one) in case of changing the b/a ratio from 1.04 to 7 .The relative length of a tube was over 40 . Then the influence of the end effects on the investigated characteristics can be neglected (8) .

The height of the experimental section with a rectangular cross-section of 76 x 85 amounted to 495 mm. Inside of the channel on both sides from the investigated tube it was mounted a pair of moving plates which could be transferred and installed on a required distance from the tube. At the top of this channel it was mounted a rotating device with a string allowing to fix the plane of tube vibration. Therefore, besides the free vibrations in the arbitrary planes in these tests the tubes could execute some bending vibrations in the strongly fixed directions. In these tests the tubes were of 8.0 mm and 12.0 mm in outer diameter. The relative length of these tubes l_{tube}/a was over 80.

The added mass was calculated with the known formula:

$$m = M_{tube}\left(\frac{\omega^2_{air}}{\omega^2_{liq}} - 1\right), \qquad (24)$$

where ω_{air} and ω_{liq} are the frequencies of the tube vibrating in air and liquid, respectively; M_{tube} is the tube mass per unit length. According to the oscillograms obtained the values of a logarithmic decrement of vibrations have been defined as:

$$\beta = \beta_M - \beta_K , \qquad (25)$$

where

β_k is the logarithmic decrement of the tube vibrating in air that characterizes a constructive damping. In case of the free vibrations while the damping force is proportional to the first power of the velocity, the damping factor of vibrations ξ is associated with the logarithmic decrement by means of the following expression:

$$\xi = \frac{\beta\omega}{\pi}\left(m + M_{tube}\right) \qquad (26)$$

The experiments have showed that in case of the tubes vibrating in "infinite" volumes of fluids (b/a=7) the added mass is rapidly growing with increase in a relative thickness of the boundary layer.

As is seen from the fig.1 the formula (9) describes satisfactory increase in the added mass with growing in the $\sqrt{\frac{2\nu}{\omega}}/a$ parameter in a range of the vibration frequencies and fluid viscosities being investigated, while this has been obtained under the assumption of a thin boundary layer.

A hydrodynamic damping of vibrations is also depended upon the fluid viscosity and the cylinder vibration frequency. Fig.2 demonstrates the experimental results for dimensionless damping factor from the $\sqrt{\frac{2\nu}{\omega}}/a$ parameter.

In a range of the large parameter values the experimental results are adequately approximated by the formula

$$\frac{\xi_\infty}{\mu} = 9{,}4\,\frac{a}{\sqrt{2\nu/\omega}} \qquad (27)$$

differed from the formula (17) at $b \to \infty$ by a constant multiplier. A large spread of the experimental results at the small magnitudes of the $\left(\sqrt{\frac{2\nu}{\omega}}/a < 0{,}1\right)$ parameter is apparently connected with both the commensurability of a hydrodynamic and constructive damping and the adopted methods of an experimental results treatment.

In case of the cylinder vibrating in a concentric shell the added mass and the damping factor of vibrations are significantly depended not only on the $\sqrt{\frac{2\nu}{\omega}}/a$ parameter but also on the value of a gap between the cylinder and the shell (figs. 3,4). The experimental data on the added mass ratios for the cylinder vibrating in a concentric shell and in an infinite fluid are fell out, on average, by 20% below the Wambsgans's and Chen's design relationship, ξ/ξ_∞ (9) and the Stokes' theoretical one for ideal fluid (8) . When treating the experimental results on the vibration damping as a function of b/a ,the damping factor of vibrations, ξ_∞ , is calculated from the semiempirical formula (27). As will be apparent from the fig.4 the formula (18) describes the experimental results satisfactory.

In the experimental section of a rectangular cross-section there have been obtained the adedd masses and the damping factors of vibrations when the cylinder is equally spaced from both the walls (the vibration proceeds within a plane gap) and when one of the walls is spaced at a distance of $y_0 > 3a$, while the influence of this wall on the characteristics investigated is not appeared substantially (the vibration proceeds near a plane wall). An analysis of the obtained results presented in figs. 5,6 in terms of m/m_∞ and ξ/ξ_∞ from y_0/a shows that the added mass and the damping factor are substantially dependent not only on the shape of bounding surfaces but also on the direction of vibrations.

As would be expected the minimum quantities of the added mass and the damping factors of vibrations have been obtained at the cylinder vibrating in the neighbourhood of the plane wall. In this case in an investigated range of the relative distance change, y_0/a , the experimental data on the added masses and the damping factors, when the direction of vibrations is parallel or normal relative to the wall are close in values. As may be seen from the figures there is a satisfactory agreement between the experimental and the theoretical results.

It is interesting to note that as the distance between the cylinder and the wall is decreased the design values of m/m_∞ tend to some limiting values and amount to ~ 2.5 at $y_0/a = 0$ this is consistent with the theoretical data obtained in (10)

Note the character of changing a damping factor in this case. As the distance between the cylinder and the wall is decreased, at vibrations being parallel to the wall, the value of ξ/ξ_∞ increases to 4.1 at y_0/a =0.045 and, when contacting with the surface, it became equal to ~ 1.4. It is associated with a sharp change of the cylinder flow-around character. When the cylinder and the plane surface are in contact, then in the neighbourhood of the contact point the stagnant zones are occured in which the energy dissipation is very small. Apparently, it should be considered that as the cylinder is approaching the surface a damping will increase until coupling between the boundary layers of a cylinder and the plane wall will occur , whereupon the ratio of ξ/ξ_∞ should decrease.

Unlike the case of the cylinder vibrating near by one wall, in case of the cylinder vibrating in a plane gap the ratios of m/m_∞ and ξ/ξ_∞ are depended upon the direction of vibrations. At $y_0/a \to 0$ for the case of vibrating along the walls the added mass tends to infinity. The damping factor at $y_0/a \to 0$ for the longitudinal and transverse direction of vibrations tends to infinity.

It is interesting to note that in case of the cylinder vibrating in an arbitrary plane when the direction of vibrations is not given the values of m/m_∞ and ξ/ξ_∞ are close to the appropriate ones obtained in case of vibrating in the direction being perpendicular to the wall, i.e., the minimum quantities of the added mass and the damping factor of vibrations.

As is seen from the fig.5 in case of the cylinder vibrating in the direction parallel to the walls the experimental results on the ratio of m/m_∞ are about 35% below the appropriate design values. Similarly to the case of vibrating along the single plate this is apparently associated with the bending shape of vibrations ignored in the calculations.

When the values of y_0/a are equal, the added masses and the damping factors for the cases of the cylinder vibrating near the single plate, within the flat gap, and in the concentric shell are substantially differed (figs. 5,6). When the values of y_0/a are small the added masses and the damping factors can be differed by the order of magnitude and more.

Of large practical interest is the determination of the added masses and the damping factors for the bundles of rods.

The authors of this paper have carried out the calculations of these characteristics for cophasal vibrations of the trigonal and tetragonal packed rods bundles with the help of a hydrodynamic analogy method (EHAM) using the formulae (20) and (23).

The figs. 7,8 represents the calculation results, the diagrams of these rods arranging within a bundle, and the direction of vibrations for each case.

The greatest values of the added masses and the damping factors have been obtained for the case of the trigonal packed rods bundle when a plane of the vibrations is directed along the bisectrix of a triangle angle formed by the centres of these rods.

The added masses and the damping factors for the trigonal and tetragonal packed rods are strongly distinguished in case of the small values of y_0/a . This is mainly associated with a different restraint of these bundles. The differentiation of the added masses and the damping factors for various planes of vibrations of the similar bundles is conditioned by the difference in distributing some velocities around the cylinders.
It should be noted that the dependencies of m/m_∞ and ξ/ξ_∞ upon a relative pitch for the bundles of rods as well as the similar dependencies for the cylinder vibrating in the plane gap near a wall and in the concentric shell are satisfactory approximated with the use of the power expressions in the form

$$m/m_\infty = 1 + A(y_0/a)^k \quad (28)$$

$$\xi/\xi_\infty = 1 + B(y_0/a)^n \quad (29)$$

The values for factors A, k, B and n are presented in the Table I.

Unlike a case of the cylinder vibrating near a plane wall or within a plane gap as to the bundle of rods being cophasal vibrated in fluid within a certain range of y_0/a it is of importance to consider the model of equivalent cells, i.e., to reduce the problem of determining the added mass and the damping factor from an appropriate formulae to the equivalent circular gap problem. The approximate method of calculating an added mass from the model of equivalent (over an area) cells was proposed in the work (11). In this work the experimental data for the multiserial packets of the tetragonal packed rods are given.

An added mass for the bundles of rods is calculated from the Stokes' formula (8) where as a radius of the surrounding shell should be taken the value of b being equal:

$$b = \frac{2}{\sqrt{\pi}}(a + y_0), \quad (30)$$

for the trigonal and tetragonal packed rods

$$b = \sqrt{\frac{3{,}464}{\pi}}(a + y_0). \quad (31)$$

As is seen from the fig.7 the calculational results for the added masses of the rod bundles from the Stokes' formula and appropriate relations of (30) and (31) are sufficiently consistent with the results obtained by the EHAM method in a range of $y_0/a > 0{,}1$ and $y_0/a > 0{,}2$ for the trigonal and tetragonal packing rods, respectively. In the more close bundles the calculation performed by the equivalent cells model underestimates the added mass values significantly. In these cases a calculation should be carried out from the formula (28). In the same manner, when using an equivalent cells model, it can be obtained the formula for calculating the damping factor. Because of lack of solid walls on the external boundaries of cells surrounding each cylinder and, respectively, the boundary layers do not occur, thereupon, when replacing the appropriate cell by the equivalent circular cell it should be considered the energy dissipation only on the surface of the cylinder vibrated. In this case the damping factor will take the form

$$\frac{\xi}{\xi_\infty} = \frac{b^4}{(b^2 - a^2)^2} \quad (32)$$

It is seen from fig.8 that the formula (32) describes satisfactory the results of calculation by the EHAM method up to $y_0/a > 0{,}1$ and $y_0/a > 0{,}2$ for the trigonal and tetragonal packing of the rods, respectively. For the more close bundles it should be used the formula (29).

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TABLE I THE VALUES OF FACTORS IN THE FORMULAE OF (28) AND (29)

Boundary shape	Direction of vibrations	A	k	B	n	Field of application
Concentric shell		1	-1	0.5	-2	Limiting values at $\frac{y_0}{a} \to 0$
		o.9	-1	0.65	-1.95	In a range of $0.01 < y_0/a < 0.1 \div 0.2$
Plane gap	Parallel to walls	1.14	-0.64	1.30	-1.24	Calculation $0.01 < y_0/a < 0.1 \div 0.2$
	Perpendicular to walls	0.30	-0.60	0.77	-0.83	Experiment $0.1 < y_0/a < 0.6$
Trigonal package of rods	Along bisectrix	1.18	-0.79	0.58	-1.60	$0.01 < y_0/a < 0.1 \div 0.2$
	Along side	1.18	-0.71	0.74	-1.35	
Tetragonal package of rods	Along bisectrix	0.93	-0.69	0.58	-1.35	$0.01 < y_0/a < 0.1 \div 0.2$
	Along side	1.04	-0.70	0.64	-1.35	

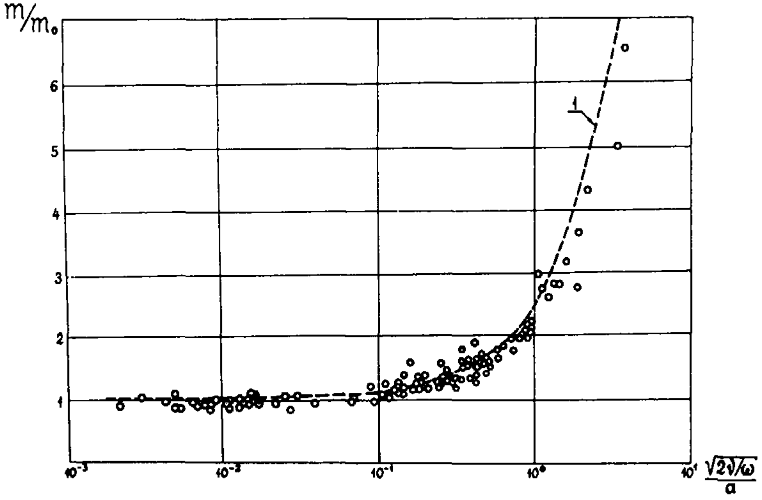

Fig.1. The dependence of dimensionless added masses on the $\sqrt{2\nu/\omega}/a$ parameter for single tubes vibrating in "infinite" volumes of fluids.
1 - calculation from the formula (9)
o - experimental data belonging to authors.

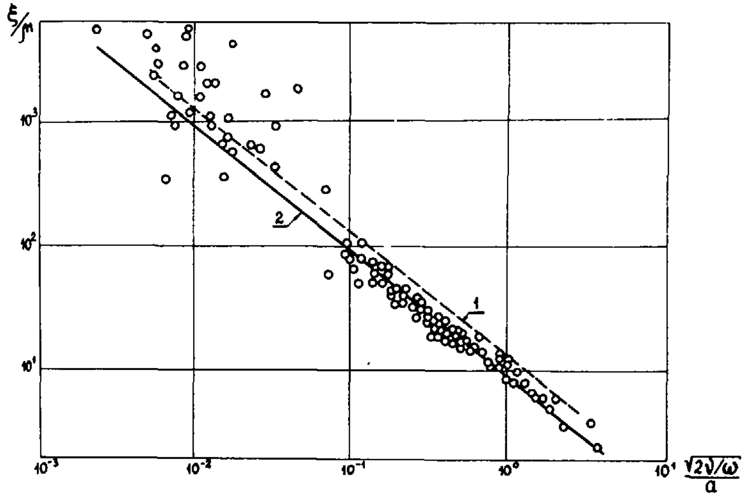

Fig.2. The dependence of damping factors on the $\sqrt{2\nu/\omega}/a$ parameter for single tubes vibrating in "infinite" volumes.
1 - calculation from the formula (17),
2 - calculation from the formula (27),
o - experimental data belonging to authors.

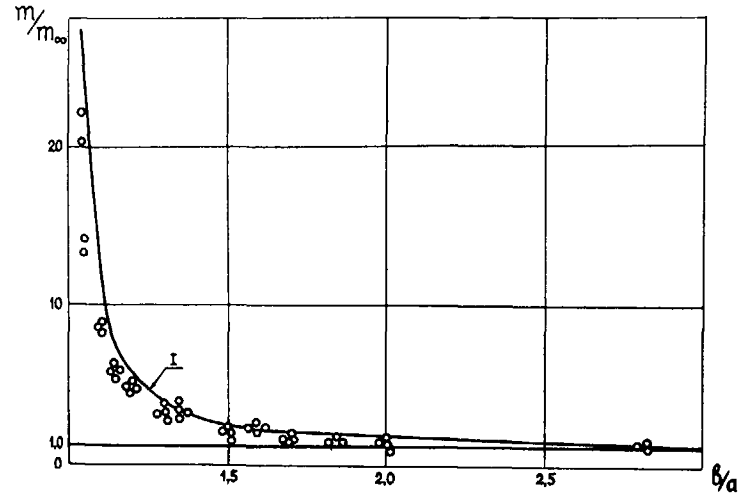

Fig.3. The dependence of dimensionless added masses of tubes in annular channel on the b/a ratio.
1 - calculation from the formula (8) and data in[9],
o - experimental data belonging to authors (3).

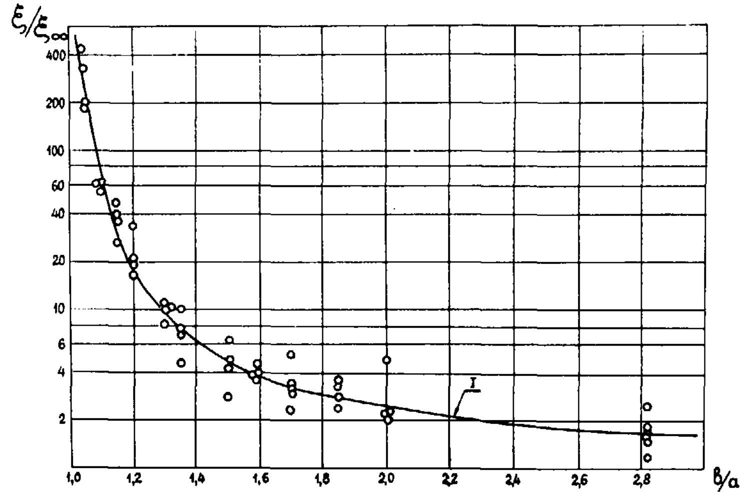

Fig.4. The dependence of dimensionless damping factors of tubes in annular channel on the b/a ratio.
1- calculation from the formula (18),
o - experimental data belonging to authors.

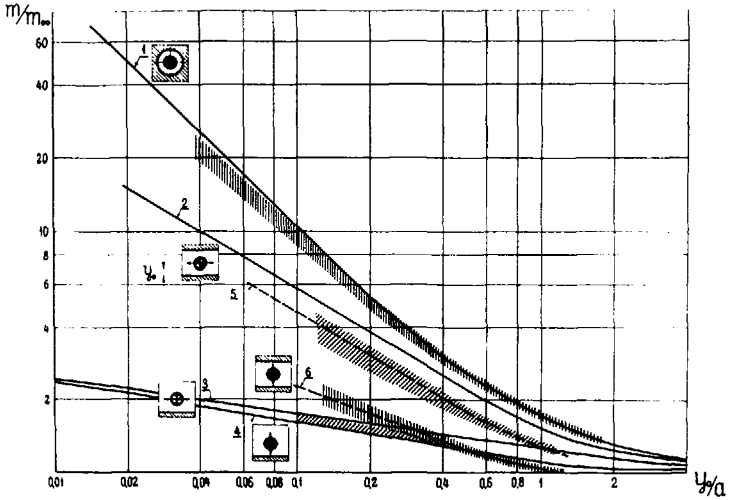

Fig.5. The dependence of added masses on y_0/a when vibrating tubes near different boundaries.
1 - calculation from the formula (8)
2,3,4 - calculation performed by the EHAM method;
5,6 - approximation of experimental data belonging to authors.

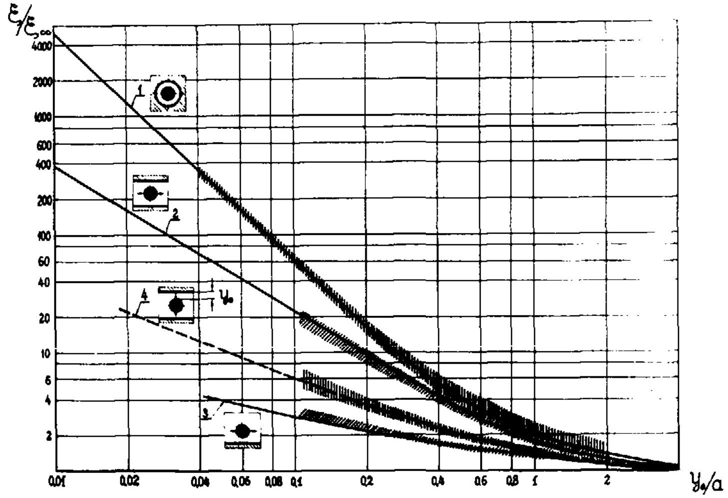

Fig.6. The dependence of damping factors on y_0/a when vibrating tubes near different boundaries.
1 - calculation from the formula (18);
2,3 - calculation by the EHAM method;
4 - approximation of data belonging to authors. In figs.5 and 6 the shaded regions present experimental data of authors.

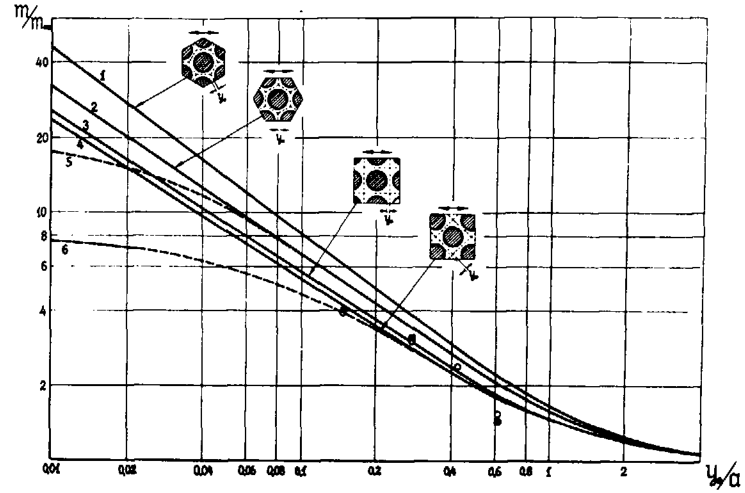

Fig.7. The dependence of added masses on y_0/a for tubes placed in trigonal and tetragonal packages.
1,2 - calculation by the EHAM method for trigonal package; 3,4 - calculation by the EHAM mthod for tetragonal package; 5 - calculation from the formulae (8) and (31) for trigonal package; 6 - calculation from the formulae (8) and (30) for tetragonal package; o,• - experimental data (11).

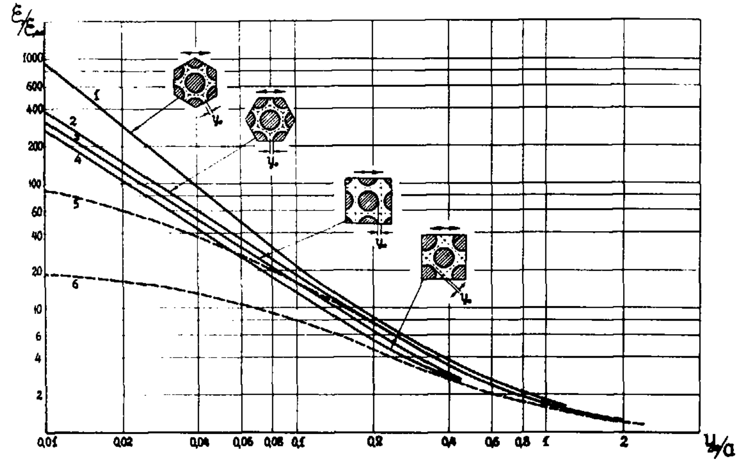

Fig.8. The dependence of damping factors on y_0/a, for tubes placed in trigonal and tetragonal packages.
1,2 - calculation by the EHAM method for trigonal package; 3,4 - calculation by the EHAM method for tetragonal package; 5 - calculation from the formulae (32) and (31) for trigonal package; 6 - calculation from the formulae (32) and (30) for tetragonal package.