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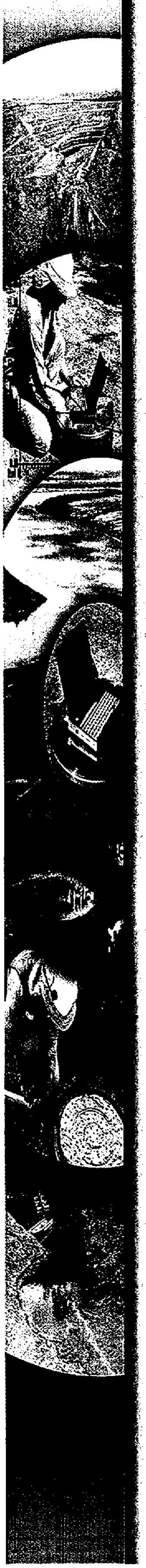

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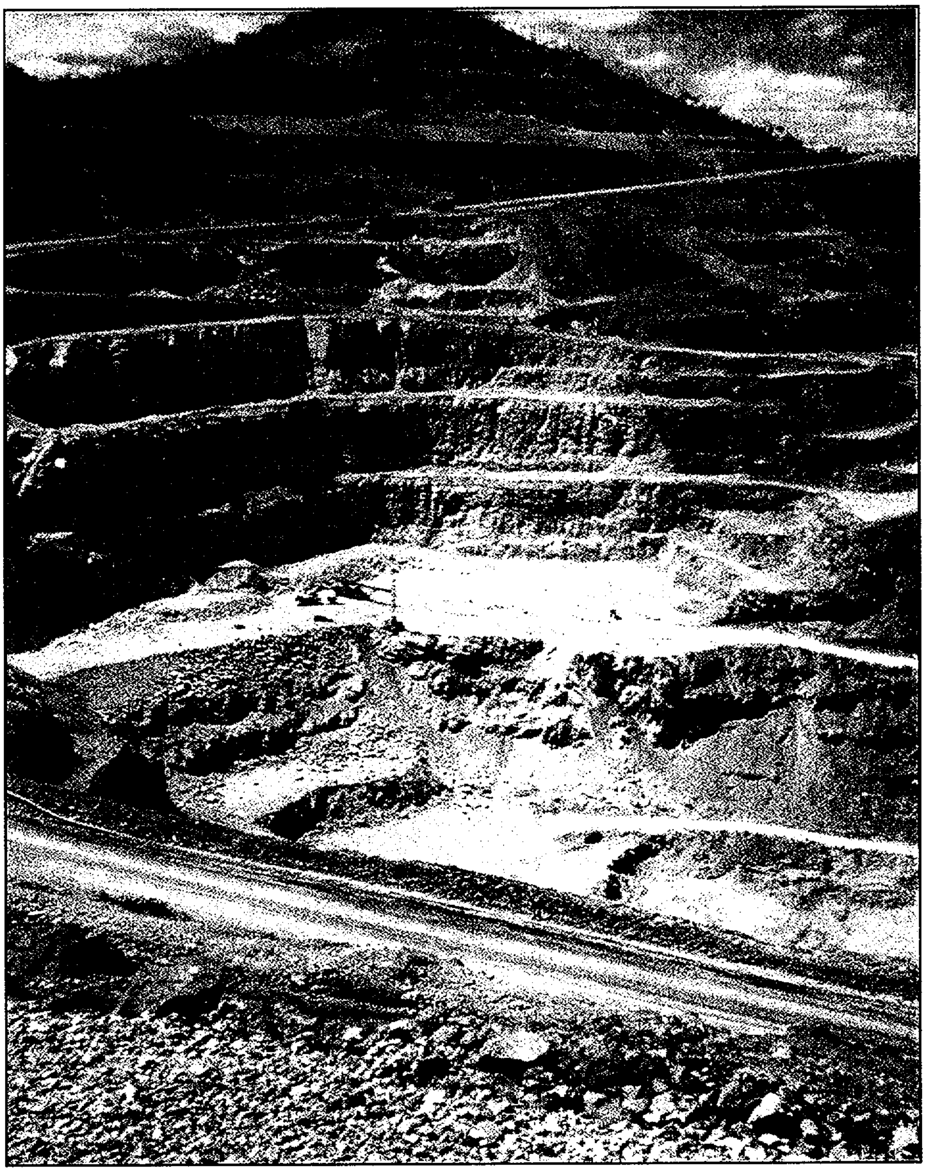

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APPLICATION OF ACOUSTICAL METHODS TO THE MEASUREMENT OF WATER CONTENT IN SAND

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APPLICATION OF ACOUSTICAL METHODS TO THE MEASUREMENT OF WATER CONTENT IN SAND

by

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Abstract

Results of laboratory experiments on the propagation of high-frequency acoustic waves ($f = 100$ kHz) in a glass tube, filled with river sand are presented. Several sand samples have been used with different water content: dry, unsaturated and completely water saturated. It is shown that the dissipative coefficient of acoustic waves decreases with increasing wave amplitude. This 'self-brightening' phenomenon takes place over the whole range of moisture content, from zero to 100%, but its degree of manifestation depends on the moisture content. The exponent of the dissipative nonlinearity α, is found to be the most sensitive parameter to the moisture content and is determined on the basis of measurements. It is considered to be a good indicator of water content in porous media and provides an opportunity to measure water content in such materials indirectly by means of an acoustic method. A simple phenomenological model is presented to explain the experimental results.

TABLE OF CONTENTS

1. Introduction

One of the most serious problems in the mining industry (particularly for opencut mines) is the production of large amounts of waste rock. Waste rock usually is stored in large dumps of several hundred metres in diameter and up to 100 m height. In the open air, sulfidic materials in these waste rock dumps undergo chemical reactions which produce sulfates that are highly soluble in water. Rainfall and other precipitation lead to water flow through the dump, transporting chemical contaminants to the surrounding environment (ground and surface waters). Water also affects the temperature regime within the dump both directly as a cooling mechanism (drainage of water at the base of the dump is a very effective heat remover from the system) and implicitly by means of change of gas permeability in the porous material. For these reasons the water content is a very important characteristic when considering the practical control of pollutant generation, transport and effluents from dumps, and in the successful development of theoretical models that describe physical and chemical processes occurring within dumps.

Examination of numerous measurements has shown that waste rock materials within dumps are usually at unsaturated conditions. For artificially covered dumps, a typical degree of water saturation is approximately 20 – 25% (i.e. the ratio of the volumetric water content to the volume of void is about 0.2 – 0.25).

Various instruments are currently used for the measurement of water content in waste rock dumps [see e.g. (Fredlund & Rahardjo 1993)], however all of them have definite limitations. The development of new measurement techniques and new instruments is topical, particularly those which can provide improved non-contacting and non-destructive methods of measurement.

Recent publications by Nazarov and Zimenkov (1993) and Nazarov (1994) have shown that the propagation of acoustic waves in some porous materials (e.g. river sand) may be accompanied by an interesting non-linear effect which is known in general wave theory as self-brightening. In this phenomenon, the decay coefficient of an acoustic wave is dependent on its intensity, decreasing with increasing wave amplitude. The laboratory experiments, reported in those papers, were carried out for two limiting cases: dry sand and completely water saturated sand. It was discovered that manifestation of the self-brightening phenomenon depends on the water content in sand. Some complementary non-linear parameters were also determined in these experiments and their dependencies on water content established.

The results reported by Nazarov and Zimenkov (1993) stimulated the idea of conducting similar measurements in the intermediate range of water content to examine the dependencies of the non-linear parameters on water content in detail and to decide how such dependencies may be used in practice. The idea was explored as a cooperative project, the results of which are presented in this Report. We give a description of the laboratory set-up, results obtained and their interpretation on the basis of semi-phenomenological theory.

2. Experimental Set-Up and Procedure

The experimental set-up for the procedure described below is shown in Figure 1. A glass cylinder (1) with internal diameter 9 mm, external diameter 11 mm and length $L = 370$ mm was filled with river sand. The lower and upper ends of the tube were tightly sealed by metallic stoppers (2). The lower stopper was glued to the acoustic radiator of longitudinal waves (3), which received high-frequency pulses from the amplifier (4). These pulses had a carrier frequency $f = 100$ kHz, duration $\tau = 300\ \mu s$ and repetition rate $F = 30$ Hz.

After passing through the sand in the glass cylinder, the acoustic pulses were received by the piezo-accelerometer (5) glued to the upper stopper. The signal from the piezo-accelerometer then passed through the preliminary amplifier (6) to the first input terminal of the double-beam oscilloscope (7). The output signal from the power amplifier arrived at the second input terminal. The amplitudes of the radiated,

A_1, and received, A_2, signals were measured by the oscilloscope (relative accuracy of the measurement was 0.015 dB or about 17%). In the experiments described below, these amplitudes were proportional to the amplitudes of displacement U_0 and U_L for radiated and received acoustic pulses. Two openings (8) of diameter approximately 2.5 mm were located near the top and the bottom of the glass cylinder; to each of which was glued a flexible tube (9).

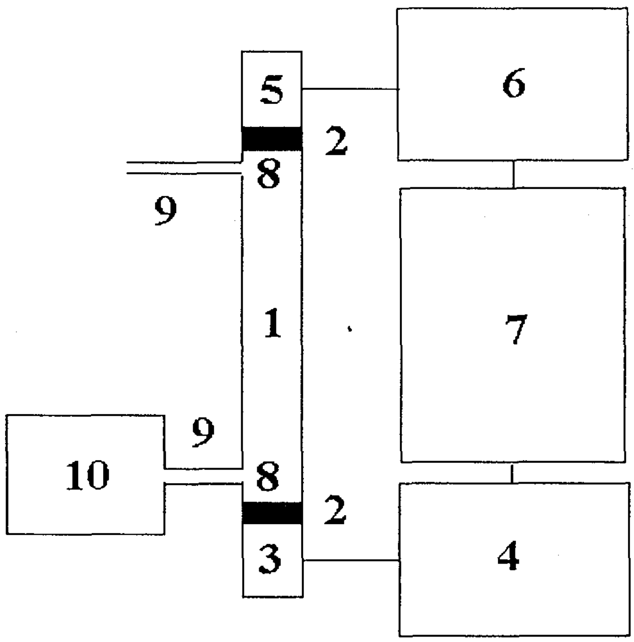

Figure 1. The experimental setup

1. glass cylinder with sand;
2. metallic stoppers;
3. acoustic radiator;
4. amplifier;
5. piezo-accelerometer;
6. preliminary amplifier;
7. double-beam oscilloscope;
8. openings;
9. flexible tubes;
10. medical syringe.

At first, the glass cylinder was filled with compacted dry sand having an average particle diameter of about 0.2 mm. The water saturation of the sand in the glass cylinder was changed with the help of the flexible tubes and a medical syringe (10). The initial volume of water in the glass cylinder was recorded, and the volume of water drawn into the medical syringe from the cylinder was carefully measured. Below a certain degree of water content, it was found to be impossible to draw water into the syringe. Because the water content remaining in the sand was so low, the water phase became discontinuous. The medical syringe filled with air passing though the pore space of the sand rather than with water. The quantitative limit of the lower boundary of water saturation in sand is indicated in the next Section.

A preliminary test of the radiator–receiver system showed that it was linear, i.e. the amplitude of the output signal was proportional to the amplitude of the input signal, $A_2 \propto A_1$. This relationship was also observed in two control experiments with a glass rod and empty glass cylinder.

3. Experiment

A series of experimental studies of the dependence of the amplitude of the received signal on that of the input signal was carried out. Several sand samples with different degrees of water saturation were investigated. The first experiment was conducted with dry sand. The measured received amplitude A_2 is presented in Figure 2 as a function of the input amplitude, A_1.

The figure clearly shows that as the amplitude of the radiated pulse increased, so the amplitude of the received pulse increased in the following way: at small and large amplitudes, a linear dependence of A_2 on A_1 is observed whereas at the intermediate range of amplitudes, A_2 grows faster than A_1. From Figure 2, it is also seen that an increase in the amplitude of the radiated pulse by a factor of 300 leads to an increase in the amplitude of the received pulse by a factor 1400, i.e. the coefficient

G of the relative growth of the received signal with respect to the radiated signal is about 4.7. In the general theory of non-linear waves, this effect is known as *self-brightening* [a similar effect in non-linear optics is known as *self-induced transparency* (Ablowitz & Segur 1981)]. In our case this phenomenon is caused by the dissipative non-linearity of the medium and becomes apparent when the damping coefficient of the acoustic wave decreases as the amplitude increases.

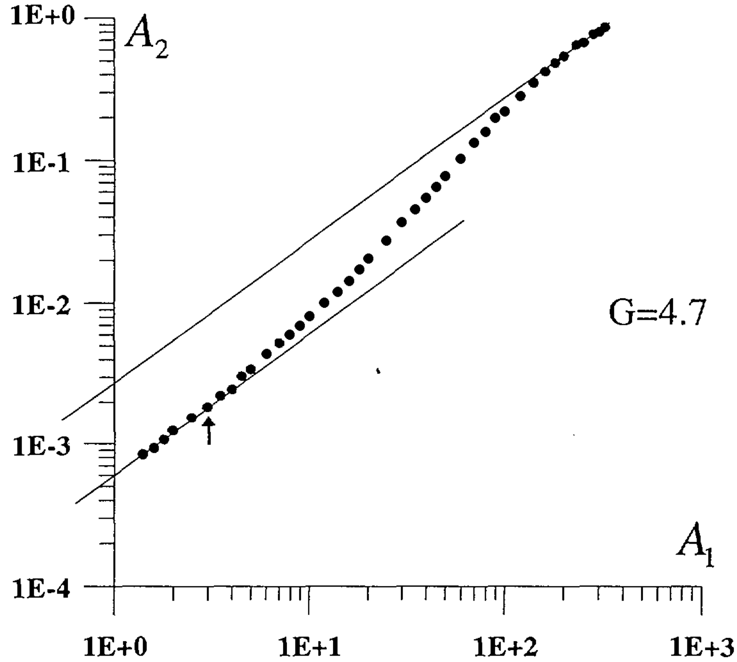

Figure 2. Amplitude of the output signal versus amplitude of input signal in Volts for a dry sample (zero degree of water saturation, $s = 0$).

Similar measurements were performed on the same glass cylinder with the river sand being either completely or partially saturated with water. In the completely saturated case, the volumetric content of the water in the cylinder was 5.9 cm^3 (absolute accuracy of water volume measurements was 0.01 cm^3), and the sand porosity was about $\varepsilon = 0.31$. The degree of water saturation, s, of the sand has been defined experimentally as the ratio of the actual water volume in the glass cylinder to its maximum possible value, 5.9 cm^3. This ratio varied within the range from 0.44 to

1.0. Unfortunately, as noted above, it was impossible to reduce the degree of water saturation below this lower boundary (0.44) by means of syringe suction.

In Figure 3 one can see a dependence of A_2 on A_1 similar to that observed in Figure 2 but for completely water saturated sand.

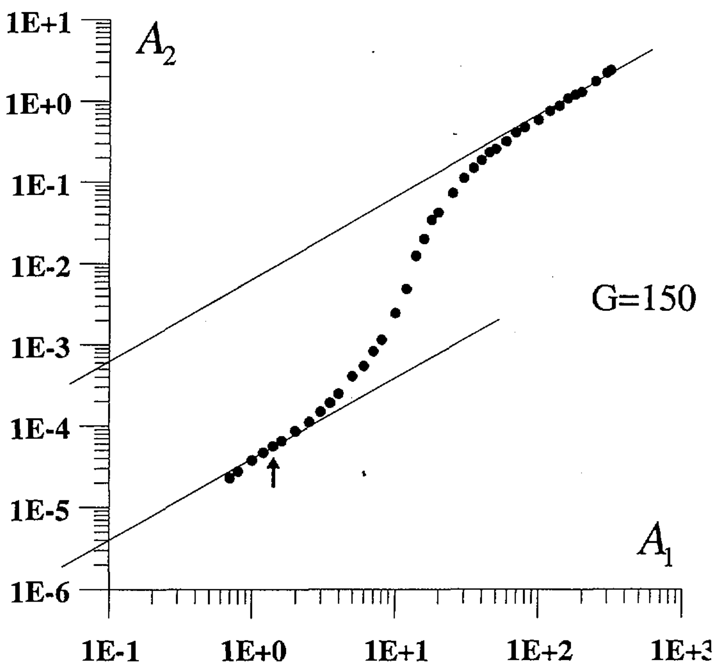

Figure 3. Amplitude of the output signal versus amplitude of the input signal in Volts for a completely water saturated sand sample ($s = 1$).

This dependence was qualitatively the same as for the dry sand, however the coefficient of relative growth of the received signal was significantly larger: $G = 150$. This is evidence that the self-brightening effect was more pronounced in the case of saturated sand.

A series of measurements was carried out for several intermediate values of the degree of water saturation. As was expected, the self-brightening effect took place in all cases and the dependence of A_2 on A_1 was qualitatively the same as for that of the two limiting cases. The coefficient of relative growth of the received signal was not a monotonic function of s. At $s = 0$, it had a minimum value, $G = 4.7$, then reached a maximum value, $G = 7000$ at $s = 0.49$ (we reiterate that measurements were not conducted within the range $0 < s < 0.44$), and then, G decreased as s increased to 1. This experimentally obtained dependence is presented in Figure 4. However, relatively large data scattering with respect to an averaged curve having a wide and almost flat maximum, as well as the non-linear character of the dependence of G on s, make this parameter impractical for solving the inverse problem, i.e., for determining s from the measurement of G.

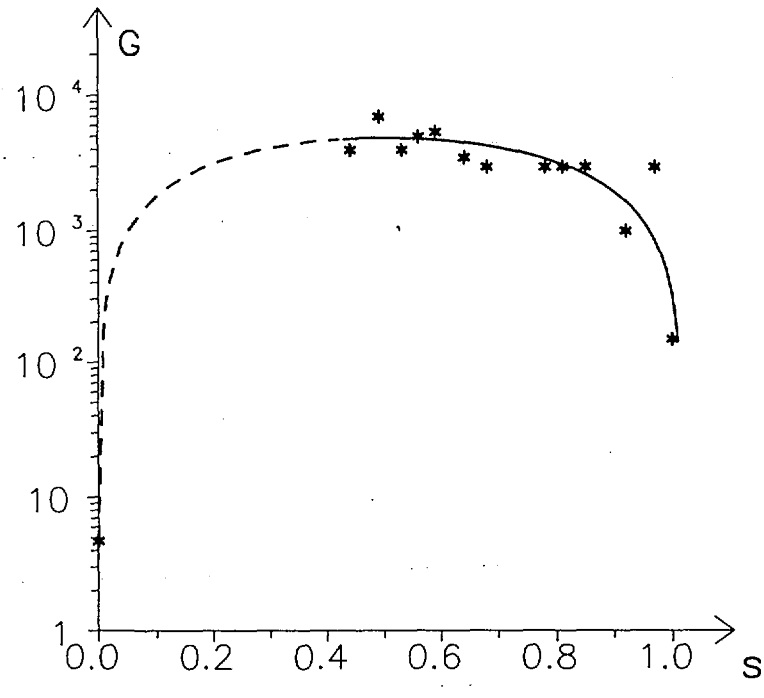

Figure 4. Dependence of the coefficient of relative growth of the received signal, G, on the degree of water saturation, s (asterisks); the dashed–solid curve is a smooth data approximation by a third-order polynomial.

One more important parameter, the exponent of dissipative non-linearity, α, has also been determined experimentally as a function of the degree of water saturation, s, based on the dependence of A_2 on A_1. It is depicted in Figure 7, but prior to its discussion, some preliminary theoretical consideration must be presented.

4. Phenomenological Theory

In the one-dimensional case, propagation of longitudinal acoustic waves in non-linear media can be described by the following equation of motion (Landau & Lifshitz 1987):

$$\rho U_{tt} = \sigma_x(\xi, \xi_t), \tag{1}$$

where ρ is the effective density of a medium, $U(t,x)$ is the longitudinal displacement, $\sigma(\xi,\xi_t) \equiv \sigma_1(\xi) + \sigma_2(\xi_t)$ is the longitudinal stress with $\sigma_1(\xi)$ and $\sigma_2(\xi_t)$ being the elastic and inelastic (viscous) parts of the stress, respectively, $\xi \equiv U_x$ is the longitudinal strain, and ξ_t is the rate of the strain (indices t and x denote temporal and spatial derivatives, respectively).

The elastic part of the stress is assumed to be linear in the subsequent analysis, so that

$$\sigma_1(\xi) = E\xi, \tag{2}$$

where E is the linear modulus of elasticity.

As was evidenced in the experiments, acoustic waves propagating in sand undergo a dissipation which has a non-linear character. This means that the viscous stress is characterised by a non-linear dependence on the rate of the strain. In hydrodynamics, such media are called non-Newtonian or Bingamian [see, e.g. (Wilkinson 1960)].

For a description of the dissipative features of Bingamian media, equations having phenomenological character are usually used (Knopoff 1960; Pal'mov 1967, 1968; Nikolaevsky, 1968). Here the fairly general and commonly used power approximation of the non-linear dependence $\sigma_2 = \sigma_2(\xi_t)$ is applied:

$$\sigma_2(\xi_t) = \rho\left(\beta + \gamma|\xi_t|^\alpha\right)\xi_t, \tag{3}$$

where α, β and γ are constant coefficients defining linear and non-linear viscous stresses in the system.

The dissipative function behaves in essentially different ways depending on the coefficient γ and exponent α. For $\gamma > 0$, the effective viscosity of a medium grows with the growth of the strain rate at $\alpha > 0$ and falls at $\alpha < 0$. Conversely, for $\gamma < 0$ the effective viscosity of a medium falls with the growth of the strain rate at $\alpha > 0$ and grows at $\alpha < 0$.

It should be noted that, generally speaking, at $\alpha < 0$, equation (3) is not a valid description of the non-linear viscous stress near the point $\xi_t = 0$ owing to the singular character of the equation ($\sigma_2(\xi_t) \to \infty$ at $\xi_t \to 0$). Therefore, at $\alpha < 0$ a different approximation of the function $\sigma_2(\xi_t)$ should be used in the vicinity of zero strain rate ($|\xi_t| < \xi_t^*$, with ξ_t^* being a certain critical value of the strain rate) to avoid the singularity. However, as is seen below, this additional approximation is only necessary for $\alpha \leq -3$, whereas at $\alpha > -3$, equation (3) can be used at any ξ_t because there are no singularities in the final expression.

On the radiator of sinusoidal waves of amplitude U_0 and frequency ω, the following boundary condition applies:

$$U(t,0) = U_0 \sin \omega t . \tag{4}$$

By substituting expressions (2) and (3) into equation (1) one obtains the non-linear wave equation for the displacement $U(t,x)$,

$$U_{tt} - c_s^2 U_{xx} = \beta U_{txx} + \gamma\left(|U_{tx}|^{\alpha} U_{tx}\right)_x , \tag{5}$$

where $c_s^2 \equiv E/\rho$ is the square of the sound velocity in the considered medium. For $\gamma = 0$ or for $\alpha = 0$, this equation becomes the usual linear wave equation for describing acoustic wave propagation in dissipative Newtonian media (Landau & Lifshitz 1987).

Small-amplitude perturbations, for which the non-linear term in equation (5) is small with respect to the linear terms, are now considered. In this case, the equation can be approximately solved using the perturbation method. It can be readily shown that the requirement for the last term to be small leads to the following condition:

$$\left| \frac{(\alpha+1)\gamma\, U_0^{\alpha}\, \omega^{2\alpha+1}}{c_s^{\alpha+2}} \right| << 1 . \tag{6}$$

The solution of equation (5) can be searched for in the form of a quasi-harmonic wave with slowly varying amplitude, $A(x)$, and phase, $\Phi(x)$, and with a fundamental frequency, ω, of the same value as the frequency of the radiator:

$$U(t,x) = A(x) \sin[\omega t - kx + \Phi(x)], \tag{7}$$

where $k = \omega / c_s$ is the wavenumber (corresponding spatial period of the wave $\lambda = 2\pi / k$). The dispersion relation, following from (5) for infinitesimal perturbations ($A \to 0$), links the frequency, ω, and the wavenumber, k:

$$\omega^2 = c_s^2 k^2 + i\beta\omega k^2 \to k \approx \frac{\omega}{c_s}\left(1 - i\frac{\beta\omega}{2c_s^2}\right). \qquad (8)$$

It is assumed that the dissipation is relatively small in the system, so that $|\beta\omega / 2c_s^2| << 1$. The dependence $k(\omega)$ is plotted qualitatively in Figure 5.

Substituting (7) into (5), expanding the nonlinear term in the right-hand side of the equation into Fourier series, and keeping only the terms at the fundamental frequency, ω, one obtains

$$\left(A_x + \delta A + \mu A^{\alpha+1}\right)\cos\theta - A\Phi_x \sin\theta = 0, \qquad (9)$$

where

$$\theta = \omega t - kx + \Phi(x), \quad \delta = \frac{\beta\omega^2}{2c_s^3}, \quad \mu = \frac{\gamma\,\Gamma\left(\frac{\alpha+3}{2}\right)\omega^{2(\alpha+1)}}{\sqrt{\pi}\,\Gamma\left(\frac{\alpha+4}{2}\right)c_s^{\alpha+3}}, \quad \alpha > -3,$$

and $\Gamma(x)$ is the Euler gamma-function.

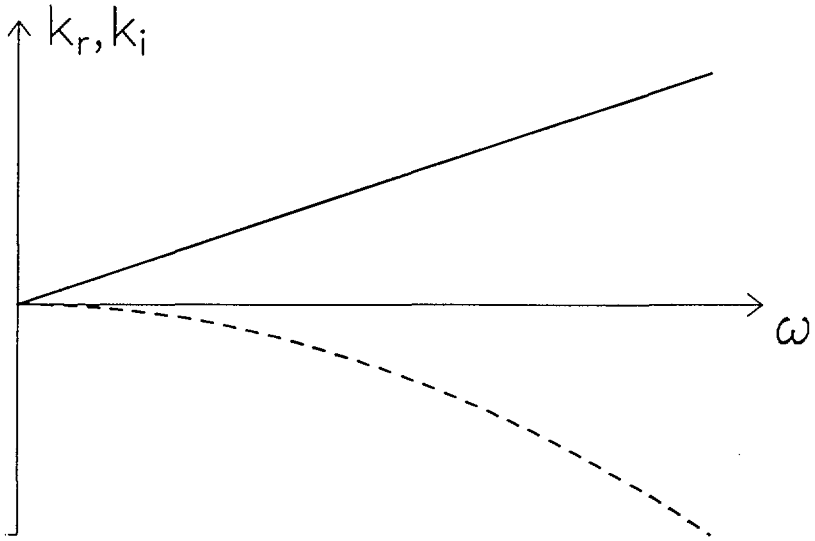

Figure 5. Qualitative sketch of the dispersion relation (8): the solid line represents the real part of the wavenumber, k_r, and the dashed line represents the imaginary part of the wavenumber, k_i.

It follows from equation (9) that the non-linear friction does not lead to a correction to the total wavenumber, $-\theta_x$, because $\Phi_x = 0$. The amplitude decay is described by the Bernoulli equation:

$$A_x = -\delta A - \mu A^{\alpha+1}. \qquad (10)$$

The solution of this equation with the boundary condition (4) has the form

$$A(x) = \frac{A_0 e^{-\delta x}}{\left[1 + \frac{\mu}{\delta} A_0^{\alpha}\left(1 - e^{-\alpha\delta x}\right)\right]^{\frac{1}{\alpha}}}. \qquad (11)$$

By comparing this solution with the experimental results (see, e.g. Figure 2), one can define the parameters of the system considered: the coefficient γ and the exponent α of the dissipative non-linearity. However, as the measurements are relative rather than absolute, only the exponent α could be obtained practically. It follows from the experiments that the coefficient of the dissipative non-linearity, γ, is negative (as is $\mu < 0$), and the exponent, α, is positive.

At very small wave amplitudes, when the non-linear effect may be neglected, formula (11) is reduced to the usual linear law of wave decay:

$$A^*(x) = A_0^* e^{-\delta x} . \tag{12}$$

Let us take the ratio of solutions (11) and (12) at the end of glass cylinder, $x = L$. By denoting $M \equiv \dfrac{A(L)}{A^*(L)}$, $N \equiv \dfrac{A_0}{A_0^*}$, $b \equiv \dfrac{\mu}{\delta}\left(A_0^*\right)^\alpha \left(1 - e^{-\alpha\delta L}\right) < 0$, one can obtain

$$\frac{N}{M} = \left(1 + bN^\alpha\right)^{-1/\alpha} . \tag{13}$$

By taking the logarithm of this equation twice and assuming that $|bN^\alpha| << 1$, one obtains the equation for defining the exponent of the dissipative nonlinearity, α, in equation (3):

$$\ln\left(\ln\frac{M}{N}\right) = \ln\left(-\frac{b}{\alpha}\right) + \alpha \ln N . \tag{14}$$

5. Comparison with Experiments

For the results of experiments described in the previous section the dependencies $\ln\left(\ln\frac{M}{N}\right)$ on $\ln N$ have been constructed for sand samples with different degrees of water saturation. Examples of such plots are depicted in Figure 6 a) and b) for two limiting cases, $s=0$ and $s=1$, which correspond to Figures 2 and 3. The arrows in Figures 2 and 3 show the points corresponding to the values A_0^* in the linear solution (12). From these figures, the exponent of the dissipative non-linearity, α, can be readily found; its values for the two particular cases are indicated in Fig. 6 a) and b).

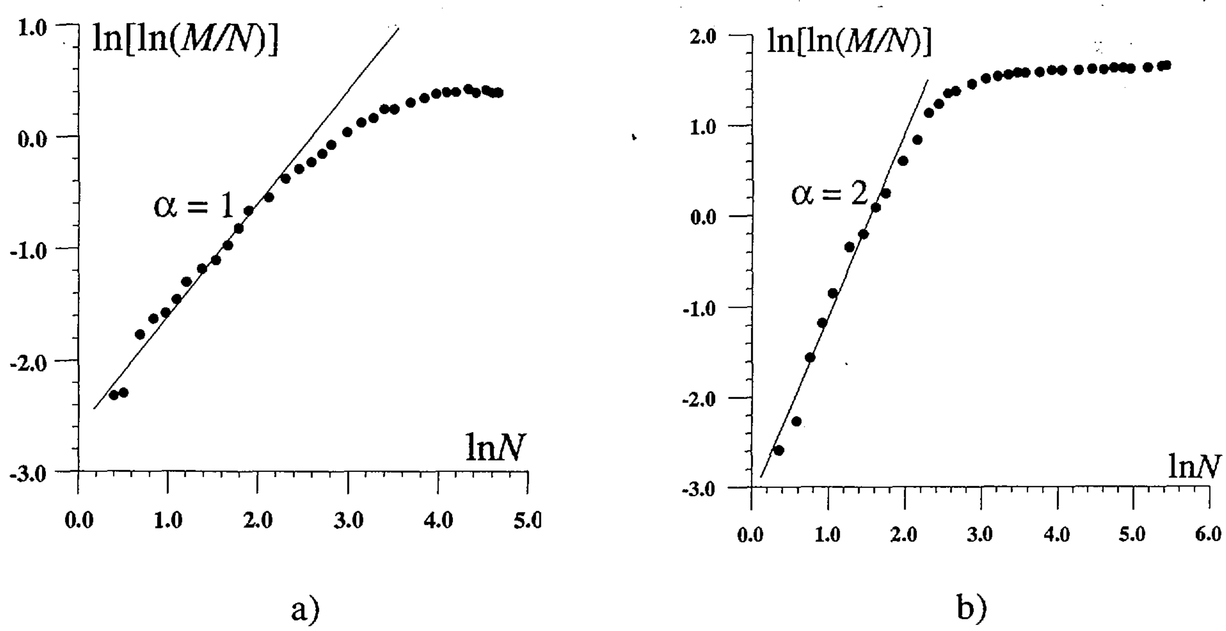

Figure 6. Dependence of $\ln\left(\ln\frac{M}{N}\right)$ on $\ln N$ for samples of (a) dry sand and (b) completely water saturated sand.

Using the same approach, the exponent α has been determined for different intermediate values of degree of water saturation within the range $0.44 \le s \le 1$. The plot of the dependence of α on s is given in Figure 7. The straight line in the figure, $\alpha = 2.2s - 0.07$, is the best-fit line within this range of s.

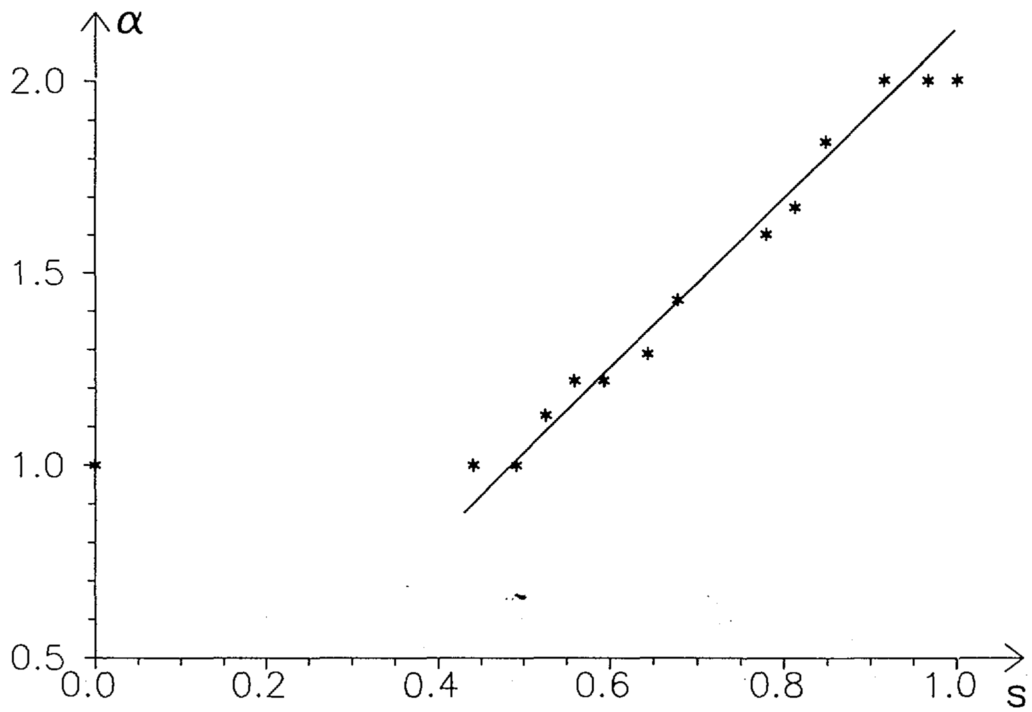

Figure 7. Dependence of the non-linear parameter α on the degree of water saturation, s, of sand samples.

It follows from the results obtained that in these experiments α ranges from 1 to 2, with variation in the water content of the sand from $s = 0$ (dry) to $s = 1$ (completely water saturated). The dependence of α on s is linear within the indicated range of s and is well pronounced; there are no particularly large deviations of experimental data from the best-fit line. In addition, the experimental results are easy to reproduce in different trials at the same conditions. This makes the parameter α a good indicator of water content in sand samples. In other words, having measured this parameter, the water content in any given sand sample can be readily assessed.

6. Conclusions and Recommendations

Laboratory experiments, have shown that the self-brightening effect takes place for strong acoustic waves over the entire range of water content in sand samples, from dry to completely water saturated sand. However, the magnitude of the effect depends on the degree of moisture content in sand. Some non-linear parameters were experimentally determined and their dependence on water content in sand has been obtained. One of them, the exponent of dissipative non-linearity, α, was shown to be the most sensitive to the water content (at least, within the experimental range of the degree of water saturation, $0.44 \leq s \leq 1$). The dependence of α on s has a pronounced linear form in the indicated range, is reproducible in experiments and can be easily measured. From a practical point of view, the parameter α is a good indicator of the water content in river sand. The inverse problem can also be solved experimentally and the water content determined by measurement of α. A simple phenomenological theory has been developed to explain the main features of non-linear acoustic wave propagation and dissipation in sand. The theory gives a clear physical interpretation of the parameter α and predicts a way for its practical determination by means of experimental data processing.

It is natural to assume that there may be analogous dependencies not only for sand samples, but also for other porous materials, including soil, clay, waste rock, etc. The combination of the proposed acoustical method of water content measurement with the measurement of seepage velocity in porous materials (e.g. by the tracer method), provides an opportunity to evaluate the water flux, which is an important characteristic both in waste rock management and in many other applications. In our opinion, the acoustical method may be developed for use not only in the laboratory but also under field conditions. Further optimism in this direction has been stimulated by recent experiments conducted under field conditions by Zaitsev et al. (1998), in which seismo-acoustic waves were generated in sand soil on a river bank and then received 165 m away from the source. Different non-linear effects were observed, but their sensitivity to water content was not investigated.

We recommend that a series of laboratory tests should be carried out to test if the acoustical method can be developed for use in waste rock dump management. The experiments should be carried out on sand and waste rock samples having a water content range of $0 \leq s \leq 0.44$. A further series of tests should be performed under field conditions to gain experience in practical applications of the method. The long-term perspective is for its application in the extended monitoring of waste rock dumps using remote sensors.

7. Acknowledgments

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8. References

1. Ablowitz, M.J. & Segur, H. 1981, Solitons and the Inverse Scattering Transform, SIAM, Philadelphia.
2. Fredlund, D.G. & Rahardjo, H. 1993, Soil Mechanics for Unsaturated Soils, John Wiley & Sons, Inc., New York.
3. Knopoff, L. & Mac Donald, G.S. 1960, 'Models for acoustic loss in solids', J. Geoph. Res., vol. 65, n. 7, pp. 2191–2197.
4. Landau, L.D. & Lifshitz, E.M. 1987, Theory of Elasticity, Nauka, Moscow. (English translation: Landau, L.D. & Lifshitz, E.M. 1970, Theory of Elasticity, Pergamon Press, Oxford.)
5. Nazarov, V.E. 1994, 'Self-action of elastic waves in media with dissipative non-linearity', Acoust. Lett., vol. 17, n. 8, pp. 145–149.
6. Nazarov, V.E. & Zimenkov, S.V. 1993, 'Self-action of acoustic waves in rocks', Acoust. Lett., vol. 16, n. 10, pp. 218–221.
7. Nikolaevsky, V.N. 1968, 'Monochromatic waves in elastic medium with local dry friction', Izv. Akad. Nauk SSSR. Mekh. Tverd. Tela, vol. 4, pp. 85–92 (in Russian).
8. Pal'mov, V.A. 1967, 'Vibration propagation in nonlinear dissipative medium', J. Appl. Math. Mech., vol. 31, n. 4, pp. 749–755. (English translation from the Russian journal Prikladnaya Matematika i Mekhanika).
9. Pal'mov, V.A. 1968, Vibration of Elastic-Plastic Solids, Nauka, Moscow (in Russian).
10. Wilkinson, W.L. 1960, Non-Newtonian Fluids: Fluid Mechanics, Mixing and Heat Transfer, Pergamon Press, London & Oxford.

11.Zaitsev, V.V., Nazarov, V.E., & Talanov, V.I. 1998, 'Experimental study of self-action of seismo-acoustic waves', Preprint, No 469, 21 p., Inst. Appl. Phys., Russ. Acad. Sci., Nizhny Novgorod (in Russian).

9. Glossary of Technical Terms

Degree of water saturation, s, – is the ratio of the water volume in a given volume of porous medium with respect to its maximum possible value in the same volume.

Infiltration – water percolation in a soil or in waste rock dumps (usually from rainfall).

Infiltration rate ($m^3\ m^{-2}\ s^{-1} \equiv ms^{-1}$) – the bulk rate of flow of water infiltrating the pile through a unit area in a unit time.

Non-Newtonian or Bingamian media – media in which viscous stress, in contrast to the usual Newtonian media, is a non-linear function rather then the linear one of the rate of strain.

Porosity, ε, – the fraction of the void volume in the sample of porous material to the total volume of this material.

Self-brightening – a wave process which occurs when the damping coefficient of an acoustic wave decreases as its amplitude increases. It is caused by the dissipative non-linearity of the medium.

Strain (the longitudinal strain), ξ, – a spatial derivative of the displacement, U in solids or in porous media: $\xi \equiv U_x$.

Stress (the longitudinal stress), σ, – is a function of the strain and its growth rate, whose gradient determines a force acting on elementary particle within the volume of solids or porous media.