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PP-6 REB-Instability with Magneto-Active Inhomogeneous Warm Plasma

Kh. H. El-Shorbagy
Plasma Physics and Nuclear Fusion Dept.
Nuclear Research Centre
Atomic Energy Authority
Cairo - Egypt.

ABSTRACT

The beam-plasma heating due to a relativistic electron beam (REB) under the effect of an external static magnetic field is investigated. It is considered that a longitudinal 1-D oscillations exist in the plasma, which is inhomogeneous and bounded in the direction of the beam propagation . It is found that the variation in the plasma density has a profound effect on the spatial beam-plasma instability. Besides, the external static magnetic field and warmness of plasma electron leads to more power absorption from the electron beam, and consequently an auxiliary plasma heating.

INTRODUCTION

REB has many applications in areas like material studies, compact torus formation, generation of x - ray and microwave, ion acceleration, etc.... where it is desirable to have energy source supplied over a long duration of time.

As a potential application, use of a REB to heat magnetically confined plasma to a high temperature has attached a lot of attention, both theoretically (e.g., [1-3]) and experimentally (e.g., [4-6]). When a REB propagates through a plasma, its kinetic energy is transferred into plasma thermal energy, thereby heating the plasma [1].

Different from previous works on beam - plasma instability [7-12], we study the effect of both plasma inhomogeneity and the thermal electron motion on the quenching of the beam - plasma instability. Investigation of beam - plasma interaction present a great interest for development of effective methods via plasma stability, amplification and generation of electromagnetic waves, acceleration of charged particles in plasma, high frequency heating of plasma and so on [13-14].

Many authors studied the problem of nonrelativistic electron beam interaction with unmagnetized plasma, where this interaction takes the form of an amplification of waves by beam (e.g., [7,9,11,12,15]). It is shown that due to the resonance rise of the wave field with plasma dielectric permeability is reduced to zero, and the power absorbed by the plasma is finite and independent of the value of the dissipation. In this case the beam not only amplifies the waves in plasma, but also provides an effective absorption of these waves by the plasma (plasma heating).

In the present work we study the influence of the variable plasma density and plasma thermal motion under the effect of the external static magnetic field directed along z-direction, ($\vec{H}_{ext.} = H_0 \vec{e}_z$) (under the condition of the smallness of phase velocity of waves compared to the beam velocity) on the quenching of the beam - plasma instability. A semi - infinite beam - plasma system $(x \geq 0)$, in which the unperturbed plasma density $n_0(x)$ is an arbitrary function of x $[n_0(x) = N_0(1-x)\; ; N_0$ is constant] is considered. It is assumed that ions are sleeping, plasma electrons have a finite temperature, and that the relativistic electron beam is cold and homogeneous.

FUNDAMENTAL WAVES

The equation of motion and the continuity equation for a relativistic electron beam, which travels along the magnetic field, are: -

$$\frac{\partial (m\vec{V}_b)}{\partial t} + (\vec{V}_b \cdot \vec{\nabla})(m\vec{V}_b) = -e\vec{E} \quad ; \quad \vec{V}_b = V_{0b} + \vec{V}_{1b} \tag{1}$$

$$\frac{\partial N_b}{\partial t} + \vec{\nabla}\cdot(N_b \vec{V}_b) = 0 \quad ; \quad N_b = n_{0b} + n_{1b} \tag{2}$$

Where; $m = m_0\,\gamma^{-1}$, $\gamma = (1 - V^2/C^2)^{1/2}$ is the relativistic factor of the beam electrons, and V_{1b} is the component of beam velocity in the z-direction.

The equation of motion, and the continuity equation for inhomogeneous plasma electrons in the oscillating electric field and a static magnetic field $\vec{H}_{ext.}$ perpendicular to the plasma density gradient are given by: -

$$\frac{\partial \vec{V}_P}{\partial t} + (\vec{V}_P \cdot \vec{\nabla})\,\vec{V}_P = -\frac{e}{m}[\vec{E} + \frac{1}{C}(\vec{V}_P \times \vec{H}_{ext.})] - \nu V_P - \frac{1}{NM}\vec{\nabla}P \tag{3}$$

$$\frac{\partial N_P}{\partial t} + \vec{\nabla}\cdot(N_P \vec{V}_P) = 0 \quad ; \quad N_P = n_{0P} + n_{1P} \tag{4}$$

In equations (1-4), V_{0b}, n_{0b} are the unperturbed velocity and density of the beam while n_{0P}, n_{1P} are the unperturbed and perturbed density of the plasma respectively. P is the plasma pressure and ν is the collision frequency of plasma electrons with other plasma particles. All other terms have their usual meaning.

In the case of weak nonlinearly ($|n_{0b}| << n_{1b}$, $|V_{1b}| << V_{0b}$) then we have $\left|V_{1P}\frac{\partial}{\partial x}\right| < \left|\frac{\partial}{\partial t}\right|$.

From (1)-(4), we can derive the following expressions for the perturbed densities: -

$$n_{1b} = (\frac{e\gamma^2 n_{0b}}{mV_{0b}^2})e^{i(\omega/V_{0b})x}\int e^{-i(\omega/V_{0b})x}[E(x) + (i\omega/V_{0b})e^{i(\omega/V_{0b})x}\int e^{-i(\omega/V_{0b})x}E(x')dx']dx \tag{5}$$

$$n_{1P} = -(\frac{e}{m})\frac{1}{\omega\,\tilde{\omega}}\frac{\partial}{\partial x}[n_{0P}\,E(x)] - \frac{V_{T_e}^2}{\omega\,\tilde{\omega}}\frac{\partial^2}{\partial x^2}[n_{0P}\,(x)] \tag{6}$$

Where, $V_{T_e} = \sqrt{\frac{kT_e}{m}}$ is the electron thermal velocity and $\tilde{\omega} = ((\omega + i\nu)^2 - \omega_c^2)^{1/2}$, $\omega_c = \frac{eH_0}{mc}$ is the electron cyclotron frequency.

Using Poisson's equation

$$\frac{dE}{dx} = -4\pi e(\,n_{1P} + n_{1b}\,)\,, \tag{7}$$

In commination with (5) and (6) in (7), the following second order differential equation, which describes the electric field due to beam-plasma interaction, is obtained:

$$\frac{d^2F(x)}{d\,x^2} + \kappa^2 F(x) = C_1(T_e) \tag{8}$$

Where,

$$F(x) = \kappa^{-2}\,(x)E(x)e^{\frac{-i\,\omega}{V_{0b}}x} \tag{9}$$

$$C_1(T_e) = \frac{4\,\pi\,e\,\omega^2 N_0}{\tilde{\omega}V_{0b}^3}V_{T_e}^2 \quad ,\kappa^2(x) = \frac{\omega_{Rb}^2}{V_{0b}^2}\frac{\omega\,\tilde{\omega}}{(\omega\,\tilde{\omega} - \omega_{P_e}^2)}\,,\ \omega_{Rb} = \gamma\,\omega_b\ ,\ \omega_{P_e,b}^2 = \frac{4\,\pi\,e^2 n_{0,(P_e,b)}(x)}{m}$$

The term $C_1(T_e)$ in the right hand side of equation (8) represent the effect due to electrons warmness. An equation similar to (8) has been obtained in the past by many authors [7-12], however the thermal effect $C_1(T_e)$, which is of importance for analysis of plasma instability and heating, is neglected. Different from previous works [7-12]; the wave number κ(x) contains the effect of the external static magnetic field and relativistic electron beam through $\tilde{\omega}$ and ω_{Rb}.

The solution of equation (8) in the region $x \le 0$ gives the following spatially growing modes (upstream):

$E_1(x,t) = E_1(\omega)\exp(ikx - i\,\omega\,t)$, $(\mathrm{Im}\,k_1 < 0)$
Where, $k_1 = (\omega / V_{0b}) + \kappa_1$, κ_1 is given by relation (10) in region $x < 0$.

The most important mode is the one for which $|\,\mathrm{Im}\,\kappa_1(\omega)\,|$ is a maximum. Providing that the discontinuity at $x = 0$ has no influence on the solution in the region $x < 0$, the following solutions of equation (8) in the regions $x \le 0$ and $x \ge 0$ can be derived as:

$$\begin{aligned} F_1 &= A_1 e^{i\kappa_1 x} + R(x) \quad ; \quad x \le 0 \\ F_2 &= A_2 e^{i\kappa_2 x} + A_3 e^{-i\kappa_2 x} + R(x) \quad ; \quad x \ge 0 \end{aligned} \tag{10}$$

Where both $\mathrm{Im}\,\kappa_1$ and $\mathrm{Im}\,\kappa_2$ are negative, and $R(x)$ is a new addition due to the thermal motion of electrons;

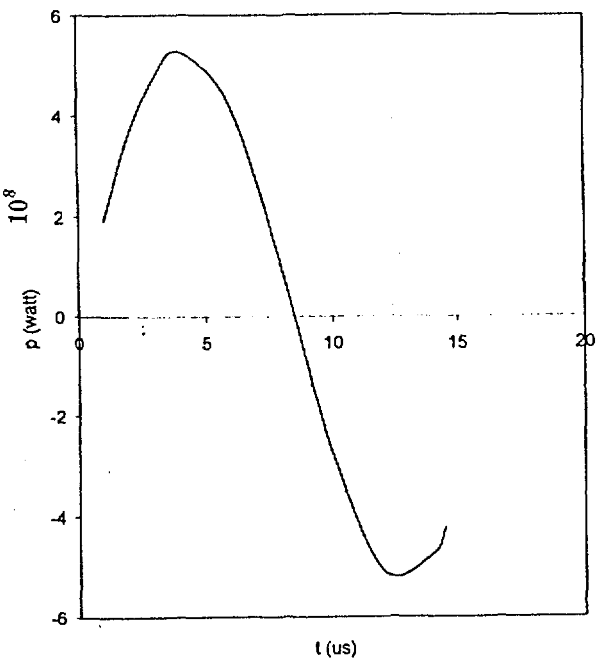


Fig (3a) Relation between P_1 and t

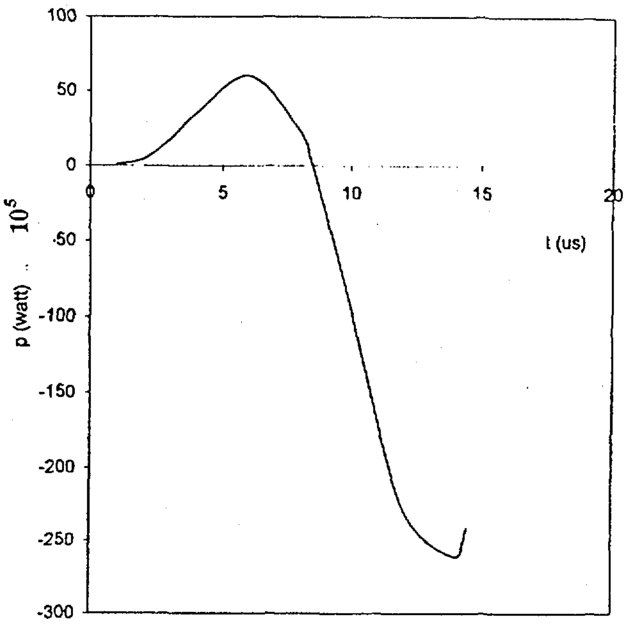


Fig (3b) Relation between P_2 and t

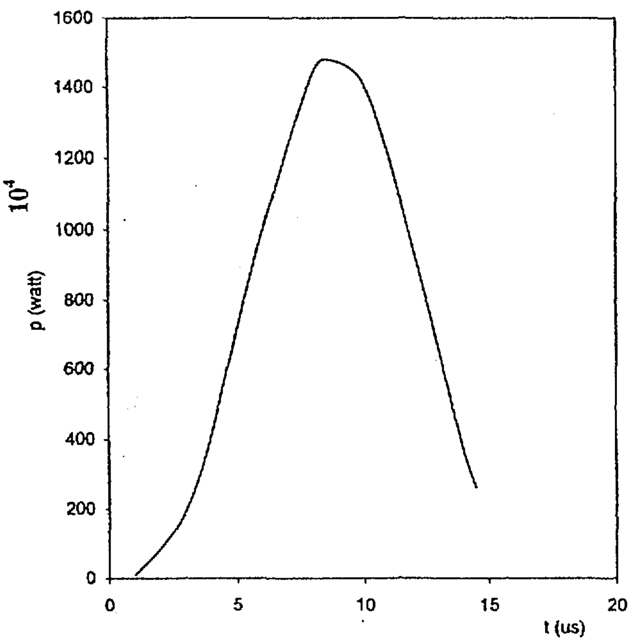


Fig (3c) Relation between P3 and t

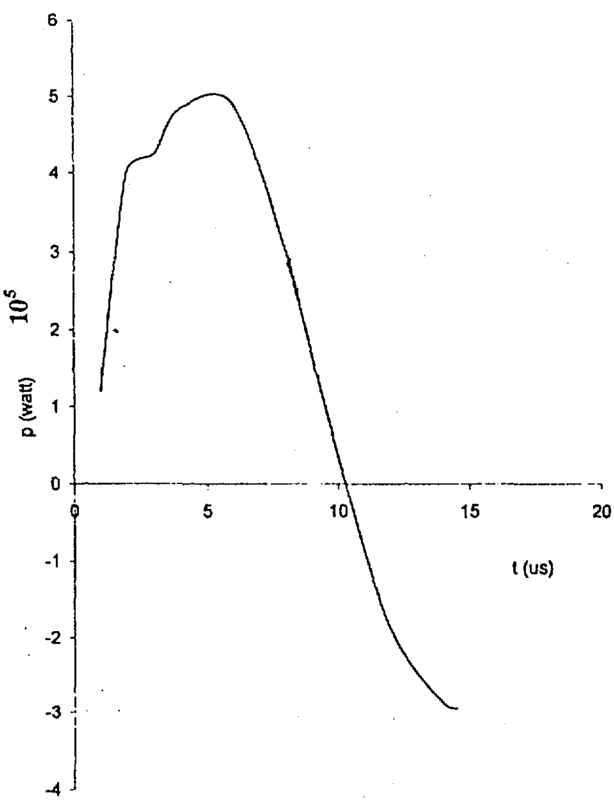


Fig (3d) Relation between P4 and t

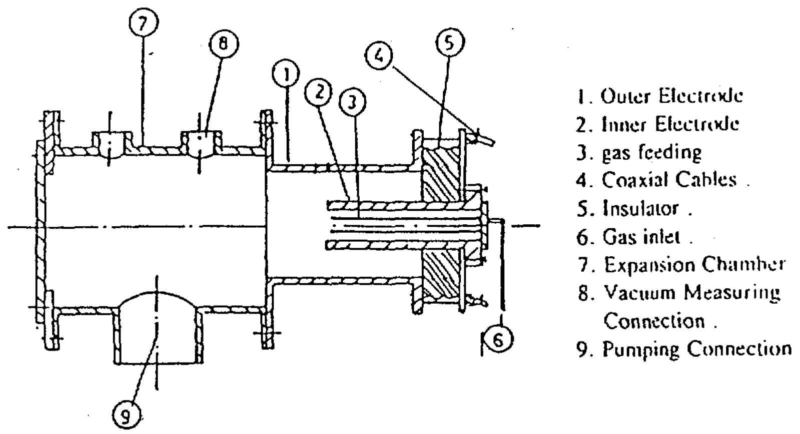


Fig (1) Cross section of coaxial electrodes.

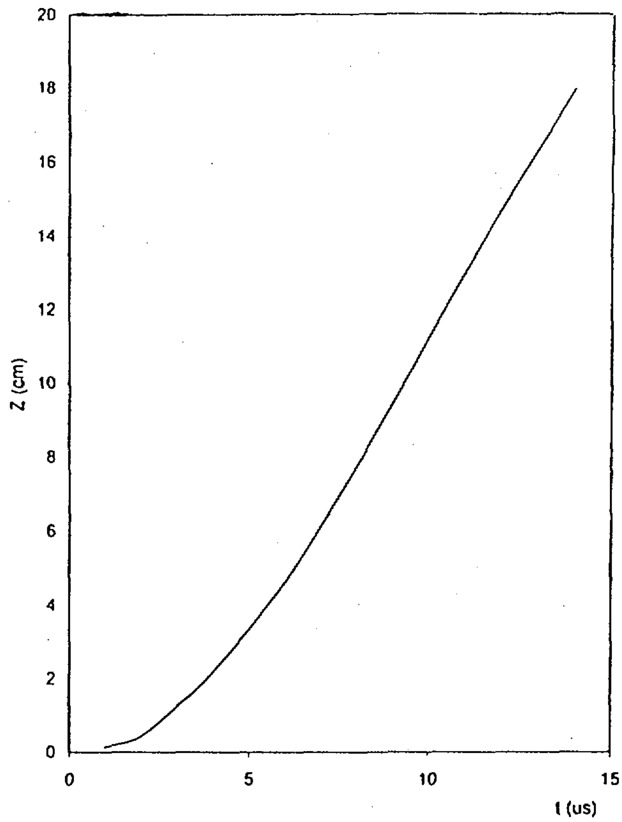


Fig (2a) Axial plasma current sheath position versus position

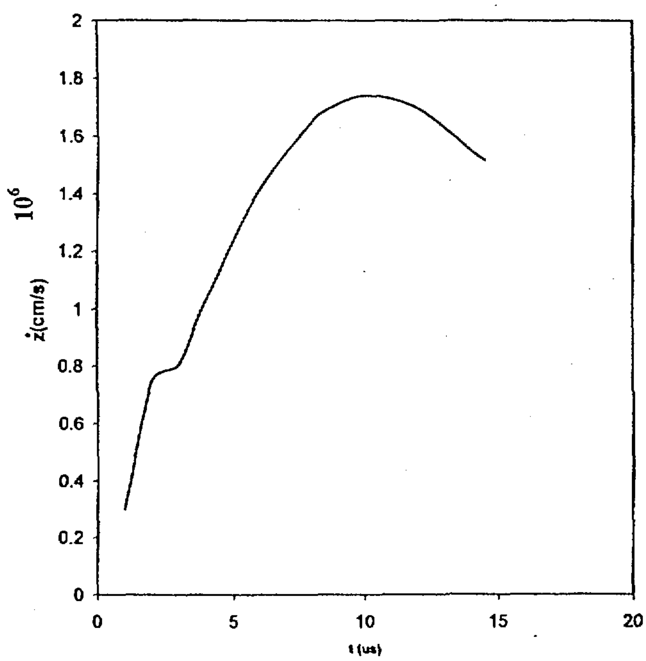
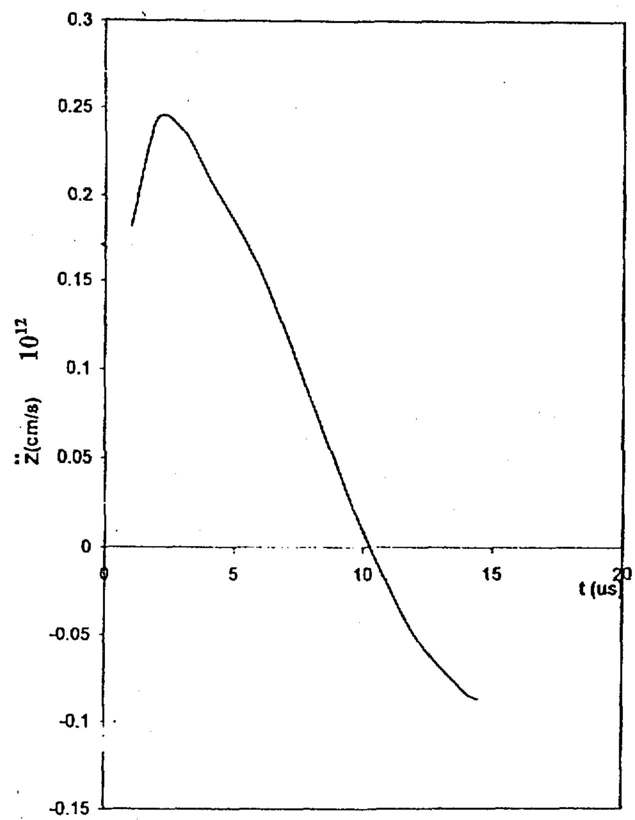


Fig (2b) Axial plasma current sheath velocity versus time



Fig(2c) Axial plasma current sheath acceleration versus time

It is the power consumed to accelerate the mass, which has been swept up into the plasma current sheath,

$$P_6 = P_7 = 0.5m\dot{Z}^3 \quad (8)$$

P_6 is the power, which produces directed kinetic energy for the plasma current sheath, and
P_7 is the power, which produces internal energy in the plasma sheath,

$$P_8 = I^2 R = R I^2 \sin^2 \omega t \quad (9)$$

It represents, the Joule heating

The axial position, Z, velocity, Z, and acceleration, Z are calculated theoretically, as function of time and they are plotted as shown in Fig (2a), Fig (2b), and Fig (2c) respectively. These figures indicate that, the plasma current sheath, reaches the coaxial electrodes muzzle at 14.5 µs, with axial velocity of 1.52 cm/us and acceleration of 0.0876×10^{12} cm/s^2 respectively.

The time variation of the power flow, p_1, p_2, p_3, p_4, p_5, p_6, p_7 and p_8 are plotted as shown in Figs (3a, 3b, 3c, 3d, 3e, 3f, 3g) respectively.

Table (1) shows the relation between the peak value of the power consumed and the corresponding values of time, for p_1, p_2, p_3, p_4, p_5, p_6, p_7 and p_8

Table (1)

Power flow	Peak value (watt)	Time (µs)
P_1	5.277×10^8	4
P_2	-261.4×10^5	14
P_3	148×10^5	8.5
P_4	4.9×10^5	6
P_5	-182.26×10^5	14.5
P_6	195×10^5	10
P_7	195×10^5	10
P_8	1.1×10^8	8.5

Fig (4) shows the calculated values of time variation of the total power P_t flowing from the capacitor bank. This figure demonstrates that, the peak value of $P_t=6\times10^8$watt at t=4 µs

Fig (5) shows the measured values of time variation of total power =IV, where I and V are the discharge current and voltage respectively. This figure shows that, the peak value of $P=3.5\times10^8$watt at t=7µs.

The total electrical energy flow to the coaxial accelerator is evaluated numerically as the integral $\int$ pdt.

Fig.(6) and Fig.(7) show, the time variation of the calculated and the measured values of the total energy flow, E_t respectively. These figures clear that, the peak value of E_t (measured) = 3.54×10^2 J at t=7.5µs and the peak value of E_t (calculated) =5.92×10^2 J at t =5.5µs .

Conclusion

The power and the energy flow from a condenser bank to a coaxial plasma accelerator are theoretically studied, under the assumption of a snow plough model action and they are experimentally measured from the analysis of the simultaneous discharge current and voltage waveforms.The calculated value of the total power, $p_{t\ (max)}$ is classified into p_1=88.33% of p_t, p_2=0.586% of p_t, p_3=0.7% of p_t, p_4=0.081% of p_t, p_5=0.0582% of p_t, p_6=p_7=0.68% of p_t and p_8=8.37% of p_t.

In fact, p_t depends on the discharge current, the pressure of filling gas, the dimensions of the coaxial electrodes, the external inductance and the resistance. Then, the values of these parameters must be modified, to produce a more useful uniform condition, for optimum operation .

A comparison of the experimental results of the total power and the energy flow, with those predicated by the theoretical calculations, indicates that, the measured values are smaller than predicated, this may be due to, a dissipation of energy in the stray impedence of the circuit, a dissipation of energy in heat conduction wall, dissipation of wave radiation energy losses, the gas is may be not completely snow ploughed by the sheath and wall interaction.

$$R(x) = \frac{1}{W}\int_0^x [Z_1(x)Z_2(t) - Z_2(x)Z_1(t)]C_1(T_e)dt$$

Where, $Z_1 = e^{i\chi_1 x}$; $Z_2 = e^{-i\chi_2 x}$; $W = Z_1' Z_2 - Z_1 Z_2' = const.$

The constants of integration A_i $(i = 1-3)$ are determined under the boundary conditions that both F and $\frac{dF}{dx}$ are continuous at $x = 0$, hence,

$$A_2 = \frac{1}{2}\frac{\kappa_1 + \kappa_2}{\kappa_2}A_1 \quad , A_3 = \frac{1}{2}\frac{\kappa_2 - \kappa_1}{\kappa_2}A_1$$

Using the definition (9) the electric field $E_2(x)$ is derived in terms of $E_1(0)$ as:

$$E_2(x) = \{(\frac{E_1(0)\kappa_2}{2\kappa_1^2})[(\kappa_1 + \kappa_2)e^{i\kappa_2 x} + (\kappa_2 - \kappa_1)e^{-i\kappa_2 x}] + R(x)\}e^{i\frac{\omega}{V_{0b}}x} \tag{11}$$

E_2 yields a power of the form:

$$\begin{aligned}|E_2(x)|^2 = [&|E_1(0)|^2|\kappa_2|^2/4|\kappa_1|^4][|\kappa_2 + \kappa_1|^2 e^{i(\kappa_1 - \kappa_1^*)x} + |\kappa_2 - \kappa_1|^2 e^{-i(\kappa_2 - \kappa_2^*)x} + \\ &2(|\kappa_2|^2 - |\kappa_1|^2)\cos(\kappa_2 + \kappa_2^*)x - 2i(\kappa_2\kappa_1^* - \kappa_1\kappa_2^*)\sin(\kappa_2 + \kappa_2^*)x] + |R(x)|^2 + \\ &R^*(x)\{(E_1(0)\kappa_2/2\kappa_1)[(\kappa_2 + \kappa_1)e^{i\kappa_2 x} + (\kappa_2 - \kappa_1)e^{-i\kappa_2 x}]\} + \\ &R(x)\{(E_1^*(0)\kappa_2^*/2\kappa_1^*)[(\kappa_2^* + \kappa_1^*)e^{-i\kappa_2^* x} + (\kappa_2^* - \kappa_1^*)e^{i\kappa_2^* x}]\}\end{aligned} \tag{12}$$

The 3rd and 4th terms on the right hand side of equation (12) are due to the mixing (spatial beats) between the growing and decaying modes in the region $x \geq 0$, while the last three terms are due taking into consideration the thermal motion of electrons. It is clear that the electric field power given by (12) is strongly affected by both mixing, static magnetic field, relativistic electron beam and thermal effect, (i.e., the power of electric field at $V_{T_e} \neq 0, H_0 \neq 0$ and $\gamma < 1$ is greater than the power in the case of $V_{T_e} = 0, H_0 = 0$ and $\gamma = 1$). Mixing produce a noticeable effect on $|E_2(x)|^2$ under the conditions $\kappa_1 \neq \kappa_2$; $|\mathrm{Re}\,\kappa_2| >> |\mathrm{Im}\,\kappa_2|$, which are necessary in order for the trigonometric terms in (12) to vary rapidly, compared with the exponential growth terms. The (*) represent the conjugate values. These conditions are not necessary for the thermal effect.

From (12), we get:

$$|E_2(0)|^2 = |\kappa_2/\kappa_1|^4|E_1(0)|^2 \tag{13}$$

Hence the electric field is discontinuous at $x = 0$.

Let's now analyze the solution (11) for a realistic plasma model, i. e., an inhomogeneous plasma with a finite gradient in $n_0(x)$. For this we assume:

$$\omega_{P_e}^2(x) = \omega_{P_1}^2(x)[1 + \varepsilon(x/L)] \quad ; \quad (L \geq x \geq 0; \varepsilon > -1) \tag{14}$$

Corresponding to a constant density gradient in the transition region. It can be shown that the linear approximation is valid in this case provided that

$$1 >> |\varepsilon|(\omega_b / \omega_{P_e})^2 (\omega_{P_e} / \nu)(V_{0b} / \omega_{P_e} L) \tag{15}$$

Which indeed requires that $L \neq 0$.

In order to prove that expression (11) is essentially correct, a solution of wave equation (8) requires the use of density profile (14), which yields the equation

$$\frac{d^2 F}{d \xi^2} + b^{-2} \xi^{-1} F = C_2(T_e) \tag{16}$$

Where;

$$\xi = a - b(\frac{\omega_{Rb}}{V_{0b}} x) \quad ; \quad a = \frac{\omega \tilde{\omega} - \omega_{P_1}^2}{\omega \tilde{\omega}} \quad ; \quad b = \frac{\varepsilon}{L} \frac{V_{0b} \omega_{P_1}^2}{\omega \tilde{\omega} \omega_{Rb}}$$

Solution of (16) is:

$$F(z) = \frac{z^2 b^4}{4} C_2(T_e) + A z J_1(z) + B z N_1(z) \quad ; \; 0 \le x \le L \tag{17}$$

Where, $z = 2\xi^{1/2} / b$ and $J_1(z)\,;\, N_1(z)$ are the Bessel function of the first and second kind, respectively, and $C_2(T_e) = (\frac{V_{0b}}{b\omega_{Rb}})^2 C_1(T_e)$.

Bohmer, et al. [7] result is in agreement with that obtains solution (17), except the presence of a new term $C_2(T_e)$ on right hand side and static magnetic field H_0 and relativistic effect γ, which strongly proportional to the electron thermal velocity V_{T_e}.

$$F(x) = \sum_{\pm} A_{\pm} e^{\pm i\kappa_2 (x-L)} + \frac{b^4 z^2}{4} C_2(T_e)\,; x \ge L$$

Where

$$A_{\pm} = \frac{\pi}{4} z_1 F(0) \{[(N_0(z_0) J_1(z_1) - J_0(z_0) N_1(z_1)) \pm (J_1(z_0) N_0(z_1) - N_1(z_0) J_0(z_1))] + \\ i[(N_1(z_0) J_1(z_1) - J_1(z_0) N_1(z_1)) \pm (J_0(z_1) N_0(z_0) - N_0(z_1) J_0(z_0))]\} \tag{18}$$

Such that

$$z_0 = \frac{2\omega_{Rb}}{bV_{0b}} \frac{1}{\kappa_1} \quad ; \quad z_1 = \frac{2\omega_{Rb}}{bV_{0b}} \frac{1}{\kappa_2}$$ are correspond to $(x = 0\,, x = L)$ and:

$$\kappa_1 = \frac{\omega_{Rb}\,\omega}{V_{0b} \sqrt{\omega \tilde{\omega} - \omega_{P_1}^2}}, \qquad \kappa_2 = \frac{\omega_{Rb}\,\omega}{V_{0b} \sqrt{\omega \tilde{\omega} + \omega_{P_1}^2 (1 + \varepsilon)}}.$$

Equation (18) can be re-written as

$$A_{\pm} = (\frac{\omega_{Rb}}{bV_{0b}})^2 \frac{F(0)}{\kappa_1 \kappa_2^2} [(\kappa_1 \mp \kappa_2) \pm i \frac{V_{0b}}{\omega_{Rb}} b \kappa_1 \kappa_2] \ln \frac{\kappa_2}{\kappa_1}$$

The case of interest is when ε is not too small and (L/λ) is not very large [large rapid changes in $n_0(x)$] which is the opposite extreme from the WKB situation. From the definition following (15) where $\lambda = V_{0b}/\omega_{P_e}$ it is noted that $\max(\nu/\omega_{P_e};\varepsilon) > |\xi|$ and $|b| \approx \varepsilon(\omega_{P_e}/\omega_b)(\lambda/L)$. Therefore, if (L/λ) is not too large, and ε is not too small, then b is large and $|\xi|$ will be fairly small. Consequently, z is small in this case and Bessel function in (18) may be expanded for small argument. When this is done, one finally obtains the approximate result

$$E_2(x) = \left(\frac{\omega_{Rb}}{bV_{0b}}\right)^2 \frac{F(0)}{\kappa_1\kappa_2^2}\left[(\kappa_1 \mp \kappa_2) \pm i\frac{V_{0b}}{\omega_{Rb}} b\kappa_1\kappa_2\right]\ln\frac{\kappa_2}{\kappa_1} e^{\pm i\kappa_2(x-L)+\frac{i\omega}{V_{0b}}x} + b^2 C_2(T_e) e^{\frac{i\omega}{V_{0b}}x} \qquad (19)$$

Where $x \geq L$.

This equation is in agreement with that obtained by Sahyouni et al. [12] but in case of vanishing magnetic field (i.e., $H_0 = 0$) and nonrelativistic electron beam (i.e., $\gamma = 1$). Since the power of electric field in this case is greater than the power in the case of vanishing magnetic field an auxiliary plasma heating may be obtained. The result (19) may be compared with result (11) for the simple discontinuous model. It can be see that provided b is large and (16) is satisfied (L not too large and not too small), the result (11) is a good approximation to equation (19).

CONCLUSIONS

Existence of an external static magnetic field and plasma warmness leads to wave amplification and accordingly to plasma heating in beam-plasma system (solutions (11) and (19)). From (12), we could conclude that power absorbed from the beam into plasma is strongly affected by both mixing and plasma warmness. The variation in the plasma density does have a profound effect on the spatial beam-plasma instability. This effect indicates that the resulting drop in intensity of electric field is a sensitive function of the plasma discontinuity. It also growing modes, only if the plasma density decreases.

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