A FAST AND OPTIMIZED DYNAMIC ECONOMIC LOAD DISPATCH FOR LARGE SCALE POWER SYSTEMS

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ABSTRACT

This paper presents LaGrangian Multipliers (LM) and Linear Programming (LP) based dynamic economic load dispatch (DELD) solution for large-scale power system operations. It is to minimize the operation cost of power generating units subject to the considered constraints. After individual generator units are economically loaded and periodically dispatched, fast and optimized DELD has been achieved. DELD with period intervals has been taken into consideration. The results found from the algorithm based on LM and LP techniques appear to be modest in both optimizing the operation cost and achieving fast computation time.

Keywords
Dynamic Economic Load Dispatch, LaGrange Multipliers, Linear programming, Convergence, Constraints, Objective Function.

INTRODUCTION

Economic dispatch may some times classified into two types [1]. One is static optimization problem in which costs associated with the changing time period of the unit outputs are not considered. A dynamic dispatch on the other hand is considered as the one in which the operating cost changes with the time periods and power output in the load dispatching process [2]. The use of the steady state operating costs in the static optimization, poor transient behavior result when these solutions are incorporated in the feedback control of dynamic electric power networks. The dynamic dispatch method uses load forecasts of the system to enable the system operator experts to develop optimal generator output trajectories. Generators are driven along the optimal trajectories by the control performance of the feedback controller [1, 3].

In dynamic economic load dispatch, it is habitual to divide dispatch horizon into different time periods and form a model to meet the power generation and load demand balance at the end of each selected period [2]. Since the constraints under consideration are very large scale and complex, after changing non-linear mathematical modeling into linear ones, it is assumed in this work that the load changes with its related period linearly.

in the case of linear load curves, two types may be taken into consideration; step approximation curve and trapezoid approximation curve. According to reference [2] The above figure shows all the constraints which involve DELD and objective function (total operating cost) of the system.

The problem has been decomposed into different sub-models, which contain the original model of the problem. A partition of the real problem into smaller sub-problems simplifies the multi-constrained problems to a manageable one. The applied algorithm is two-stage solution technique that solves DELD...
problems with LM initially and interfaces LP technique finally. This facilitates real time DELD scheduling profile in real generation systems. The methods used to this large-scale problem show the required convergence with both variables and constraints. An optimized total objective function has been gained with a very short computation time.

The DELD can be decomposed into the following way:

<table>
<thead>
<tr>
<th>Thermal Unit Constraints</th>
<th>System Operational Limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Start-up Costs</td>
<td>Total On-Line Generation Costs</td>
</tr>
<tr>
<td></td>
<td>System Generation Limit Constraints Group Import/Export Constraints</td>
</tr>
<tr>
<td>Objective Function</td>
<td></td>
</tr>
<tr>
<td>Generating Unit Limitations</td>
<td>Operating Cost</td>
</tr>
<tr>
<td>Upper and Lower Gen. Limits Reloading Rate Limit Minimum Up-time and Down-Times Loading Rate Limit</td>
<td></td>
</tr>
<tr>
<td>Running Costs</td>
<td>Shut-down Cost and Start-up Costs</td>
</tr>
<tr>
<td>Total Demand</td>
<td>Total Spin. Reserve Requirement Transmission Loss</td>
</tr>
</tbody>
</table>

**FIGURE 1: Flow Diagram DELD Constraints and Objective Function**

**PROBLEM FORMULATION**

The solution of this problem consists of two portions of solution techniques.

1) Standard LaGrange Multipliers Method is used for solving initial conditions of dispatch system

2) The basic model is the result of using linear mathematical model of DELD problem already performed in the first model. Then LP is selected as an application technique for the problem.

**LaGrangian Multipliers**

As mentioned above Standard LaGrange Multiplier Method is used for the initial economic dispatch. The following solution method has been adopted. First of all an initial Standard LaGrange Method is formulated and used for the preliminary run of the dispatch system. The solution method adopted is based on the following equations:

1) Power produced by the units must at least equal to the power demand by the consumer, \( P_D \) plus the network losses \( P_L \) and. System Spinning Reserve, SR as:

\[
\sum P_i \geq P_D + P_L + SR
\]

2) The following relation determines LaGrange multiplier, \( \lambda \) which is:

\[
\lambda = \frac{2 \times P_D + \sum_{i=1}^{m} (\beta_i/\gamma_i)/\sum_{i=1}^{m} \gamma_i^i}{2}
\]

Where

\( \beta_i, \gamma_i \) are cost coefficient of unit i

Since the aim is to find a solution to the constrained values of power demand and power generated by the units. Lambda is in effect a conversion factor that accounts for the dimensional incompatibility of the cost function ($/h) and constraints (MW) and resulting problem is unconstrained one [4].

3) Quadratic equation can be stated as:

\[
F_i (P_i) = \alpha_i + \beta_i P_i + \gamma_i^2
\]
The above equation is changed to linear one by differentiating it for optimality condition and is reduced to:

\[ \beta_i + 2\gamma P_i - \lambda = 0 \]  

(4)

By reaching the optimum solution of operating cost "Eqn. (4)" output power is determined.

Subsequently, optimum incremental a cost (\(\lambda\)), unit output levels and load balance and spinning reserve are aimed at their solutions.

Different prototype examples have been used and the same data have also been applied to the second portion of linear programming technique.

**Linear Programming Method**

Generation levels of each unit changes linearly with the related periods sequentially until it reaches the last generation level. The dispatch system is dynamic one in that sense.

Energy cost of a unit with respect to its periods is expressed as:

\[ Z_{ij} = b_i P_{ij} T_j \]  

(5)

Where

- \(Z_{ij}\) = energy cost produced by unit \(i\)
- \(b_i\) = incremental cost of unit \(i\)
- \(P_{ij}\) = power generated by unit \(i\) in interval \(j\)
- \(T_j\) = time interval of period \(j\)

Since overall goal of economic dispatch is to reduce the total objective function with reasonable computation, Eqn. (6) is formulated.

Minimize \(Z_t = \sum_x \alpha_{ij} P_{ij}\)  

(6)

Where

- \(Z_t\) = total objective function
- \(\alpha_{ij}\) = \(1/2b_i(T_j+T_l)\)

and \(P_{ij} = 1/2(P_{ij} + P_{il})\) (average of two output powers of two different periods \(j\) and \(l\))

Since spinning reserve of the system is important here, the following equality condition must be met whereby output power of unit \(i\) must maintain that constraint in Eqn. (7).

\[ P_i \geq P_D + S_T \]  

(7)

where

- \(P_i\) = output power of unit \(i\)
- \(S_T\) = total system spinning reserve

Power generated by the units must be within the minimum and maximum capacity limits of the units themselves.

\[ P_{i_{\text{min}}} \leq P_i \leq P_{i_{\text{max}}} \]  

(8)

where

- \(P_{i_{\text{min}}}\) = minimum generation limit of unit \(i\)
- \(P_{i_{\text{max}}}\) = maximum generation limit of unit \(i\)

Units must be loaded within a specific hour and can be changed in the next hour, the following condition must be found:

\[ P_{i_{\text{min}}} \leq P_{il} \leq P_{i_{\text{max}}} \]  

(9)

\[ \delta P_{i_{\text{min}}} \leq \delta P_{i_{\text{max}}} \]  

(10)

where

- \(P_{i_{\text{min}}}\) = maximum loading rate of unit \(i\)
- \(P_{i_{\text{min}}}\) = minimum deloading rate of unit \(i\).
Spinning reserve contribution of a steam thermal unit can be modeled as:

If the unit is operating in the lower region, then its reserve contribution can be written as:

\[ S_y = k_y P_y \]

If not, then it can be expressed as:

\[ S_y = P_{y,\text{max}} - P_y \]

Convexity permits the overall contribution to be stated as:

\[
\sum S_y \leq k_y P_y \tag{11}
\]

\[
S_y \leq P_{y,\text{max}} - P_y \tag{12}
\]

The Total Spinning reserve constraint can be stated as:

\[
\sum S_y \geq S_{TJ} \tag{13}
\]

These Equations represent the objective function and constraints undertaken by the formulations used in this research, as the objective function is a subject to each constraint while the objective function is a minimization type problem (operating cost). To simplify the dispatch problem used in this work, fig. 2 shows a decomposition flow chart about DELD objective function and the constraints under consideration [5].

**RESULTS**

A number of case studies have been carried out using SUN WORKSTATIONS, SPARC10 as well as PC for an ongoing research. This research has performed in different case studies using the aforesaid techniques. Many different data types have been tested for both experiment and verification purposes [5]. The following six tables show some samples from those case studies. Three sample unit characteristics and related output results and an other for example of forty units are and their output results shown below.

According to this research, the adopted formulation towards economic load dispatch and unit commitment problems is based on optimizing the operating cost of generation units using LaGrangian Multipliers. This technique depends on formulating cost coefficients in such a way that all the units in a station will have only one incremental cost. This incremental cost represents the optimum value of operating cost of all generating units in any particular time interval in a power generation system.

As different data engagements have been carried out by this research, one of the examples applied in this work includes the following three units proposed in [6].

In table 2, results shown are calculated by the developed program using LaGrangian Multipliers for the aim of minimizing the common incremental cost of the generation units. After getting optimum incremental cost of all units in an interval and the required unit output levels then operating cost can be determined as the product of the two variables.

Three main constraints are involved in this part which are; one constraint deals with the balance between unit output levels and power demand without considering network losses of the system another one involves the system spinning reserve allocation and the last one maintains unit lower and upper limitations. All of the constraints have met their requirements.
### TABLE 1: Three Units Characteristics and Their Cost Coefficients.

<table>
<thead>
<tr>
<th>Unit Number</th>
<th>$P_{\text{max}}$ (MW)</th>
<th>$P_{\text{min}}$ (MW)</th>
<th>$\alpha$ (S/h)</th>
<th>$\beta$ (S/h)</th>
<th>$\gamma$ (S/h)</th>
<th>Demand (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>600</td>
<td>150</td>
<td>561</td>
<td>7.92</td>
<td>0.003124</td>
<td>850</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>100</td>
<td>310</td>
<td>7.85</td>
<td>0.00388</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>50</td>
<td>78</td>
<td>7.97</td>
<td>0.00964</td>
<td></td>
</tr>
</tbody>
</table>

$P_{\text{max}}$ = Maximum capacity i, $P_{\text{min}}$ = minimum capacity of unit i, $\alpha =$ first cost coefficient expressed in “Eqn. (4)”, $\beta =$ second cost coefficient expressed in “Eqn. (4)”, $\gamma =$ third portion of the cost coefficient, demand = power demanded by the consumer.

### TABLE 2: Results Achieved from the Three Units System.

<table>
<thead>
<tr>
<th>Periods</th>
<th>Unit No.</th>
<th>Unit Output Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>396.458</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>327.822</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>125.721</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>10.394</td>
</tr>
<tr>
<td>Total output</td>
<td>850.000</td>
<td>850.000</td>
</tr>
<tr>
<td>Period Cost</td>
<td>8834.817</td>
<td>8834.817</td>
</tr>
<tr>
<td>Total Cost</td>
<td>53008.902</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 3: Fifteen-unit characteristics [7]

<table>
<thead>
<tr>
<th>Unit</th>
<th>$\gamma$</th>
<th>$\beta$</th>
<th>$P_{\text{min}}$</th>
<th>$P_{\text{max}}$</th>
<th>$S^i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000299</td>
<td>10.07</td>
<td>150</td>
<td>455</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>0.00183</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.001265</td>
<td>8.8</td>
<td>20</td>
<td>130</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>0.001265</td>
<td>8.8</td>
<td>20</td>
<td>130</td>
<td>0.0</td>
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<tr>
<td>5</td>
<td>0.00205</td>
<td>10.4</td>
<td>150</td>
<td>470</td>
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<tr>
<td>6</td>
<td>0.00301</td>
<td>10.1</td>
<td>135</td>
<td>460</td>
<td>0.0</td>
</tr>
<tr>
<td>7</td>
<td>0.003649</td>
<td>9.87</td>
<td>135</td>
<td>465</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>0.00338</td>
<td>11.5</td>
<td>60</td>
<td>300</td>
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<tr>
<td>9</td>
<td>0.00807</td>
<td>11.21</td>
<td>25</td>
<td>162</td>
<td>30.0</td>
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<tr>
<td>10</td>
<td>0.01203</td>
<td>10.72</td>
<td>20</td>
<td>160</td>
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</tr>
<tr>
<td>11</td>
<td>0.03586</td>
<td>11.21</td>
<td>20</td>
<td>80</td>
<td>20.0</td>
</tr>
<tr>
<td>12</td>
<td>0.05513</td>
<td>9.97</td>
<td>20</td>
<td>80</td>
<td>0.0</td>
</tr>
<tr>
<td>13</td>
<td>0.00371</td>
<td>13.12</td>
<td>25</td>
<td>85</td>
<td>20.0</td>
</tr>
<tr>
<td>14</td>
<td>0.01929</td>
<td>12.12</td>
<td>15</td>
<td>55</td>
<td>40.0</td>
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<tr>
<td>15</td>
<td>0.04447</td>
<td>12.41</td>
<td>15</td>
<td>55</td>
<td>40.0</td>
</tr>
</tbody>
</table>
TABLE 4: Fifteen-unit Output power levels [7].

<table>
<thead>
<tr>
<th>Periods</th>
<th>Units</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>420.730</td>
<td>420.730</td>
<td>420.730</td>
<td>420.730</td>
<td>420.730</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>409.188</td>
<td>409.188</td>
<td>409.188</td>
<td>409.188</td>
<td>409.188</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>63.657</td>
<td>63.657</td>
<td>63.657</td>
<td>63.657</td>
<td>63.657</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>83.429</td>
<td>83.429</td>
<td>83.429</td>
<td>83.429</td>
<td>83.429</td>
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<tr>
<td>4</td>
<td>5</td>
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<td>418.529</td>
<td>418.529</td>
<td>418.529</td>
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<td></td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>368.099</td>
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<td>368.099</td>
<td>368.099</td>
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<tr>
<td>6</td>
<td>7</td>
<td>378.875</td>
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<td>378.875</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>255.455</td>
<td>255.455</td>
<td>255.455</td>
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</tr>
<tr>
<td>8</td>
<td>9</td>
<td>156.009</td>
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<td>156.009</td>
<td>156.009</td>
<td>156.009</td>
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</tr>
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<td>9</td>
<td>10</td>
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<td>49.708</td>
<td>49.708</td>
<td>49.708</td>
<td>49.708</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>40.912</td>
<td>40.912</td>
<td>40.912</td>
<td>40.912</td>
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</tr>
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<td>11</td>
<td>12</td>
<td>38.237</td>
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<td>38.237</td>
<td>38.237</td>
<td>38.237</td>
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<tr>
<td>12</td>
<td>13</td>
<td>80.623</td>
<td>80.623</td>
<td>80.623</td>
<td>80.623</td>
<td>80.623</td>
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</tr>
<tr>
<td>14</td>
<td>15</td>
<td>47.507</td>
<td>47.507</td>
<td>47.507</td>
<td>47.507</td>
<td>47.507</td>
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</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

in tables 1, 2, 3, and 4 respectively. The problems are solved using both Standard LaGrange Multipliers and Linear Programming methods are shown in these tables. Respective models used also envisage the flow chart shown in figure 1.

CONCLUSION

Two steps of modeling have been made for the solution of DELD problem; they are:
1) Standard LaGrange Multipliers
2) Linear Programming technique

1) In Standard LaGrange Multipliers Method, initial economic load dispatch has been formulated using Standard LaGrange Multipliers, which is intended to be optimized for the next stage by using linear programming DELD problem. Initial balance of power generation outputs and total load demand and incremental cost have been determined and referred to the second stage.

3) In the case of LP model, balanced unit output and total demand, Inequality, Spinning reserve contribution, Loading and De-loading rate, Import/export and other constraints are double-checked and solved. Linking technique is also applied by LaGrange Dual Function. All selected constraints shown in fig. 2 are considered in both stages. The aim is to optimize the operation cost of generation units in a short processing time. The algorithm appears to be fast and meets the deserved near optimum solution.
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