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REPHASING-INVARIANT *CP* VIOLATING PARAMETERS WITH MAJORANA NEUTRINOS

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Abstract

We analyze the dependence of the squared amplitudes on the rephasing-invariant *CP*-violating parameters of the lepton sector, involving Majorana neutrinos, for various lepton-conserving and lepton-violating processes. We analyze the conditions under which the *CP*-violating effects in such processes vanish, in terms of the minimal set of rephasing invariants, giving special attention to the dependence on the extra *CP*-violating parameters that are due to the Majorana nature of the neutrinos.

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1 Introduction

Sometime ago [1], we presented a rephasing-invariant formulation of CP violation in the standard model, paying particular attention to the CP violation in the lepton sector involving Majorana neutrinos. In that reference we generalized the previous work by Greenberg [2], and by Dunietz, Greenberg and Wu [3], on the quark sector. We offered a well-defined prescription for identifying a minimal set of rephasing-invariant CP-violating parameters for any number of generations of Dirac fermions, which is applicable to the quark sector, and the method was extended to the lepton sector with Majorana neutrinos.

However, the question of how those rephasing invariants enter in the amplitudes or rates for physical processes was touched upon only briefly there. It is certainly of interest to see how different physical quantities may depend on the rephasing invariants in a given physical process, since this is necessary in order to understand the conditions under which the CP-violating effects in a given process may vanish.

The question is more pressing when we consider the case of Majorana neutrinos. In this case, there are more independent rephasing invariant CP-violating parameters than in the quark sector or in a lepton-number conserving lepton sector with the same number of families, and are due to the Majorana nature of the neutrinos [4, 5, 6]. Accordingly, there is a larger number of rephasing invariants for this case [1]. It is generally believed that these parameters show up only in lepton number violating processes [7], and the rephasing invariant formulation has proven to be useful in analyzing a variety of such processes [8, 9, 10]. A particularly important question is whether those additional parameters can show up in lepton number conserving processes as well.

Another interesting question arises from the following. In the neutrinoless double beta decay process, the CP-violating terms in the amplitude squared consist of products of the real and the imaginary part of the relevant rephasing invariant parameters[8, 10]. Thus it would seem, at least in that context, that the potential observability of any effect associated with the CP-violating part of the rephasing invariants requires also that the real part of those parameters be non-zero. Therefore, an important question to ask is whether this is a general feature, of this and other processes, or whether it is possible to have observable effects due to the CP-violating part of the rephasing invariants even if their real parts are zero.

To this end, we consider various generic physical processes, classified according to whether they conserve total lepton number, or by how many units they violate it. We find their generic dependence on the rephasing-invariant CP-violating parameters, from which the conditions under which the CP-violating effects may vanish follow. This procedure allows us to consider the questions that we have posed, and opens the way to make more general statements, divorced from the kinematical aspects of the processes considered.

In Section 2 we introduce the notation, conventions and other ingredients needed for our

analysis. They are used in Section 3 to consider various classes of processes with regard to the questions mentioned above and, finally, our conclusions are summarized in Section 4.

2 Basic vertices and amplitudes

2.1 The rephasing invariants

As shown in Ref. [1], a suitable set of rephasing invariants to describe CP violation in the lepton sector with Majorana neutrinos, is constructed out of the products

$$s_{\alpha ij} = V_{\alpha i} V_{\alpha j}^* K_i^* K_j^* , \tag{2.1}$$

where the $V_{\alpha i}$ are the elements of the mixing matrix that appears in the charged-current part of the Lagrangian,

$$\mathscr{L}_{\rm cc} = \frac{g}{\sqrt{2}} W_\lambda \sum_{\alpha,i} V_{\alpha i} \bar{\ell}_\alpha \gamma^\lambda L \nu_i + \text{H.c.} , \tag{2.2}$$

while the K_i are the phases that appear in the Majorana condition[4]

$$\nu_i^c = K_i^2 \nu_i . \tag{2.3}$$

We have denoted by ℓ_α and ν_i the charged lepton and the neutrino mass eigenfields, respectively, and $L = \frac{1}{2}(1 - \gamma_5)$. The sum in Eq. (2.2) goes over all charged leptons and all neutrino eigenstates.

For processes which do not involve lepton number violation on any fermion line, it is convenient to use the rephasing invariant combinations[5] [2, 3]

$$t_{\alpha i \beta j} = V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^* . \tag{2.4}$$

In these cases, the independent CP-violating parameters for N lepton generations can be taken to be

$$\text{Im}(t_{\alpha i e N}), \quad \alpha \leq i, \quad \alpha \neq 1, \quad i \neq N . \tag{2.5}$$

For processes that violate lepton number, there are two ways to proceed[1]. One way is to take the minimal set given in Eq. (2.5) and append it with the set

$$\text{Im}(s_{1iN}), \quad i \neq N . \tag{2.6}$$

An alternative is to use the fact that the t-invariants can be expressed in terms of the s-invariants as

$$t_{\alpha i \beta j} = s_{\alpha ij} s_{\beta ij}^* , \tag{2.7}$$

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[4]We take this opportunity to note that Eq. (16) of Ref. [1] contains a typographical error. The correct formula, consistent with the notation of that paper, is given here in Eq. (2.3). We thank Serguey Petcov for bringing this to our atention.

[5]In connection with these and later formulas in this paper, it should be remarked that the flavor indices, both for charged leptons and neutrinos, are not assumed to be summed over whenever they are repeated. Summation is always explicitly indicated wherever applicable.

so that we only need to make reference to the s-invariants. If we follow this option, a minimal set of independent CP-violating parameters, which are rephasing invariant, are

$$\text{Im}(s_{\alpha i N}), \qquad \alpha \leq i, \quad i \neq N. \tag{2.8}$$

For example, with this choice, the three independent rephasing invariants in the case of three families of leptons are

$$s_{113}, \ s_{123}, \ s_{223}. \tag{2.9}$$

while in the first approach, the set is

$$s_{113}, \ s_{123}, \ t_{2213}. \tag{2.10}$$

2.2 Basic amplitudes

We consider processes such as $nn \to ppee$ (neutrinoless double-beta decay) and $\mu \to 3e$, in which the individual lepton flavors are not conserved, and which do not involve neutrinos in the external states. Since the Standard Model conserves baryon number, and since the total number of external fermions in any given process must be even, then the number of external charged leptons must be even also. This means that, in such processes, the total lepton number is either conserved, or it is violated in multiples of two units. Since the only source of flavor violation in the Standard Model is the charged current, it follows that any process of the kind we are considering must contain the basic subprocesses shown in Fig. 1 as building blocks. Apart from the kinematical factors, the amplitudes associated with these two subprocesses involve the following elements of the neutrino mixing matrix

$$A_{\beta\alpha i} \ = \ V_{\beta i} V_{\alpha i}^* \tag{2.11}$$

$$B_{\beta\alpha i} \ = \ V_{\beta i} V_{\alpha i} (K_i^*)^2, \tag{2.12}$$

corresponding to the diagrams 1a and 1b, respectively.

$$\ell_\alpha \quad \nu_i \quad \ell_\beta \qquad\qquad \ell_\alpha \quad \nu_i \quad \ell_\beta$$
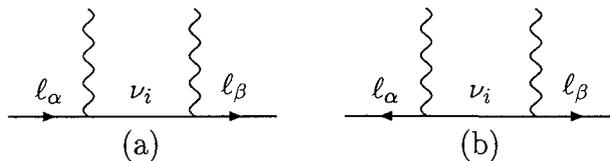$$\text{(a)} \qquad\qquad\qquad \text{(b)}$$

Figure 1: The two basic subdiagrams for processes that do not involve neutrinos in the external states. Diagram (a) conserves total lepton number, while Diagram (b) changes it by 2 units.

The appearance of the $(K_i^*)^2$ factor in Eq. (2.12) originates in the contraction of the form $\langle 0 | \nu_i \nu_i | 0 \rangle$ that appears in the Wick contraction of the interaction terms, which can be turned into the neutrino propagator $\langle 0 | \nu_i \bar{\nu}_i | 0 \rangle$ with the help of the Majorana condition of Eq. (2.3). An alternative treatment, which amounts to a redefinition of the neutrino field, is to associate

a factor of $V_{\alpha i} K_i^*$ with each W vertex in a diagram. In any case, the underlying reason is that, since the processes do not involve neutrinos in the external states, then the amplitudes (and not just the rates) must be invariant under a rephasing of the neutrino field alone, which requires that the mixing matrix elements appear in the combination $V_{\alpha i} K_i^*$.

The precise way in which the factors $A_{\beta\alpha i}$ and $B_{\beta\alpha i}$, and/or their products, may enter in the total amplitude for a process depends on the particular process under consideration. We consider such processes in the following section, and identify the CP-violating parts of their amplitudes in terms of the rephasing invariants.

3 Analysis of specific processes

In this section, we study how the amplitudes for various processes depend on the parameters $s_{\alpha ij}$. As we will see, there are processes for which the CP violating terms vanish if either the real or the imaginary part of the relevant s-invariants are zero. However, this result is not general, and in particular it does not hold if we consider higher order corrections. In the general case, the CP violating terms do not vanish when the real parts of the s-invariants are zero, although, as expected, they do vanish when the imaginary parts of the s-invariants are zero. Furthermore, there are processes that conserve lepton number for which the CP violating terms depend on the s-invariants and cannot be expressed in terms of the t-invariants alone.

3.1 Classification scheme

In order to organize the analysis, we divide the processes into two groups, according to whether the total lepton number changes or not. These two groups are further subdivided into smaller groups depending on the change of the individual lepton flavors, ΔL_α. It is then useful to introduce the following device. In a given diagram, the subamplitude $A_{\beta\alpha i}$ increases the lepton flavor β and decreases the flavor α, both by one unit, and leaves unchanged all the other flavors. To use this feature in a precise way, we introduce a flavor counting operator L_γ, for each flavor γ, and write

$$L_\gamma\{A_{\beta\alpha i}\} = \delta_{\gamma\beta} - \delta_{\gamma\alpha} \,. \tag{3.1}$$

Similarly,

$$L_\gamma\{B_{\beta\alpha i}\} = \delta_{\gamma\beta} + \delta_{\gamma\alpha} \,, \tag{3.2}$$

and the total lepton number operator is defined as

$$L = \sum_\gamma L_\gamma \tag{3.3}$$

where the sum runs over all the lepton flavors. This definition, together with Eqs. (3.1) and (3.2) imply

$$
\begin{aligned}
L\{A_{\beta\alpha i}\} &= 0 \\
L\{B_{\beta\alpha i}\} &= 2 \,.
\end{aligned}
\tag{3.4}
$$

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These definitions are complemented by the rules for the complex conjugate amplitudes

$$
\begin{aligned}
L_\gamma\{A^*_{\beta\alpha i}\} &= -L_\gamma\{A_{\beta\alpha i}\} \\
L_\gamma\{B^*_{\beta\alpha i}\} &= -L_\gamma\{B_{\beta\alpha i}\}\,,
\end{aligned}
\tag{3.5}
$$

which imply

$$
\begin{aligned}
L\{A^*_{\beta\alpha i}\} &= 0 \\
L\{B^*_{\beta\alpha i}\} &= -2\,.
\end{aligned}
\tag{3.6}
$$

The amplitude M for any given process is a sum of terms

$$
M = \sum_X M_X f_X\,,
\tag{3.7}
$$

where the f_X are kinematical factors and each M_X is made of a product of the subamplitudes $A_{\beta\alpha i}$, $B_{\beta\alpha i}$ and/or their complex conjugates. We indicate this schematically by writing

$$
M_X = C_1 C_2 ... C_{N_X}\,,
\tag{3.8}
$$

where each C_x stands for one of the subamplitudes or its complex conjugate. The flavor counting rules are amended to include the rules for the product

$$
L_\gamma\{M_X\} = \sum_{x=1}^{N_X} L_\gamma\{C_x\}\,.
\tag{3.9}
$$

When building the amplitude for a given process out of products of the subamplitudes $A_{\beta\alpha i}$, $B_{\beta\alpha i}$ (and their complex conjugates), the basic requirement is that all the terms M_X for that process must have the same lepton flavor; i.e.,

$$
L_\gamma\{M_1\} = L_\gamma\{M_2\} = ... = L_\gamma\{M_{N_X}\}
\tag{3.10}
$$

for all flavors γ.

3.2 $\Delta L = 2$ processes

We subdivide this group of processes according to whether only one flavor changes or two of them change.

3.2.1 One flavor change

In this kind of process, the amplitude is such that, for some flavor α,

$$
\begin{aligned}
L_\alpha\{M\} &= 2 \\
L_\gamma\{M\} &= 0 \qquad (\gamma \neq \alpha)\,.
\end{aligned}
\tag{3.11}
$$

An example of this kind of process is neutrinoless double beta decay, or simply $W^- W^- \to e^- e^-$, which is of the type given in Fig. 1b, with $\ell_\alpha = \ell_\beta = e$.

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It is then easy to see that, in the notation of (2.12), the amplitude for this type of process is of the form

$$\mathcal{M} = \sum_i B_{\alpha\alpha i} f_i, \tag{3.12}$$

where f_i are functions that depend on the kinematic variables of the problem, including the mass of the internal neutrino line. Thus, [8]

$$\left|\mathcal{M}\right|^2 = \sum_{ij} (B_{\alpha\alpha i} f_i)(B_{\alpha\alpha j} f_j)^* = \sum_{ij} \left(s_{\alpha ij}\right)^2 f_i f_j^*, \tag{3.13}$$

where we have used Eqs. (2.1) and (2.12). Therefore, any CP violating effect is proportional to the imaginary part, Im $\delta_{\alpha\alpha ij}$, of the interference terms

$$\delta_{\alpha\alpha ij} = \left(s_{\alpha ij}\right)^2. \tag{3.14}$$

Since Im $(s_{\alpha ij})^2 = 2(\text{Re } s_{\alpha ij})(\text{Im } s_{\alpha ij})$, then the following statement can be made for this type of process: any CP violating effect due to a non-zero imaginary part of an element $s_{\alpha ij}$ vanishes unless the real part of the same element is non-zero.

3.2.2 Two flavors change

In this kind of process the amplitude is such that, for some two flavors α, β,

$$\begin{aligned} L_\alpha\{M\} = L_\beta\{M\} &= 1 \\ L_\gamma\{M\} &= 0 \qquad (\gamma \neq \alpha, \beta). \end{aligned} \tag{3.15}$$

An example is the process $nn \leftrightarrow ppe\mu$, or simply $WW \to e\mu$. For this case,

$$\mathcal{M} = \sum_i B_{\alpha\beta i} f_i, \tag{3.16}$$

and it is easy to see that

$$\left|\mathcal{M}\right|^2 = \sum_{ij} s_{\alpha ij} s_{\beta ij} f_i f_j^*. \tag{3.17}$$

The interference terms are

$$\delta_{\alpha\beta ij} = s_{\alpha ij} s_{\beta ij}, \tag{3.18}$$

and the CP violating effects are proportional to

$$\text{Im } \delta_{\alpha\beta ij} = (\text{Re } s_{\alpha ij})(\text{Im } s_{\beta ij}) + (\alpha \leftrightarrow \beta). \tag{3.19}$$

Therefore, in these processes the CP-violating effects due to a non-zero Im$(s_{\beta ij})$ or Im$(s_{\alpha ij})$ vanish unless the real part of Re$(s_{\alpha ij})$ or Re$(s_{\beta ij})$ is non-zero, respectively.
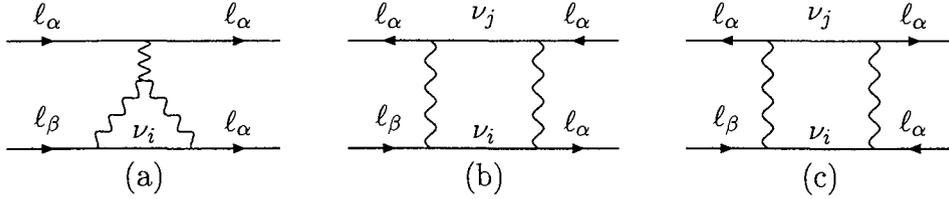
Figure 2: Different kinds of diagram for the process $\ell_\beta \ell_\alpha \rightarrow \ell_\alpha \ell_\alpha$. The actual number of diagrams to be calculated are larger because of crossed diagrams etc. In diagram (a), the vector boson connected to the upper fermion line can be the photon or the Z-boson. All other vector boson lines are W lines.

3.3 $\Delta L = 0$ processes

We consider two possibilities, depending on whether two flavors change (in an opposite way), or three flavors change.

3.3.1 Two flavors change

In this case the amplitude is such that, for two given flavors α, β,

$$
\begin{aligned}
L_\alpha\{M\} = -L_\beta\{M\} &= 1, \\
L_\gamma\{M\} &= 0 \qquad (\gamma \neq \alpha, \beta).
\end{aligned}
\tag{3.20}
$$

The simplest form of an amplitude for this type of process is

$$
\mathcal{M} = \sum_i A_{\alpha\beta i} f_i,
\tag{3.21}
$$

and it corresponds to, for example, processes such as $\mu + p \rightarrow e + p$. In this case,

$$
|\mathcal{M}|^2 = \sum_{ij} t_{\alpha i \beta j} f_i f_j^*,
\tag{3.22}
$$

and the CP violating effects are proportional to

$$
\text{Im}\,(t_{\alpha i \beta j}) = (\text{Im}\,s_{\alpha ij})(\text{Re}\,s_{\beta ij}) - (\alpha \leftrightarrow \beta),
\tag{3.23}
$$

where we have used Eq. (2.7). Once again, this implies that the CP-violating effects due to $s_{\alpha ij}$ and $s_{\beta ij}$ disappear unless the real part of $s_{\beta ij}$ and $s_{\alpha ij}$ are non-zero, respectively.

We can consider also terms satisfying Eq. (3.20), but built out of products of the subamplitudes A, B. Such terms appear, for example, in the process $\mu + e \rightarrow e + e$. There are two such products consistent with Eq. (3.20), namely $A_{\alpha\beta i} A_{\alpha\alpha j}$ and $B^*_{\alpha\beta i} B_{\alpha\alpha j}$, and therefore the total amplitude is of the form

$$
\begin{aligned}
\mathcal{M} &= \sum_i A_{\alpha\beta i} f_i^a + \sum_{ij} \left(A_{\alpha\beta i} A_{\alpha\alpha j} f_{ij}^b + B^*_{\alpha\beta i} B_{\alpha\alpha j} f_{ij}^c \right) \\
&= \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c.
\end{aligned}
\tag{3.24}
$$

Diagrammatically, these three terms correspond respectively to the diagrams shown in Fig. 2. It is instructive to observe the analogy with the process considered previously, $\mu + p \to e + p$, which involves diagrams like Fig. 2a and another that resembles Fig. 2b. The latter involves the mixing matrix elements from the quark sector, which are implicitly taken into account in the structure given in Eq. (3.21) by means of the functions f_i.

In the case at hand, when we take the absolute square of the amplitude we obtain, first of all, the terms

$$
\begin{aligned}
\left|\mathcal{M}_a\right|^2 &= \sum_{ij} t_{\alpha i \beta j} f_i^a f_j^{a*}, \\
\left|\mathcal{M}_b\right|^2 &= \sum_{ijkl} t_{\alpha i \beta k} t_{\alpha j \alpha l} f_{ij}^b f_{kl}^{b*}, \\
\mathcal{M}_a \mathcal{M}_b^* &= \sum_{ijk} t_{\alpha i \beta j} |V_{\alpha k}|^2 f_i^a f_{jk}^{b*},
\end{aligned}
\tag{3.25}
$$

which can be expressed in terms of the t-type rephasing invariants alone since the fermion lines conserve total lepton number. This is not true for the remaining terms

$$
\begin{aligned}
\mathcal{M}_a \mathcal{M}_c^* &= \sum_{ijkl} s_{\beta ij}^* s_{\alpha ik} s_{\alpha jk} f_{ij}^b f_{kl}^{c*}, \\
\mathcal{M}_b \mathcal{M}_c^* &= \sum_{ijkl} s_{\beta ik}^* s_{\alpha il} s_{\alpha jk}^* s_{\alpha jl} f_{ij}^b f_{kl}^{c*}, \\
\left|\mathcal{M}_c\right|^2 &= \sum_{ijkl} s_{\beta ik}^* s_{\alpha ik}^* (s_{\alpha jl})^2 f_{ij}^c f_{kl}^{c*}.
\end{aligned}
\tag{3.26}
$$

It is interesting to see that the interference terms that appear in $\mathcal{M}_a \mathcal{M}_c^*$,

$$
\delta_{\alpha\beta ijk}^{(ac)} \equiv s_{\beta ij}^* s_{\alpha ik} s_{\alpha jk},
\tag{3.27}
$$

contain an odd number of s-invariants, in contrast with the other terms. In particular, their imaginary part

$$
\operatorname{Im} \delta_{\alpha\beta ijk}^{(ac)} = -\operatorname{Im}(s_{\beta ij})\operatorname{Im}(s_{\alpha ik})\operatorname{Im}(s_{\alpha jk}) + \text{other terms},
\tag{3.28}
$$

do not involve the real part of any s-invariants directly. On the other hand, if the imaginary part of the s-invariants vanish, then the imaginary part of all the interference terms certainly go to zero, as it should be. The distinctive feature of the amplitude given in Eq. (3.24) is that, as we have seen, it leads to interference terms that contain an odd number of s-invariants.

We can consider other processes where two flavors change, keeping the total lepton number conserved. For example, consider the case

$$
\begin{aligned}
L_\alpha\{M\} = -L_\beta\{M\} &= 2, \\
L_\gamma\{M\} &= 0 \qquad (\gamma \neq \alpha, \beta),
\end{aligned}
\tag{3.29}
$$

which corresponds to processes of the type $\ell_\beta \ell_\beta \to \ell_\alpha \ell_\alpha$ or, by crossing, to processes such as muonium-antimuonium oscillations. The diagrams for this case are similar to those given in

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Fig. 2b,c, where the incoming ℓ_α line has to be replaced by an incoming ℓ_β line. The corresponding amplitude is

$$
\begin{aligned}
\mathcal{M}(\ell_\beta \ell_\beta \to \ell_\alpha \ell_\alpha) &= \sum_{ij} \left(A_{\alpha\beta i} A_{\alpha\beta j} f^b_{ij} + B^*_{\beta\beta i} B_{\alpha\alpha j} f^c_{ij} \right) \\
&= \mathcal{M}_1 + \mathcal{M}_2 ,
\end{aligned}
\tag{3.30}
$$

and squaring we obtain

$$
\begin{aligned}
\left| \mathcal{M}_1 \right|^2 &= \sum_{ijkl} t_{\alpha i \beta k} t_{\alpha j \beta l} f^b_{ij} f^{b*}_{kl} , \\
\mathcal{M}_1 \mathcal{M}_2^* &= \sum_{ijkl} s_{\alpha il} s_{\alpha jl} s^*_{\beta ik} s^*_{\beta jk} f^b_{ij} f^{c*}_{kl} , \\
\left| \mathcal{M}_2 \right|^2 &= \sum_{ijkl} (s_{\alpha jl})^2 (s^*_{\beta ik})^2 f^c_{ij} f^{c*}_{kl} .
\end{aligned}
\tag{3.31}
$$

It is easy to see that the CP violating terms contain products of the real and imaginary parts of the s-invariants, and therefore the possible effects of a particular element $s_{\alpha ij}$ vanish unless the real part of a corresponding element is non-zero.

3.4   Three flavors change

This case corresponds to an amplitude such that for three given flavors α, β, γ, we have

$$
\begin{aligned}
L_\alpha\{M\} = L_\beta\{M\} &= -1 \\
L_\gamma\{M\} &= 2 \\
L_\delta\{M\} &= 0 \qquad (\delta \neq \alpha, \beta, \gamma) .
\end{aligned}
\tag{3.32}
$$

Examples of such processes are $\mu + e \to \tau + \tau$ and $\mu + \tau \to e + e$. The products that are consistent with Eq. (3.32) are $A_{\gamma\alpha i} A_{\gamma\beta j}$ and $B^*_{\alpha\beta i} B_{\gamma\gamma j}$, and therefore the amplitude is of the form

$$
\mathcal{M} = \sum_{ij} \left(A_{\gamma\alpha i} A_{\gamma\beta j} f^a_{ij} + B^*_{\alpha\beta i} B_{\gamma\gamma j} f^b_{ij} \right) .
\tag{3.33}
$$

In this case also, the CP violating terms appear as products of real parts and imaginary parts of the s-invariants.

3.5   Higher order corrections

So far we have found that we can divide the processes into two classes. In the first class, the CP-violating terms in the squared amplitude consist of products of the real and the imaginary parts of the s-invariants. For processes in this class, the CP-violating effects of a given invariant element $s_{\alpha ij}$ vanishes unless the real of some corresponding element is non-zero. In the second class, which contains, for example, the process $\ell_\beta \ell_\alpha \to \ell_\alpha \ell_\alpha$ considered in Section 3.3.1, the vanishing of all the CP-violating terms requires that the imaginary parts of the invariants be zero. As already hinted at in the argument that lead to Eq. (3.28), the feature that distinguishes
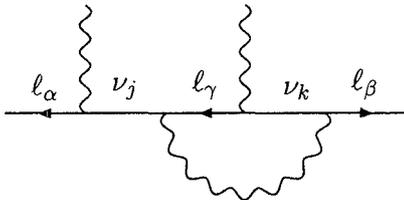
Figure 3: A higher order correction to diagram (b) of Fig. 1.

these two classes is the following. In the first class the amplitude is such that the interference terms consist of products of only an even number of s-invariants, while in the second class some interference terms contain an odd-number of them.

In fact, any process for which the corresponding amplitude is of such a form that, when it is squared, the interference terms can be expressed as products of an even number of s-invariants would fall in the first class. However, even for this class of process, in general, this property is lost when higher order diagrams are taken into account. We now show that the higher order terms in the amplitude lead to interference terms that contain an odd number s-invariants, and hence CP is strictly conserved in those processes only if the imaginary part of the relevant invariant elements vanish.

As an example, we reconsider the processes such as $WW \to e\mu$ which were discussed in Section 3.2.2. In addition to the term $B_{\alpha\beta i}$ considered in Eq. (3.16), the amplitude can contain the term

$$\sum_{\gamma jk} A^*_{\alpha\gamma j} B_{\gamma\beta k} f_{\gamma jk} \tag{3.34}$$

which is consistent with the condition given in Eq. (3.15). Such a term can arise, for example, from the diagram given in Fig. 3. When the total amplitude, containing the terms given in Eqs. (3.16) and (3.34), is squared, there will be an interference term

$$\sum_{\gamma ijk} \delta_{\alpha\beta\gamma ijk} f^*_i f_{\gamma jk} \tag{3.35}$$

where

$$\delta_{\alpha\beta\gamma ijk} = B^*_{\beta\alpha i} A^*_{\alpha\gamma j} B_{\gamma\beta k}$$
$$= s_{\beta ki} s_{\gamma kj} s_{\alpha ji} . \tag{3.36}$$

Therefore, the CP violating part of the amplitude involves products of the type $\mathrm{Im}(s_{\beta ki})\mathrm{Im}(s_{\gamma kj})\mathrm{Im}(s_{\alpha ji})$, which do not necessarily vanish even if some of the $\mathrm{Re}(s_{\alpha ik})$ are zero.

Similar examples can be constructed for other processes discussed here. For example, consider the process discussed in Sec. 3.4. The higher order corrections would include the following terms in the amplitude

$$\sum_{\delta ijk} A_{\alpha\delta k} B_{\beta\delta i} B_{\gamma\gamma j} f_{\delta ijk} . \tag{3.37}$$

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Here the interference terms will also contain products of the imaginary parts several s-invariants, similarly to the previous case.

4 Conclusions

We have presented a variety of examples to illustrate how the mixing and CP violating parameters of the lepton sector appear in rephasing invariant combinations in the squared amplitudes for various types of processes. As we have already discussed in Section 3.5, in general the CP-violating terms contain products of an odd number of rephasing invariants, and hence CP is strictly conserved only if the imaginary part of the relevant invariant elements vanish. However, there are processes in which CP-violating terms contain products of an even number of the invariants, and hence the CP-violating effects may vanish if the real part of the appropriate rephasing invariants are zero, neglecting higher order corrections.

It is well known that, in the presence of Majorana neutrinos, there are additional CP violating parameters as compared to, for example, the quark sector. While it is sometimes believed that these extra parameters appear only in lepton number violating processes [7], we have shown that they can, and in fact they do, appear in lepton number conserving processes as well. The true condition for the occurrence of these parameters in a given process seems to be the violation of lepton number on any fermion line in the corresponding diagrams, and not necessarily that total lepton number be violated by the process as a whole. In processes that conserve the total lepton number, there are in general diagrams in which the individual fermion lines change the lepton number, but do so in such a way that the changes between different lines cancel in the overall diagram. The interference terms produced by such diagrams would contain the extra CP-violating parameters that exist due to the Majorana nature of the neutrinos.

References

[1] J. F. Nieves and P. B. Pal, Phys. Rev. D **36**, 315 (1987).

[2] O. W. Greenberg, Phys. Rev. D **32**, 1841 (1985).

[3] I. Dunietz, O. W. Greenberg and D. Wu, Phys. Rev. Lett. **55**, 2935 (1985).

[4] S. M. Bilenky, J. Hosek and S. T. Petcov, Phys. Lett. B **94**, 495 (1980).

[5] J. Schechter and J. W. Valle, Phys. Rev. D **22**, 2227 (1980).

[6] M. Doi, T. Kotani, H. Nishiura, K. Okuda and E. Takasugi, Phys. Lett. B **102**, 323 (1981).

[7] J. Schechter and J. W. Valle, Phys. Rev. D **23**, 1666 (1981).

[8] P. J. O'Donnell and U. Sarkar, Phys. Rev. D **52**, 1720 (1995) [hep-ph/9305338].

[9] R. Rangarajan, U. Sarkar and R. Vaidya, Phys. Lett. B **442**, 243 (1998) [hep-ph/9809304].

[10] S. M. Bilenky, S. Pascoli and S. T. Petcov, hep-ph/0102265, hep-ph/0104218.