CONSTITUTIVE EQUATIONS USING THE BACK STRESS INTERNAL VARIABLE TO MODEL CWSR ZIRCALOY PLASTIC DEFORMATION

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Abstract

A variety of engineering correlations have been used to describe thermal creep deformation of Zr alloys. The primary creep, which occurs after the initial loading, is typically characterized by a decreasing creep strain rate and thus, formulations with time at a power close to 0.5 have been fitted to experimental data. By differentiating such an expression, a strain rate constitutive equation can be obtained for the plastic strain rate, \( \dot{\varepsilon} \), as dependent on \( \sigma \), the applied stress and on \( T \), the temperature. This equation is used in conjunction with a multi-axial anisotropy model whereby the equivalent plastic strain rate calculated by relation (1) is distributed along the three principal directions according to generalized Hill anisotropy model. This type of constitutive plastic strain rate equation was successfully applied in fuel rod behavior codes to model cladding plastic deformation by incorporating the effect of irradiation hardening. However, this type of relation is unable to model the steady-state creep regime and also it is difficult to account for the previous deformation history. A more mechanistically based model is needed to describe both the primary and the steady-state stages of the creep deformation and also to capture the effect of microstructural processes underlying the plastic deformation. A set of two coupled equations is proposed to describe the plastic deformation of Zircaloy tubes including both the tensile and creep regimes. It is the simplest form that uses the concept of internal variables to represent the microstructural processes occurring during plastic deformation. For the creep regime it provides a natural transition from primary to secondary creep and also allows for simulation of inverted primary creep. The “back stress,” \( \sigma_b \), is included in the model as a macroscopic variable representing the dislocation sub-structure resistance to deformation. The evolution of this back stress is the result of the competition between the opposing hardening and recovery processes. Hardening is created during deformation by the emerging dislocation sub-structure and by internal defects and impurities which act as obstacles to deformation. Recovery is due to thermal processes that enable dislocations to be released from the walls of the dislocation sub-structure by climbing and other mechanisms. The pool of data used to build the model consisted of both unirradiated and irradiated CWSR Zry4 cladding. Results from tensile and creep tests of both uniaxial or biaxial type have been used to obtain the fitting constants of the model and the effect of fast fluence on some of them. The paper presents the comparison of calculated and measured deformation to support the favorable conclusion of applicability of the back-stress constitutive equations to modeling Zry plastic deformation.

1. INTRODUCTION

The engineering thermal creep model, implemented in numerous fuel codes, models the plastic deformation through a constitutive plastic strain rate equation of the form [1]:

\[
\frac{d\varepsilon}{dt} = \frac{A \exp\left(-\frac{Q}{kT}\right) \sinh(a\sigma)}{2\varepsilon} \tag{1}
\]

which is used in conjunction with a multi-axial anisotropy model (the equivalent plastic strain rate calculated by relation (1) is distributed along the three principal directions according to generalized Hill anisotropy model).
This expression is a reformulation of the original parabolic time relation, which was developed by fitting the measured yield and creep data. The sinh law for the stress dependence follows the model proposed by Garofalo [2] which unifies the low stress domain, traditionally present in creep tests, and the high stress region, typically involved in the tensile tests. Thus, relation (1) is a unified inelastic deformation type of constitutive equation and it is easy to use in fuel codes.

However, Equation (1) models only the primary creep behavior. Also, it cannot reproduce inverted primary creep, which is sometimes observed at high stresses for a class of particle-strengthened materials, Zr alloys included [3]. A more sophisticated model is needed to capture the effect of microstructural processes underlying the plastic deformation. A model with one internal parameter, as the next level up in order of complexity, is presented below.

2. PROPOSED BACK-STRESS PLASTIC DEFORMATION MODEL

A set of two coupled equations is proposed to describe the plastic deformation of Zircaloy tubes including both the tensile and creep regimes. It is the simplest form that uses the concept of internal variables to represent the microstructural processes occurring during plastic deformation. This feature permits a more realistic modeling of complex loading histories which accounts for previous history. For the creep regime it provides a natural transition from primary to secondary creep and also allows for simulation of inverted primary creep.

The "back stress", \( \sigma_b \), is included in the model [4] as a macroscopic variable representing the dislocation sub-structure resistance to deformation. Also, particle-strengthening processes [5] that oppose to the plastic deformation, are equally represented by a back stress. The evolution of this back stress is the result of the competition between the opposing strain hardening and recovery processes. Hardening is manifested by increased strength during deformation due to the emerging dislocation sub-structure and to internal defects and impurities, all of which act as obstacles to deformation. Recovery, on the other hand, softens the material during deformation and is due, among other things, to thermal processes that enable dislocations to be released from the walls of the dislocation sub-structure by climb and other mechanisms.

The net stress that dictates the plastic deformation rate is the difference between the applied stress, \( \sigma_a \), and the back stress introduced before. The constitutive equation for the strain rate is based on the phenomenological plastic strain rate equation, as described in [6] and [7], and is formulated as follows:

\[
\frac{d\varepsilon}{dt} = C \exp\left(-\frac{Q}{kT}\right)\{\sinh\left[\frac{v_0}{s\kappa T}(\sigma_a - \sigma_b)\right]\}^\gamma
\]

where \( T \) is the temperature in K, \( Q \) is the activation energy in cal, and \( k \) is the gas constant in cal/K (equal to 1.98). The sinh function unifies the low-stress with the high-stress domains of the plastic deformation which according to all previous experience are characterized by a power law and by an exponential law, respectively.

The constitutive equation for the back stress describes the competition between hardening and recovery. The following relation was used to describe the kinetics of these two competing processes:
\[
\frac{d\sigma_b}{dt} = \frac{A\frac{d\varepsilon}{dt}}{\sigma_b^p} - \frac{B \exp\left(-\frac{Q}{kT}\right)}{T} \sigma_b^m
\]  

(3)

A static recovery term can be added to Equation (2) to represent the static recovery during annealing. Equation (3) is similar to the equation proposed for particle-strengthened materials in [5], where the equation is written in terms of the dislocation density, as:

\[
\frac{d\rho}{d\varepsilon} = k - k_2 \rho
\]

(4)

However, the relation between the back stress and the dislocation density is not the traditional square root dependence as assumed in [5], but rather a power law with an exponent of 1/3 (the exponent is \(\frac{1}{2}\) in the square root expression). In this case, Equation (4) can be re-written as Equation (3). It is interesting to note that Equation (3) can be solved analytically for constant temperature and constant average strain rate. The result is a exponentially decreasing primary creep strain, similar to the Dorn proposed model [8], and which was used by Murty [9], to formulate a primary creep relation for Zr alloys.

3. MODEL PARAMETERS DERIVATION

Two coefficients can be determined from experimental data at steady-state deformation by regression analysis: \(Q\) by linear regression of the logarithm of the strain rate versus inverse temperature for the same stress, and \(v_0\) by linear regression of the logarithm of the strain rate versus stress. The value of the \(s\) parameter was chosen to be consistent with the low-stress data which indicate a power law with exponent around 5 for dislocation creep. The remaining constants, \(C, A\) and \(B\), as well as the initial back stress remain to be determined as fitting parameters of the model.

A pool of data available at SPC, characteristic of stress-relieved cladding, was used to build the model. Creep and tensile tests of either longitudinal or biaxial type were available for unirradiated material, and irradiated material at two fluences. The data was enough to determine the \(s\) parameter (see Fig. 1) but not \(Q\). The value for \(Q\) was taken approximately equal to the self-diffusion activation energy, as indicated by the vast majority of experimental studies. The other model parameters were fit in order to obtain the best agreement possible with both the creep and tensile data. The following values were obtained for the model parameters and given in Table 1.

![Figure 1](image.png)

**FIG. 1. Linear fit to derive \(v\) from the slope at 623 K (left) and 653 K (right).**
TABLE 1. MODEL PARAMETERS FOR UNIRRADIATED AND IRRADIATED MATERIAL

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unirradiated Zry</th>
<th>Irradiated Zry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>1.E14</td>
</tr>
<tr>
<td></td>
<td>Q</td>
<td>62000</td>
</tr>
<tr>
<td></td>
<td>v₀</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>s</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>3.5E7</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>3.E21</td>
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<td></td>
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<td>p</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>1</td>
</tr>
</tbody>
</table>

The bold face values are for the parameters that change with irradiation exposure, and reflect the effect of irradiation hardening on the creep strain rate through v₀ and on the internal dislocation dynamics through the coefficients A and B of the back stress evolution equation.

4. RESULTS

Examples of the good agreement of the calculated and measured creep strain time evolution for unirradiated material are presented in Fig. 2 for uniaxial creep conditions and in Fig. 3 for bi-axial creep conditions. The agreement for the two test conditions, characterized by the hoop stress to axial stress ratio of 0 and 1, respectively, shows that the anisotropy is well represented. The values of R = 2 and P = 1 have been used for the anisotropy coefficients, according to the information on similar Zircaloy-4 material presented in References [9] and [10]. Texture measurements on the irradiated material showed no discernable change from the unirradiated condition. The good agreement for the bi-axial creep tests on Zry-4 cladding irradiated for three cycles are presented in Fig. 4.

FIG. 2. Unirradiated CWSR Zry, axial creep at 350 °C & 353 MPa (left) and 380 °C & 364 Mpa (right).
The simulation of the tensile test and its comparison with experimental data is presented in Fig. 5 for unirradiated material and in Figs 6 and 7 for irradiated material. The overall agreement obtained for both the creep and the tensile test conditions shows that the proposed constitutive equation can model the whole range of inelastic deformation.

Figure 8 shows the calculation-measurement comparison for a bi-axial creep test at 382°C and 220 MPa. The exposure of the fuel rods from which the sample was taken is lower than in the previous case. Accordingly, the model parameters affected by irradiation are different, as presented in Table 1. Also, it is remarked that the upward trend of the strain curve at large strains is reproduced by calculation. This is due to the inclusion of the geometric feedback on the stress calculation. This implies that larger stresses are generated with increasing strains even if the internal pressure remains constant.
FIG. 5. Bi-axial tensile tests on unirradiated CWSR Zry at 350 °C, and 350 °C and 5 %/h.

FIG. 6. Bi-axial tensile tests on irradiated CWSR Zry at 350°C.
FIG. 7. Bi-axial tensile tests on irradiated CWSR Zry at 380 °C.

FIG. 8. Bi-axial creep tests on irradiated CWSR Zry at 382 °C and 220 MPa.

REFERENCES


