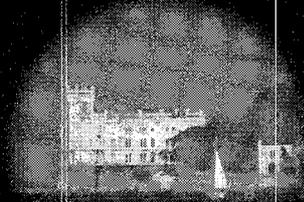


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IN FERROMAGNETIC FILM
THROUGH SOLITONS BY
ELECTROMAGNETIC FIELD**

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United Nations Educational Scientific and Cultural Organization
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**MAGNETIZATION REVERSAL IN FERROMAGNETIC FILM
THROUGH SOLITONS BY ELECTROMAGNETIC FIELD**

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Abstract

We study the reversal of magnetization in an isotropic ferromagnetic film free from charges by exposing it to a circularly polarized electromagnetic (EM) field. The magnetization excitations are obtained in the form of line and lump solitons of the completely integrable modified KP-II equation which is derived using a reductive perturbation method from the set of coupled Landau-Lifshitz and Maxwell equations. It is observed that when the polarization of the EM-field is reversed followed by a rotation, for every $\frac{\pi}{2}$ -degrees, the magnetization is reversed.

MIRAMARE - TRIESTE

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I. INTRODUCTION

Studies on developing high density magneto-optic data storage and development of ultrafast magnetic recording have attained momentum due to their high technological implications in the very recent times. It has been identified both experimentally and theoretically that magnetization reversal in magnetic films is one of the very fundamental issues in magnetic data storage [1–4]. In conventional magnetic recording the reversing field is applied anti-parallel to the direction of magnetization of the medium in which case the reversal takes place at the nano-second level. However, much shorter reversal times can be achieved through precessional reversal of the magnetization [1]. The torque developed between the magnetic moment of the medium and the external magnetic field will make the magnetic moment precess at the pico-second scale. The precessional motion is governed by the Landau-Lifshitz (LL) equation and recent experiments have shown the validity of the equation at the pico-second level [1]. Thus the switching process or magnetization reversal can be understood by solving the Landau-Lifshitz equation of motion for the applied magnetic field. In this context static and time dependent (pulse) applied field switching phenomenon have been studied in the past [3–5]. Magnetization reversal studies in the case of a more general applied magnetic field that varies both spatially and temporally is also equally interesting and needs attention. In the context of magneto-optics, it is very relevant and important to consider the applied field as the magnetic field component of the EM-field and thus the problem can be formulated in terms of the LL equation coupled with Maxwell equations. The one-dimensional version of this kind of problem has been studied recently in the case of isotropic and anisotropic ferromagnets and soliton modes were found to represent the magnetization excitations - the EM-field has also been modulated in the form of solitons [6–9]. The purpose of the present paper is to examine the reversal process of magnetization in a ferromagnetic film by generating soliton modes during precession of magnetization in the presence of an EM-field. By assuming that there are no free electric charges in the medium, we solve analytically the LL equation coupled with Maxwell equations in two spatial dimensions using a reductive perturbation method.

The paper is organized as follows. In section II, we formulate the model and the dynamical equations governing the magnetization excitations in the ferromagnetic film in the presence of an external EM-field. Using a reductive perturbation method in section III, we obtain soliton modes for the magnetization of the ferromagnetic film and the EM-field. The process of magnetization reversal is explained in section IV and the results are concluded in section V.

II. MODEL AND DYNAMICAL EQUATIONS

The dynamics of magnetization density in an isotropic charge-free ferromagnetic film under the influence of an external EM-field can be expressed in terms of the LL equation [10] $\partial_t \mathbf{M} = \mathbf{M} \wedge [\nabla^2 \mathbf{M} + A \mathbf{H}]$, $\mathbf{M}^2 = 1$, where $\mathbf{M}(x, y, t) = (M^x, M^y, M^z)$ is the magnetization density and $\mathbf{H}(x, y, t) = (H^x, H^y, H^z)$ is the magnetic field component of the electromagnetic field. The first term on the right-hand side of the LL equation represents the contribution due to spin-spin exchange interaction between the nearest neighbours and the term proportional to A ($= g\mu_B$; g = gyromagnetic ratio, μ_B = Bohr magneton) corresponds to the interaction between the magnetization of the medium and the magnetic field component of the EM-field (Zeeman term). When \mathbf{H} is a constant or a time dependent field set along a specific direction, it can be transformed away using the transformation $\mathbf{M}^\pm \equiv (M^x \pm iM^y) = \tilde{\mathbf{M}}^\pm e^{\mp i \int_{-\infty}^t \mathbf{H}(t') dt'}$ and the dynamics remains the same as in the case without any external field. However, when the field \mathbf{H} varies spatially, the effect due to the field cannot be transformed away in this fashion. It may be noted that in the LL equation we have not included the phenomenological Gilbert damping term because we have assumed that the medium does not contain any free charges which on interaction with magnetic electrons will introduce damping.

The interaction between the magnetic field component of the EM-field and matter or material medium can be expressed in terms of Maxwell equations [11]. In the absence of static and moving charges, Maxwell equations can be written as $\nabla^2 \mathbf{H} - \nabla(\nabla \cdot \mathbf{H}) = \frac{1}{c^2} \partial_t^2 [\mathbf{H} + \mathbf{M}]$, where $\nabla (= \hat{x} \partial_x + \hat{y} \partial_y)$ and $\nabla^2 (= \partial_x^2 + \partial_y^2)$ are the two-dimensional gradient and Laplacian operators respectively, and $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ is the velocity of propagation of the EMW and ϵ_0 and μ_0 are the dielectric constant and permeability of the medium, respectively. An untreated ferromagnetic material has the constitutive relation $\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$ where $\mathbf{B}(x, y, t) = (B^x, B^y, B^z)$ is the magnetic induction. Now using this constitutive relation, the LL and Maxwell equations can be rewritten as

$$\partial_t \mathbf{M} = \mathbf{M} \wedge \left[\nabla^2 \mathbf{M} + \frac{A}{\mu_0} \mathbf{B} \right], \quad \mathbf{M}^2 = 1, \quad (1a)$$

$$\tilde{\Delta} \mathbf{B} = \frac{1}{\epsilon_0} \left[\nabla(\nabla \cdot \mathbf{M}) - \nabla^2 \mathbf{M} \right]. \quad (1b)$$

where the operator $\tilde{\Delta} = (\partial_t^2 - c^2 \nabla^2)$. Thus, the set of coupled equations (1a) and (1b) completely describe the interaction of the EM-field with the ferromagnetic film and the dynamics of the associated fields namely magnetization, magnetic induction and magnetic field from the EM-field.

III. SOLITON MODES IN FERROMAGNETIC FILM

The nonlinear character of the LL equation (1a) makes the analysis difficult in the present form and hence we try to solve Eqs.(1a) and (1b) using a reductive perturbation method [12]. Assuming

that the plane wave of the EM-field travels along x -direction in the ferromagnetic film (xy -plane) we introduce the wave variable $\hat{\xi} = (x - vt)$ where v is the group velocity. We also introduce slow space and time variables through the stretching $\xi = \varepsilon\hat{\xi}$, $\zeta = \varepsilon^2 y$ and $\tau = \varepsilon^3 t$, where ε is the small perturbation parameter. The solutions of Eqs.(1) are then expanded asymptotically about uniform values

$$\mathbf{F}(\xi, \zeta, \tau) = \mathbf{F}_0(\xi, \zeta, \tau) + \varepsilon\mathbf{F}_1(\xi, \zeta, \tau) + \varepsilon^2\mathbf{F}_2(\xi, \zeta, \tau) + \dots, \quad (2)$$

where \mathbf{F} stands for the magnetic induction \mathbf{B} and the magnetization \mathbf{M} . We substitute the slow variables and the expansions in the component forms of Eqs.(1a) and (1b), collect terms corresponding to similar powers of ε and solve the equations at different orders. At $O(\varepsilon^0)$ from the component forms of Maxwell equations (Eq.(1b)), we obtain the results $B_0^x = 0$, $B_0^\alpha = kM_0^\alpha$, where $\alpha = y, z$ and $k = [\varepsilon_0(c^2 - v^2)]^{-1}$ and from the LL equation (Eq.(1a)) after using the above results, we get $M_0^x = 0$. The results at $O(\varepsilon^0)$ thus show that a planar anisotropy is developed in the magnetization ($M_0^y - M_0^z$), the magnetic induction ($B_0^y - B_0^z$) and in the magnetic field ($H_0^y - H_0^z$) space at the lowest order of expansion. At $O(\varepsilon^1)$, from Eq.(1b) we obtain $B_1^x = 0$, $B_1^\alpha = kM_1^\alpha$ and from Eq.(1a) we find that $\partial_\xi M_0^y = \frac{kA}{v} M_0^z M_1^x$ and $\partial_\xi M_0^z = -\frac{kA}{v} M_0^y M_1^x$. At $O(\varepsilon^2)$, after using the results from the lower orders, we find from Eq.(1b) that $B_2^x = 0$ and

$$[B_2^\alpha - kM_2^\alpha] = -\frac{1}{\varepsilon_0 k} \left[2v \int_{-\infty}^{\xi} d\xi' \partial_\tau B_0^\alpha + c^2 \int_{-\infty}^{\xi} d\xi' \int_{-\infty}^{\xi} d\xi'' \partial_{\zeta'}^2 B_0^\alpha \right], \quad (3)$$

and from Eq.(1a), we get

$$\partial_\xi M_1^x = \frac{1}{v} \left\{ M_0^z \partial_\xi^2 M_0^y - M_0^y \partial_\xi^2 M_0^z - A [M_0^y B_2^z + M_2^y B_0^z - M_2^z B_0^y - M_0^z B_2^y] \right\}. \quad (4)$$

We now try to solve the set of equations (3) and (4) to evaluate the complete set of solutions at the lowest nonvanishing order. For this it is advantageous to switch to the polar coordinate representation. As the results at $O(\varepsilon^0)$ show that the magnetic field is restricted to the ($H_0^y - H_0^z$) plane, we consider a circularly polarized or rotating magnetic field component of the EM-field in this plane by considering $\mathbf{H}_0 = (0, \sin\theta, \cos\theta)$. This makes us also choose $\mathbf{M}_0 = (0, \sin\theta, \cos\theta)$, where $\theta = \theta(\xi, \zeta, \tau)$ is the angle made between the direction of the applied EM-field and the uniform magnetization of the film. Thus Eq.(4) in the above polar co-ordinate system after using Eq.(3) becomes

$$\begin{aligned} \mu \partial_\xi^2 \theta &= 3\gamma \left[\sin\theta \int_{-\infty}^{\xi} d\xi' \int_{-\infty}^{\xi} d\xi'' \partial_{\zeta'}^2 \cos\theta - \cos\theta \int_{-\infty}^{\xi} d\xi' \int_{-\infty}^{\xi} d\xi'' \partial_{\zeta'}^2 \sin\theta \right] \\ &+ \frac{2vA}{\varepsilon_0} \left[\sin\theta \int_{-\infty}^{\xi} d\xi' \partial_\tau \cos\theta - \cos\theta \int_{-\infty}^{\xi} d\xi' \partial_\tau \sin\theta \right], \end{aligned} \quad (5)$$

where $\mu = (\frac{v}{kA} - 1)$ and $\gamma = \frac{c^2 A}{3\varepsilon_0}$. Differentiating Eq.(5) twice with respect to ξ , we get

$$\begin{aligned} \partial_\xi \left[\frac{F(\theta)}{\partial_\xi \theta} \right] \partial_\xi \theta = & -\mu(\partial_\xi^2 \theta) \partial_\xi \theta + 3\gamma \left[\cos\theta \int_{-\infty}^\xi d\xi' \partial_\zeta^2 \cos\theta + \sin\theta \int_{-\infty}^\xi d\xi' \partial_\zeta^2 \sin\theta \right] \\ & + 3\gamma \partial_\xi \left[\sin\theta \int_{-\infty}^\xi d\xi' \partial_\zeta^2 \cos\theta - \cos\theta \int_{-\infty}^\xi d\xi' \partial_\zeta^2 \sin\theta \right], \end{aligned} \quad (6)$$

where $F(\theta) = \partial_\tau \theta + \mu \partial_\xi^3 \theta$. While writing the above equation, we have rescaled $\tau \rightarrow \frac{\epsilon_0}{2vA} \tau$. We assume that the fields vary very slowly along y -direction when compared to x -direction (i.e.), $(\partial_\zeta^2 \theta \ll \partial_\xi \theta)$. Then we substitute Eq.(5) in the resultant equation obtained after differentiating Eq.(6) once with respect to ξ and again, after successive integration and differentiation, we finally obtain $\partial_\xi [\partial_\tau f - \frac{3}{2} \mu f^2 \partial_\xi f + \mu \partial_\xi^3 f] = -3\gamma \partial_\zeta^2 f + 3\gamma [\partial_\xi^2 f + \partial_\xi f \partial_\zeta f + \partial_\xi^2 f \int_{-\infty}^\xi d\xi' \partial_\zeta f]$, where $f = \partial_\xi \theta$. When $\gamma = \mu$, this is equivalent to the completely integrable modified Kadomtsev-Petviashvili (MKP) equation

$$\partial_\tau f + \mu \partial_\xi^3 f - 3\mu \left[\frac{1}{2} f^2 \partial_\xi f - \partial_\zeta w + w \partial_\xi f \right] = 0, \quad \partial_\xi w = \partial_\zeta f. \quad (7)$$

Eq.(7) when $\mu = i$ and $\mu = 1$ are respectively known as the MKP-I and MKP-II equations. However, in our problem μ cannot be imaginary and hence we have only the MKP-II equation for further analysis. Different types of soliton solutions to the MKP-II equation such as line soliton, lump soliton and breather have been found using the $\bar{\partial}$ -dressing and inverse scattering transform (IST) methods [13]. However, as the structure of breather solution is very rich we are unable to present them here. The general N -line soliton solution is obtained using $\bar{\partial}$ -dressing method [13] and the simplest line soliton of the MKP-II equation corresponding to $N = 1$ can be explicitly written in the form $f(\xi, \zeta, \tau) = -2(\alpha_1 - \beta_1)^2 \gamma_1 / [\alpha_1 \beta_1^2 (e^{-G} - \frac{\alpha_1 \gamma_1}{\beta_1} e^G) (e^{-G} - \gamma_1 e^G)]$, where $2G = (\frac{1}{\alpha_1} - \frac{1}{\beta_1}) \xi - (\frac{1}{\alpha_1^2} - \frac{1}{\beta_1^2}) \zeta - 4(\frac{1}{\alpha_1^3} - \frac{1}{\beta_1^3}) \tau + \ln 2 |\frac{R}{\beta_1 - \alpha_1}|$ and $\gamma_1 = \text{sgn}(\frac{R}{\beta_1 - \alpha_1})$. Here R is the Kernel and α_1 and β_1 are arbitrary real constants used in the IST analysis. This solution is nonsingular only if $\gamma_1 < 0$ and $\alpha_1, \frac{1}{\beta_1} > 0$. The line is regular in ξ and ζ and is a constant along a particular direction.

Lumps are rational solutions which normally decay in all directions in the plane. Unlike the MKP-I case, in the case of MKP-II, the rational solutions of the equation are singular. For instance, the simplest ($N = 1$) lump soliton of MKP-II equation corresponding to $N=1$ can be written as $f(\xi, \zeta, \tau) = 2\alpha_1 / [\frac{\alpha_1^2}{4} - (\xi + \frac{2\zeta}{\alpha_1} - \frac{12\tau}{\alpha_1^2})^2]$, where α_1 is an arbitrary real constant. This solution describes the uniform motion of two simple poles of opposite signs along a line which are parallel to each other with a distance α_1 and move with equal velocity $12\alpha_1^{-2}$.

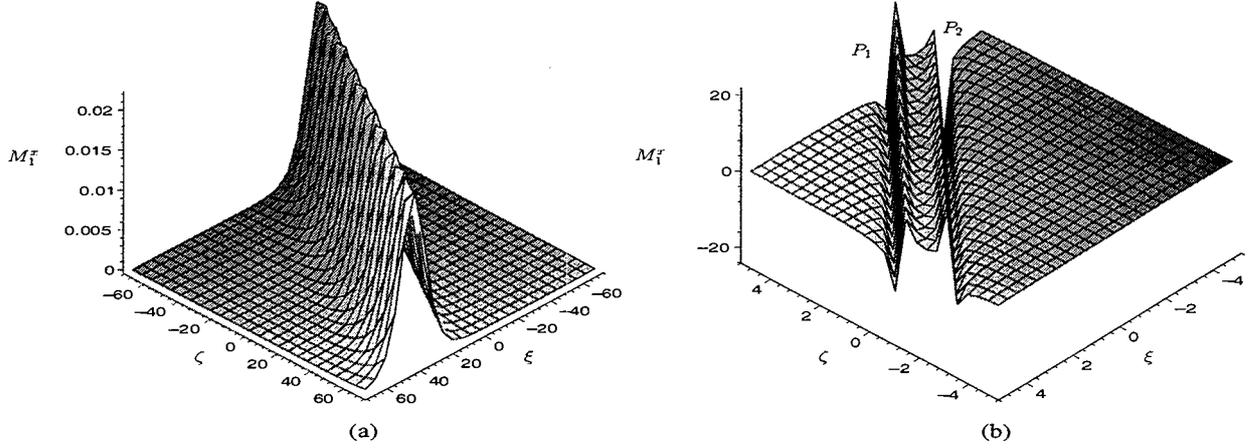


FIG. 1. (a) Line soliton of Magnetization (M_1^x) at $\tau = 1$ unit for $v = 0.5$, $k = 0.25$, $A = 1.0$, $\alpha_1 = 5.0$, $\beta_1 = 2.5$ and $\gamma_1 = -0.5$, (b) Lump soliton of magnetization (M_1^x) at $\tau = 1$ unit for $v = 0.5$, $k = 0.25$, $A = 1.0$ and $\alpha_1 = 2.05$.

Using the above line and lump solitons in the relation $f = \frac{\partial \theta}{\partial \xi}$, θ and finally after using the relation connecting M_1^x , M_0^y and M_0^z we can calculate the components of magnetization at the lowest existing order. For example, the line soliton of the x-component of magnetization at the lowest existing order (M_1^x) is found to be [13]

$$M_1^x = \frac{-2v(\alpha_1 - \beta_1)^2 \gamma_1}{kA \left\{ \alpha_1 \beta_1^2 e^{-G} - \frac{\alpha_1 \gamma_1}{\beta_1} e^G e^{-G} - \gamma_1 e^G \right\}}, \quad (8)$$

Fig.(1a) shows a snapshot of M_1^x -line soliton at $\tau = 1$ unit for $v = 0.5$, $k = 0.25$, $A = 1.0$, $\alpha_1 = 5.0$, $\beta_1 = 2.5$ and $\gamma_1 = -0.5$.

The lump 1-soliton solution of the x-component of magnetization (M_1^x) is obtained as

$$M_1^x = \frac{2v\alpha_1}{kA \left\{ \frac{\alpha_1^2}{4} - \left(\xi + \frac{2\zeta}{\alpha_1} - \frac{12\tau}{\alpha_1^2} \right)^2 \right\}}. \quad (9)$$

In Fig.(1b), a snap shot of the lump soliton of the x-component of the magnetization for $v = 0.5$, $k = 0.25$, $A = 1.0$ and $\alpha_1 = 2.05$ at $\tau = 1$ is given. In Fig.(1b) the uniform motion of the two simple poles, P_1 and P_2 separated by a distance of 2.05 units and travelling with a velocity of 2.855 units can be observed. Similarly, line and lump soliton solitons for the y and z components of magnetization can also be constructed.

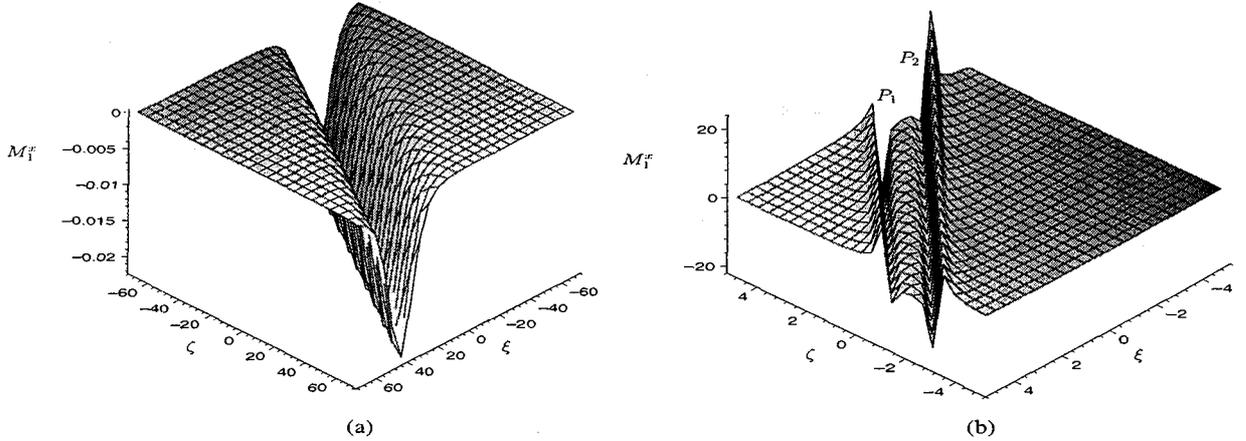


FIG. 2. (a) Reversal of line soliton of the magnetization (M_1^x) given in Fig.(1a) for the same values of the parameters, (b) Reversal of lump soliton of the magnetization (M_1^x) given in Fig.(1b) for the same values of the parameters. Both are due to change in direction of polarization (left \leftrightarrow right) followed by the rotation of the magnetic field by $\frac{\pi}{2}$ degrees.

IV. MAGNETIZATION REVERSAL IN FERROMAGNETIC FILM

The magnetization states observed in the form of line and lump solitons in the previous section correspond to the states when the rotating circularly polarized magnetic field component of the EM-field is acting on the ferromagnetic film which is magnetized perpendicular to the film at the lowest order. It is interesting to find that when the direction of polarization of the magnetic field component of the external EM-field is reversed (i.e. if the initial field is left circularly polarized, now in the present case it should be right circularly polarized and vice versa) and then rotated continuously, for every $\frac{n\pi}{2}$, $n = 0, 1, 2, \dots$ degrees of rotation, the direction of magnetization in the film is reversed. In other words, the above can be achieved by transforming $v \rightarrow -v$ and $\theta \rightarrow \theta + n\frac{\pi}{2}$. In view of these transformations, we now have $\mathbf{M}_0 = (0, \cos\theta, \sin\theta)$. We use this and the above transformations and also the results from Eq.(3) in Eq.(4) and repeating the same calculations of the previous section as found after Eq.(4), we once again obtain the MKP-II equation (7). Then we calculate the line and lump solitons of the x -component of the magnetization M_1^x using the relation connecting M_0^y , M_0^z as done before and the results are found to be $M_1^x = 2v(\alpha_1 - \beta_1)^2\gamma_1 / [kA\{\alpha_1\beta_1^2(e^{-G} - \frac{\alpha_1\gamma_1}{\beta_1}e^G)(e^{-G} - \gamma_1e^G)\}]$ for the line soliton and the lump soliton in the form $M_1^x = -2v\alpha_1 / kA\{\frac{\alpha_1^2}{4} - (\xi + \frac{2\zeta}{\alpha_1} - \frac{12\tau}{\alpha_1^2})^2\}$. On comparing the line and lump solitons for M_1^x in both cases, it can be observed that there is a sign change in the expression for solitons and hence magnetization in the present case is reversed. The new configurations of magnetization due to reversal in the case of M_1^x corresponding to the line soliton and also the lump soliton with

reference to the two poles P_1 and P_2 have been demonstrated in Figs.(2a) and (2b) respectively. Thus the reversing of the direction of polarization of the magnetic field and a rotation by $\frac{\pi}{2}$ -degrees reverses the original magnetization states given in Figs.(1a,b). As the precessional frequency of the magnetic dipole moment in ferromagnets is in the pico-seconds scale, the magnetization reversal can take place at a faster rate compared to the conventional reversal process which takes place in the nano-seconds scale by the reversal of magnetization of the domains. This has also close correspondence with the recent experimental results found in [1] on in-plane magnetized cobalt films in which a pulse-like time dependent magnetic field in the plane of the film as short as two pico-seconds is able to reverse the magnetization. It was found that when the field encompasses a right angle to the magnetization of the film, the reversal can be triggered by even very small fields. Also, our results have very close correspondence with the numerical analysis of the magnetization reversal found in refs. [3,4] using four pico-seconds magnetic field pulse which has been explained based on the LL equation with Gilbert damping in order to take into account the relaxation..

V. CONCLUSIONS

In this paper we have studied the reversal of magnetization in an isotropic charge-free ferromagnetic film when an EM-field is applied on it, by solving the coupled Maxwell equations and LL equation in two-spatial dimensions using a reductive perturbation method. The results show that at the lowest order of perturbation the ferromagnetic film is magnetized normal to the plane of the film. In the next order of perturbation it is found that the magnetization is excited and a coherent magnetization structure in the form of line and lump solitons of the MKP-II equation is obtained. Interestingly we found that, when the direction of polarization of the applied EM-field is reversed and the field is rotated continuously, the magnetization gets reversed for every addition of $\frac{\pi}{2}$ -degrees. As this effects the magnetization reversal via the precessional motion, it will decrease the reversal time to the order of pico-seconds rather than nano-seconds in conventional magnetization reversal processes. This phenomenon has very close correspondence with the recent experimental and numerical observations of magnetization reversal in cobalt film when ultrashort magnetic pulses are applied to it. This interesting phenomenon foresees applications in ultrafast magnetic recording in the future.

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