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EXOTIC MUON-TO-POSITRON CONVERSION IN NUCLEI: PARTIAL TRANSITION SUM EVALUATION BY USING SHELL MODEL

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A comprehensive study of the exotic (μ^-, e^+) conversion in ^{27}Al, $^{27}\mathrm{Al}(\mu^-, e^+)^{27}\mathrm{Na}$ is presented. The relevant operators are deduced assuming one-pion and two-pion modes in the framework of intermediate neutrino mixing models, paying special attention to the light neutrino case. The total rate is calculated by summing over partial transition strengths for all kinematically accessible final states derived with s–d shell model calculations employing the well-known Wildenthal realistic interaction.

Излагаются результаты всестороннего анализа экзотического процесса (μ^-, e^+)-конверсии на ядрах ^{27}Al, $^{27}\mathrm{Al}(\mu^-, e^+)^{27}\mathrm{Na}$. Необходимые операторы перехода вычислены в предположении однопионного и двухпионного обменов в моделях промежуточного смешивания нейтрино, при этом специально рассмотрен случай легких нейтрино. Полная вероятность вычислена путем суммирования парциальных сил переходов для всех кинематически допустимых конечных состояний на базе s–d-оболочечной модели с учетом хорошо известного реалистического взаимодействия Вилденталя.

INTRODUCTION

It is well known that many extensions of the standard model (SM), i.e., gauge models, grand unified theories, supersymmetric models etc., predict a plethora of processes which violate the lepton and/or lepton-family (flavor) quantum numbers [1–6]. Among the most interesting examples are the semileptonic processes which take place in a muonic atom [7–15]. One exotic possibility is the muon-to-positron conversion,

$$\mu^- + (A,Z) \to e^+ + (A, Z-2), \tag{1}$$

which violates the muonic (L_μ), electronic (L_e) and total lepton (L) quantum numbers [16–26]. The other anomalous process is the muon-to-electron conversion,

$$\mu^- + (A,Z) \to e^- + (A,Z), \tag{2}$$

which violates only the lepton-family (here L_μ and L_e) quantum numbers [27,28]. In the present work we will focus our attention on reaction (1). This process can be experimentally studied with the more familiar reaction (2) simultaneously, since both processes have the same intrinsic background and the same initial state (a muon at rest in the innermost 1S orbit of a muonic atom).

In recent years, continuous experimental efforts have been devoted to the measurement of the branching ratio $R_{\mu e^+}$ defined as the ratio of the (μ^-, e^+) conversion rate divided by the total rate of the ordinary muon capture reaction [29]:

$$R_{\mu e^+} = \Gamma(\mu^- \to e^+)/\Gamma(\mu^- \to \nu_\mu). \tag{3}$$

Up to now only upper limits have been set and the best limit is found for the ^{48}Ti nucleus at TRIUMF and PSI [10–12, 15] yielding the values

$$R_{\mu e^+} \leqslant 4.6 \cdot 10^{-12} \text{ [15]} \quad \text{and} \quad R_{\mu e^+} \leqslant 4.4 \cdot 10^{-12} \text{ [13]}.$$

This limit is expected to be further improved by future experiments, at PSI (SINDRUM II experiment), which aims to push the sensitivity of the branching ratio $R_{\mu e^+}$ to 10^{-14}, and at Brookhaven (MECO experiment) with expected sensitivity about four orders of magnitude below the existing experimental limits [27, 28].

Traditionally μ^-–$e^\pm$ exotic processes were searched by employing medium heavy (like ^{48}Ti and ^{63}Cu) [12, 13] or very heavy (like ^{208}Pb and ^{197}Au) [12, 14, 15] targets. For technical reasons the MECO target has been chosen to be the light nucleus ^{27}Al. The MECO experiment, which is planned to start soon at the Alternating Gradient Synchrotron (AGS), is going to use a new very intense μ^- beam and a new detector [27]. The basic feature of this experiment is the use of a pulsed μ^- beam to significantly reduce the prompt background from π^- and e^- contaminations.

The best upper limit for the $\mu^- \to e^-$ conversion branching ratio $R_{\mu e^-}$ has been extracted at PSI (SINDRUM II experiment) for ^{48}Ti target [12]:

$$R_{\mu e^-} \leqslant 6.1 \cdot 10^{-13}. \tag{4}$$

For the ^{208}Pb target the determined best limit is [13]

$$R_{\mu e^-} \leqslant 4.6 \cdot 10^{-11}. \tag{5}$$

Processes (1) and (2) are very good examples of the interplay between particle and nuclear physics in the area of physics beyond the standard model. Moreover, the (μ^-, e^+) conversion has many similarities with the neutrinoless double β decay ($0\nu\beta\beta$) represented by

$$(A, Z) \to e^- + e^- + (A, Z + 2), \tag{6}$$

which violates the lepton-flavor (L_e) and total lepton (L) quantum numbers. Both reactions (1) and (6) involve a change of charge by two units and thus they cannot occur in the same nucleon. Both of them are forbidden, if lepton number is absolutely conserved. One can show that, if either of these processes is observed, the neutrinos must be massive Majorana particles. In spite of the many similarities, however, these double charge exchange processes do have some significant differences, which can be briefly summarized as follows:

(i) Due to the nuclear masses involved, neutrinoless double beta decay can occur only in specific nuclear systems, for which single beta decay is absolutely forbidden due to energy conservation or greatly hindered due to angular momentum mismatch. These systems, with the possible exception of ^{48}Ca, have complicated nuclear structure. Neutrinoless double beta decay can lead only to the ground state and, only in exceptional cases, to few low-lying excited states of the final nucleus. Such constraints are not imposed on process (1), due to the rest energy of the disappearing muon.

(ii) From experiments, in conjunction with appropriate nuclear matrix elements as input, one may extract lepton-violating parameters, which depend on flavor. Thus in the framework of the neutrino mixing models the amplitudes for neutrinoless $\beta\beta$ decay and (μ^-, e^+) conversion, if the leptonic currents are of the same chirality, are proportional to different combination of neutrino masses. The same is true in the case of the mass-independent lepton-violating parameters η entering if the leptonic currents are of opposite chirality. One does not know *a priori* which flavor combination is favored.

(iii) The long wavelength approximation does not hold in the case of (μ^-, e^+) conversion, since the momentum of the outgoing e^+ is high. Thus, the effective two-body operator responsible for the (μ^-, e^+) conversion is strongly energy-dependent and more complicated than the corresponding one for the $0\nu\beta\beta$ decay. On the other hand, in this case one can choose a target, consistent with the standard experimental requirements, so that the nuclear structure required is the simplest possible one.

(iv) Neutrinoless double beta decay has the experimental advantage that there exists no other competing channel for the decay of the initial nucleus.

Thus, we view the two processes as providing useful complementary information and both, if possible, should be pursued.

Strictly speaking, (μ, e^+) conversion and neutrinoless double beta decay should be treated as two-step processes by explicitly constructing the intermediate states of the $(A, Z \pm 1)$ system. It has been found [30], however, that, for neutrinoless double beta decay, since the energy denominators are dominated by the momentum of the virtual neutrino, closure approximation with some average energy denominator works very well. We expect this approximation to describe the (μ^-, e^+) conversion to sufficient accuracy. We will, therefore, replace the intermediate nuclear energies by some suitable average one. By summing over all allowed final states of the nucleus $(A, Z-2)$ we obtain the total rate. This will then be compared to that obtained by invoking closure [18] with some appropriate mean energy $\langle E_f \rangle$ of the final states.

So far, theoretically the (μ^-, e^+) process has been investigated [18, 23] on the exclusive reactions

$$^{40}\text{Ca} + \mu^- \rightarrow e^+ + \ ^{40}\text{Ar (g.s.)}, \tag{7}$$

$$^{58}\text{Ni} + \mu^- \rightarrow e^+ + \ ^{58}\text{Fe (g.s.)}. \tag{8}$$

In these studies the partial g.s. $\rightarrow$ g.s. transition rate was calculated by performing microscopic calculations of these nuclear matrix elements. On the other hand, the total transition strength to all final states (inclusive process) was estimated along the lines of closure approximation and ignoring 4-body terms [18].

In the present article we apply the shell-model approach to investigate the (μ^-, e^+) conversion on the reaction

$$^{27}\text{Al} + \mu^- \rightarrow e^+ + \ ^{27}\text{Na}. \tag{9}$$

It is our purpose to calculate not only the rate to the ground state transition but the decay rates to all final nuclear states lying below some excitation energy (≈ 25 MeV) of the final nucleus ^{27}Na as well. In this work we will report only our results for the partial transition rates to the $5/2^+$ states.

The paper is organized as follows. In section 1, an extensive presentation of the relevant expressions occurring in the formal description of the $\mu^- \to e^+$ transition operators is given. In section 2, we deal with the expressions of the branching ratios. In section 3, we discuss the evaluation of the inclusive $\mu^- \to e^+$ matrix elements by means of explicit construction of the needed nuclear wave functions in the framework of the s–d shell model. In section 4, the results obtained for the Fermi and Gamow–Teller operators in the case of $^{27}\mathrm{Al}(\mu^-, e^+)^{27}\mathrm{Na}$ are presented and discussed. Also the spreading of the contributions due to the occurrence of various multipoles is described. Our conclusions are summarized in the final section of the paper.

1. BRIEF THEORETICAL FORMULATION OF THE $\mu^- \to e^+$ CONVERSION OPERATORS

1.1. Effective $\mu^- \to e^+$ Conversion Langrangian in Gauge Models. From the particle physics point of view, processes like $\mu^- \to e^\pm$ conversions, are forbidden in the SM by total-lepton and/or lepton-flavor (muonic and electronic) quantum number conservation and they have long been recognized as important probes of the lepton- and flavor-changing charged-current interactions [3–6].

There are several possible elementary particle mechanisms which can mediate the lepton-violating process (1). The mechanisms which have been studied theoretically are (i) those mediated by a massive Majorana neutrino; (ii) those accompanied by massless or light physical Higgs particles (majorons); (iii) those involving more exotic intermediate Higgs particles; (iv) those mediated by intermediate supersymmetric (SUSY) particles. In case (i) we have two possibilities. 1) The chiralities of the two leptonic currents are the same. Then the amplitude in the case of light neutrinos is proportional to some average neutrino mass or to some average of the inverse of the neutrino mass, if the neutrino is heavy. 2) The chiralities of the leptonic currents are opposite. Then the amplitude is not explicitly dependent on the neutrino mass, but it vanishes, if the neutrinos are not Majorana particles. This mechanism is significant only in the case of light neutrinos.

From a nuclear physics point of view one has to be a bit more careful when the intermediate particles are very heavy. If, in going from the quark to the nucleon level, the nucleons are treated as point-like particles, the nuclear matrix elements are suppressed due to the presence of short-range correlations. To avoid this suppression a cure has been proposed [30] which treats the nucleons as composite particles described by a suitable form factor. A different approach is to consider mechanisms which involve particles other than nucleons in the nuclear soup. Such are, e.g., mechanisms whereby the processes (1) and (6) are mediated by the decay of the doubly charged virtual Δ^{++} particle or induced by pions in flight between the two nucleons [18,22,26] according to the elementary reactions

$$\mu^- + \Delta^{++} \to n + e^+, \qquad (10)$$

$$\mu^- + \pi^+ \to \pi^- + e^+. \qquad (11)$$

The first of these to leading order does not contribute to $0^+ \to 0^+$ transitions, like in neutrinoless double beta decay, but it may contribute to (μ^-, e^+). The second may be an important mechanism for both reactions.

1.2. The Transition Operators at Nuclear Level. The current fashionable gauge models mentioned in the previous subsection give rise to a plethora of effective transition operators Ω. Their essential isospin, spin and radial structure is given as follows. The isospin structure is quite simple, i.e., of the form $\tau_-(i)\tau_-(j)$ where i and j are the participating in the process nucleons. The spin structure is given in terms of the operators:

$$W_{\mathrm{S1}}(ij) = 1 \quad \text{(Fermi)}, \tag{12}$$

$$W_{\mathrm{S2}}(ij) = \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \quad \text{(Gamow–Teller)}, \tag{13}$$

$$W_{\mathrm{S3}}(ij) = 3\,(\boldsymbol{\sigma}_i \cdot \hat{r})(\boldsymbol{\sigma}_j \cdot \hat{r}) - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \quad \text{(Tensor)}, \tag{14}$$

$$W_{\mathrm{A1}}(ij) = \imath\,\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j, \qquad W_{\mathrm{A2}}(ij) = \boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j. \tag{15}$$

The orbital part can be expressed in terms of the quantities:

(a) The momentum (p_e) of the emitted positron obtained from the kinematics of reaction (1). One finds that

$$p_e \equiv |\mathbf{p}_e| = m_\mu - \epsilon_{\mathrm{b}} + Q - E_{\mathrm{x}}, \tag{16}$$

where $Q = M(Z) - M(Z-2)$ is the atomic mass difference between the initial, (A, Z), and final, $(A, Z-2)$, nucleus, ϵ_{b} is the binding energy of the muon at the muonic atom ($\epsilon_{\mathrm{b}} \approx 0.5$ MeV), E_{x} is the excitation energy ($E_{\mathrm{x}} = E_{\mathrm{f}} - E_{\mathrm{g.s.}}$) of the final nucleus and m_μ is the muon mass ($m_\mu = 105.6$ MeV).

(b) The relative ($\mathbf{r}_{ij}$) and center-of-mass ($\mathbf{R}_{ij}$) coordinates, which are written as

$$\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j\,, \qquad \hat{r} = \frac{\mathbf{r}_{ij}}{|\mathbf{r}_{ij}|}\,, \qquad r_{ij} = |\mathbf{r}_{ij}|\,,$$

$$\mathbf{R}_{ij} = \frac{1}{2}(\mathbf{r}_i + \mathbf{r}_j)\,, \qquad \hat{R} = \frac{\mathbf{R}_{ij}}{|\mathbf{R}_{ij}|}\,, \qquad R_{ij} = |\mathbf{R}_{ij}|\,.$$

The radial part of the operator contains the spherical Bessel functions $j_l(\frac{p_e r_{ij}}{2})$ and $j_{\mathcal{L}}(p_e R_{ij})$ resulting from the decomposition of the outgoing positron and a function $f(r)$ of the relative coordinate given by

$$f(r) = \frac{R_0}{r} F(r) \Psi_{\mathrm{cor}}(r), \tag{17}$$

where the constant R_0 represents the nuclear radius. The function $\Psi_{\mathrm{cor}}(r)$ is some reasonable two-nucleon correlation function [2] of the type

$$\Psi_{\mathrm{cor}}(r) = 1 - \mathrm{e}^{-ar^2}(1 - br^2) \tag{18}$$

with $a = 1.1\ \mathrm{fm}^{-2}$ and $b = 0.68\ \mathrm{fm}^{-2}$.

As we have already indicated, the radial function $F(r)$ depends on the specific mechanism assumed for the $\mu^- \to e^+$ conversion process to occur. The following cases are of interest.

(i) In the case of light Majorana neutrinos, when the leptonic currents are left-handed, $F(r)$ takes the form [23]

$$F(r) = \frac{2}{\pi} \int_0^\infty \frac{\sin x}{x - \alpha + \imath\epsilon} dx + \frac{2}{\pi} \int_0^\infty \frac{\sin x}{x + \delta_e} dx. \tag{19}$$

The quantities δ_e and α are given in terms of the nuclear masses and the average excitation energy of the intermediate states:

$$\delta_e = [\langle E_{\rm xn} \rangle + M(Z-1) - M(Z) + p_e] r,$$

$$\alpha = [m_\mu + M(Z) - M(Z-1) - \langle E_{\rm xn} \rangle] r.$$

Note that δ_e depends on the positron momentum. The first term of $F(r)$ in (19) can be written as

$$\frac{2}{\pi} \int_0^\infty \frac{\sin x}{x - \alpha + \imath\epsilon} dx = \frac{2}{\pi} P \int_0^\infty \frac{\sin x}{x - \alpha} dx - \imath 2 \sin\alpha. \tag{20}$$

The principal value integral can be written in an equivalent form:

$$\frac{2}{\pi} P \int_0^\infty \frac{\sin x}{x - \alpha} dx = 2\cos\alpha - 1 + \frac{2}{\pi} \alpha \int_0^\infty \frac{\sin x}{x(x+\alpha)} dx. \tag{21}$$

The latter expression is more convenient for numerical integration techniques. It is worth remarking that in the case of $0\nu\beta\beta$ decay $\alpha \sim 0$, therefore $F(r) = 1$. This simplifies quite well the calculations in the $0\nu\beta\beta$ decay process.

(ii) In the case of light Majorana neutrinos, when the leptonic currents are of opposite chirality, we have $F(r) \to F'(r) = ir(d/dr)F(r)$. The same situation occurs in the context of R-parity-violating supersymmetric interactions mediated by light Majorana neutrinos in addition to other SUSY particles.

(iii) For heavy intermediate particles, e.g., heavy Majorana neutrinos, we will examine two modes:

1) Only nucleons are present in the nucleus. Then the function $F(r)$ reads

$$F(r) = \frac{1}{48} \frac{m_{\rm A}^2}{m_e m_p} x_{\rm A} (x_{\rm A}^2 + 3x_{\rm A} + 3)\, {\rm e}^{-x_{\rm A}}, \qquad x_{\rm A} = m_{\rm A} r \tag{22}$$

with m_e, m_p the masses of electron and proton respectively. It should be mentioned that in the above expression the nucleon is assumed to have a finite size adequately described [25] by a dipole shape form factor with characteristic mass $m_{\rm A}$ taking the value $m_{\rm A} = 0.85$ GeV/c^2.

2) The process is mediated by pions in flight between the two interacting nucleons. Then one distinguishes two possibilities [2,31]:

(a) The 1-pion mode represented by the reactions

$$\mu^- + p \to n + \pi^- + e^+, \qquad \pi^- + p \to n. \tag{23}$$

In this mode $F(r)$ is replaced by $F^i_{1\pi}$, $i = \mathrm{GT}$, T where

$$F^{\mathrm{GT}}_{1\pi}(x) = \alpha_{1\pi}\, \mathrm{e}^{-x}, \quad F^{\mathrm{T}}_{1\pi}(x) = \alpha_{1\pi}(x^2 + 3x + 3)\, \mathrm{e}^{-x}/x^2 \tag{24}$$

with $x = m_\pi r$ and $\alpha_{1\pi} = 1.4 \cdot 10^{-2}$. In this case the radial functions are the same as those entering the neutrinoless double beta decay.

(b) The 2-pion mode represented by the reactions

$$p \rightarrow n + \pi^+, \qquad \pi^+ + \mu^- \rightarrow \pi^- + e^+; \qquad \pi^- + p \rightarrow n. \tag{25}$$

Now the radial functions are obtained from those entering the neutrinoless double beta decay, via the substitution:

$$F(r) j_l(x_e/2) \rightarrow \int_0^1 j_l((\xi - 1/2)x_e) F^i_{2\pi}([\xi(1-\xi)x_e^2 + x_\pi^2]^{1/2}) d\xi, \tag{26}$$

where $F^i_{2\pi}$, $i = \mathrm{GT}, \mathrm{T}$ are given by [2]

$$F^{\mathrm{GT}}_{2\pi}(x) = \alpha_{2\pi}(x - 2)\, \mathrm{e}^{-x}, \quad F^{\mathrm{T}}_{2\pi}(x) = \alpha_{2\pi}(x + 1)\, \mathrm{e}^{-x} \tag{27}$$

with $\alpha_{2\pi} = 2.0 \cdot 10^{-2}$.

1.3. Irreducible Tensor Operators. In this section we are going to exhibit the structure of the various irreducible tensor operators relevant to our calculation characterized by the set of quantum numbers $l, \mathcal{L}, \lambda, \Lambda, L, S, J$, some of which may be redundant in some special cases. Some details on how these operators are combined to give the nuclear matrix elements will be discussed in the Appendix.

We will begin with operators appearing when the chiralities of the two leptonic currents involved are the same. This covers the case of minimal left-handed extensions of the SM.

One encounters Fermi-type operators of the form ($S = 0, J = L$)

$$\Omega_{\mathrm{F}} = \sum_{i<j} \tau_-(i)\tau_-(j) f(r_{ij}) j_l\left(\frac{p_e r_{ij}}{2}\right) j_{\mathcal{L}}(p_e R_{ij}) \left[\sqrt{4\pi} Y^l(\hat{r}_{ij}) \otimes \sqrt{4\pi} Y^{\mathcal{L}}(\hat{R}_{ij})\right]^J. \tag{28}$$

The Gamow–Teller operators are similarly written as ($S = 0, J = L$)

$$\begin{aligned} \Omega_{\mathrm{GT}} &= \sum_{i<j} \tau_-(i)\tau_-(j) f(r_{ij}) j_l\left(\frac{p_e r_{ij}}{2}\right) j_{\mathcal{L}}(p_e R_{ij}) \times \\ &\times \left[\left[\sqrt{4\pi} Y^l(\hat{r}_{ij}) \otimes \sqrt{4\pi} Y^{\mathcal{L}}(\hat{R}_{ij})\right]^L \otimes (-\sqrt{3}) \left[\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j\right]^0\right]^J. \end{aligned} \tag{29}$$

The first spin antisymmetric operator is ($S = 1, J = L, |L \pm 1|$)

$$\begin{aligned} \Omega_{\mathrm{A1}} &= \sum_{i<j} \tau_-(i)\tau_-(j) f(r_{ij}) j_l\left(\frac{p_e r_{ij}}{2}\right) j_{\mathcal{L}}(p_e R_{ij}) \times \\ &\times \left[\left[\sqrt{4\pi} Y^l(\hat{r}_{ij}) \otimes \sqrt{4\pi} Y^{\mathcal{L}}(\hat{R}_{ij})\right]^L \otimes (-\sqrt{2}) \left[\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j\right]^1\right]^J. \end{aligned} \tag{30}$$

Note that $\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j = -\sqrt{3}\,[\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j]^0_0$ and $i\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j = (-\sqrt{2})\,[\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j]^1$. The second spin antisymmetric operator is $(S = 1, J = L, |L \pm 1|)$

$$\begin{aligned} \Omega_{\mathrm{A2}} &= \sum_{i<j} \tau_-(i)\tau_-(j) f(r_{ij}) j_l\left(\frac{p_e r_{ij}}{2}\right) j_{\mathcal{L}}(p_e R_{ij}) \times \\ &\times \left[\left[\sqrt{4\pi} Y^l(\hat{r}_{ij}) \otimes \sqrt{4\pi} Y^{\mathcal{L}}(\hat{R}_{ij})\right]^L \otimes (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\right]^J . \end{aligned} \tag{31}$$

Note that each operator must be overall symmetric with respect to interchange of the particle indices. So, in those cases in which the spin operator is of rank unity, l must be odd. In the special case of $0^+ \to 0^+$ neutrinoless double beta decay, only the Fermi and Gamow–Teller operators occur.

We are now going to consider the case in which the theory contains both R (Right) and L (Left) currents and in particular the L–R interference in the leptonic sector. This may be important in the case of light neutrinos. As we have already mentioned, this also occurs in the context of R-parity violating supersymmetric interactions, which, in addition to other SUSY particles, involve intermediate light Majorana neutrinos. The amplitude now is proportional to the 4-momentum of the intermediate neutrino. The time component has a structure similar to that presented above, but it will not be further discussed, since it is suppressed. Its space component, after the Fourier transform, gives an amplitude proportional to the gradient of the Fourier transform of the previous case. We thus get the above operators, to be denoted by Ω'_{F} Ω'_{GT}, and Ω'_{A2} (associated with the term linear in the spin), with $f(r)$ replaced by $f'(r)$. In this case, in addition to operators of the above form, we encounter an operator of spin rank two, which is of the form $(\lambda = |l \pm 1|,\ S = 2,\ J = L, |L \pm 1|)$

$$\begin{aligned} \Omega'_{\mathrm{T}} &= \sum_{i<j} \tau_-(i)\tau_-(j) f'(r_{ij}) j_l\left(\frac{p_e r_{ij}}{2}\right) j_{\mathcal{L}}(p_e R_{ij}) \times \\ &\times \left[\left[\sqrt{4\pi} Y^\lambda(\hat{r}_{ij}) \otimes \sqrt{4\pi} Y^{\mathcal{L}}(\hat{R}_{ij})\right]^L \otimes [\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j]^2\right]^J . \end{aligned} \tag{32}$$

As has already been mentioned, in the case of heavy intermediate particles one may have to consider pions in flight between nucleons. Then one encounters only Gamow–Teller and tensor operators except that now the radial part is different (see (24)–(27)).

In the special case of $0^+ \to 0^+$ neutrinoless double beta decay mediated by light neutrinos one can invoke the long wavelength approximation. Thus to leading order one finds (up to normalization constants and possibly factors of p_e) the familiar operators:

$$\Omega_{\mathrm{F}} = \sum_{i \neq j} \tau_-(i)\tau_-(j) f(r_{ij}) \quad \text{(Fermi)}, \tag{33}$$

$$\Omega_{\mathrm{GT}} = \sum_{i \neq j} \tau_-(i)\tau_-(j) f(r_{ij}) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \quad \text{(Gamow–Teller)}, \tag{34}$$

$$\Omega'_{\mathrm{A2}} = \sum_{i \neq j} \tau_-(i)\tau_-(j) f'(r_{ij})\,(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j) \cdot (i\hat{r} \times \hat{R}), \tag{35}$$

$$\Omega'_{\mathrm{T}} = \sum_{i \neq j} \tau_-(i)\tau_-(j) f'(r_{ij})\,[3(\sigma_i \cdot \hat{r})(\boldsymbol{\sigma}_j \cdot \hat{r}) - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j] \quad \text{(Tensor)}. \tag{36}$$

2. BRANCHING RATIO

The branching ratio $R_{\mu e^+}$ of the (μ^-, e^+) reaction defined in (3) contains the LFV-parameters of the specific gauge model assumed. These parameters are entered in $R_{\mu e^+}$ via a single lepton-violating parameter n_{eff}. Under some reasonable assumptions these parameters can be separated from the nuclear physics aspects of the problem. As has been pointed out [23], the branching ratio $R_{\mu e^+}$ takes the form

$$R_{\mu e^+} = \rho |\eta_{\text{eff}}|^2 \frac{1}{A^{2/3} Z f_{\text{PR}}(A, Z)} \sum_{\text{f}} \left(\frac{p_e}{m_\mu} \right)^2 |\mathcal{M}_{\text{i}\to\text{f}}|^2. \tag{37}$$

The parameter ρ is tiny ($\rho = 1.5 \cdot 10^{-21}$) due to the fact that $\mu^- \to e^+$ conversion is a second-order weak process. In this definition, the total muon capture rate has been written in terms of the well-known Primakoff function $f_{\text{PR}}(A, Z)$ [32], which takes into account the effect of the nucleon-nucleon correlations on the total muon capture rate. $|\mathcal{M}_{\text{i}\to\text{f}}|^2$ denotes the square of the partial transition nuclear matrix element between an initial $|J_\text{i}\rangle$ and a final $|J_\text{f}\rangle$ state. This can be written as

$$|\mathcal{M}_{\text{i}\to\text{f}}|^2 = \frac{1}{2J_\text{i}+1} \sum_{M_\text{f} M_\text{i}} |\langle J_\text{f} M_\text{f} | \Omega | J_\text{i} M_\text{i} \rangle|^2. \tag{38}$$

In our case $|J_\text{i}\rangle = |\text{g.s.}\rangle$, i.e., the ground state of the initial nucleus. The summation in (37) runs over all states of the final nucleus lying up to ≈ 25 MeV. For the Fermi and Gamow–Teller contribution the square of the matrix element $|\mathcal{M}_{\text{i}\to\text{f}}|^2$ is written as

$$|\mathcal{M}_{\text{i}\to\text{f}}|^2 = \frac{1}{2J_\text{i}+1} \sum_{L} \left| \left(\frac{f_\text{V}}{f_\text{A}} \right)^2 \langle J_\text{f} || \Omega_\text{F} || J_\text{i}(\text{g.s.}) \rangle - \langle J_\text{f} || \Omega_{\text{GT}} || J_\text{i}(\text{g.s.}) \rangle \right|^2 \tag{39}$$

where f_V and f_A are the usual vector and axial vector coupling constants ($f_\text{A}/f_\text{V} = 1.25$). By combining (39) and (37) we see that, for the evaluation of the branching ratio $R_{\mu e^+}$, we have to calculate the reduced matrix elements $\langle J_\text{f} || \Omega_\text{F} || J_\text{i} \rangle$ and $\langle J_\text{f} || \Omega_{\text{GT}} || J_\text{i} \rangle$ for $|J_\text{i}\rangle = |\text{g.s.}\rangle$ and $|J_\text{f}\rangle$ of any accessible state of the final nucleus. In the present work these states have been constructed in the framework of the shell model as is described in the next section.

3. THE SHELL-MODEL NUCLEAR WAVE FUNCTIONS

The reduced matrix elements $\langle J_\text{f} || \Omega_\text{F} || J_\text{i} \rangle$ and $\langle J_\text{f} || \Omega_{\text{GT}} || J_\text{i} \rangle$ are very sensitive and their evaluation requires reliable nuclear wave functions, the derivation of which is here accomplished in the context of the shell model. Specifically, in the calculation of the $\mu^- \to e^+$ conversion matrix elements in ^{27}Al, which are required for the reaction (9) considered in this work, we choose as model space the *s–d* shell and use as effective interaction the universal *s–d* shell interaction of Wildenthal [33], which has been tested over many years. This interaction is known to accurately reproduce many nuclear observables for *s–d* shell nuclei. The Wildenthal two-body matrix elements as well as the single-particle energies are determined by least square fits to experimental data in the region of the periodic table with $A = 17{-}39$.

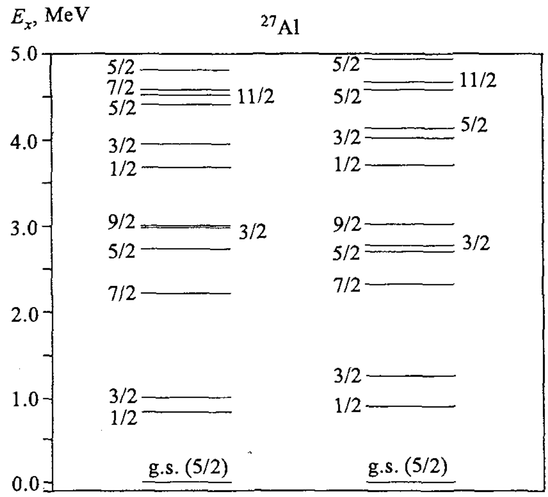

Fig. 1. The calculated (right) and measured (left) [34] energy spectrum for the lowest positive parity states of ^{27}Al

The eigenstates of the daughter nucleus ^{27}Na were evaluated in the isospin representation. The first 250 states for each spin J_f with $T = 5/2$ were calculated reaching up to $E_x = 25$ MeV, in excitation energy. On the other hand, for ^{27}Al we evaluated the ground state $(5/2)^+$ with $T = 1/2$, which plays the role of the initial state in the matrix elements of (38), as well as all the excited states up to 5 MeV. In Fig. 1 we present the calculated and measured [34] low-energy spectrum of ^{27}Al up to 5 MeV. As can be seen from this figure, the spectrum of ^{27}Al is well reproduced. In the case of the unstable ^{27}Na isotope the comparison between theory and experiment cannot be accomplished due to lack of experimental data. For the special case of the reaction (9) studied in the present work, since $M(\mathrm{Al}) - M(\mathrm{Na}) = -10.6$ MeV, the momentum transfer at which our matrix elements must be computed is given by

$$p_e = 94.5 - E_x \quad (\mathrm{MeV}). \tag{40}$$

4. RESULTS AND DISCUSSION

As we have already mentioned, the primary purpose of the present work is the calculation of the total $\mu^- \to e^+$ reaction rate by summing over partial transition strengths. We concern only with the evaluation of the Fermi-type and Gamow–Teller-type interaction discussed in section 1. To this aim, our main task is the computation of the reduced matrix elements

$$M_{\mathrm{F}} = \langle J_f || \Omega_{\mathrm{F}} || J_i(\mathrm{g.s.}) \rangle \tag{41}$$

and

$$M_{\mathrm{GT}} = \langle J_f || \Omega_{\mathrm{GT}} || J_i(\mathrm{g.s.}) \rangle \tag{42}$$

for the transitions between the initial $|J_i\rangle = (5/2)^+_{\mathrm{g.s.}}$ and all the final $|J_f\rangle = (5/2)^+$ states up to 25 MeV. We restrict ourselves to the case of light Majorana neutrinos evaluating the real part of the first term of (19).

In Fig. 2 we illustrate the distribution of the square of the reduced matrix elements of (41) and (42), i.e., the strengths $|M_F|^2$ and $|M_{GT}|^2$, for the multipolarity $L = 0$. As can be seen, the Gamow–Teller contribution (solid line) is more pronounced than the Fermi one (dotted line). The total Gamow–Teller contribution, represented by the area included between the energy axis and the histogram of Fig. 2, is almost 2.5 times greater than the Fermi one.

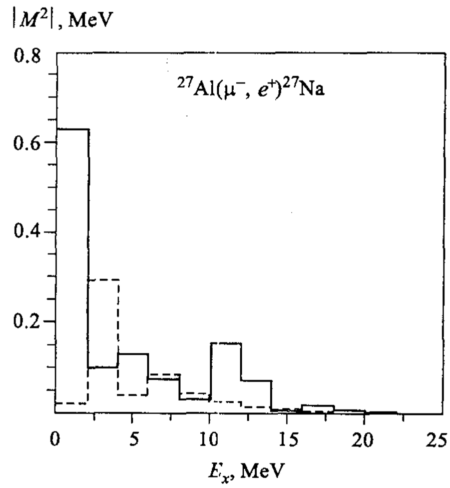

Fig. 2. Distribution of the transition strengths ($|M|^2$) for the Fermi (solid line), $|M|^2_F$, and Gamow–Teller (dotted line), $|M|^2_{GT}$, components up to 25 MeV

Specifically for the ground state transition the calculated reduced matrix elements for the Fermi and Gamow–Teller components are -0.14 and 0.79 respectively. As can be seen from Fig. 2, the main part of the Gamow–Teller contribution comes from the g.s. $\rightarrow$ g.s. transition. On the contrary, for the Fermi component the main contribution comes from the first excited $(5/2)^+$ state which appears at $E_x = 2.67$ MeV. Furthermore, 51 % of the total Gamow–Teller strength is distributed among all the excited states up to 25 MeV, while the other 49 % goes to the ground state. On the other hand, 46 % of the total Fermi strength is distributed to the first excited $(5/2)^+$ state and only 3.7 % of the total strength to the ground state.

The contribution of the remaining multipolarities $L = 2$ and $L = 4$ is, in general, quite small compared to that of $L = 0$. This becomes obvious by glancing at Table 1, where the total Fermi and Gamow–Teller strengths with respect to multipolarities L are listed.

Table 1. **Individual Fermi and Gamow–Teller transition strengths for terms of multipolarities contributing to the total rate of the process, ^{27}Al$(\mu^-, e^+)^{27}$Na, i.e., the $\mu^- \rightarrow e^+$ conversion in ^{27}Al**

	Fermi Contribution	Gamow–Teller Contribution
$L = 0$	0.528	1.283
$L = 2$	$1.615 \cdot 10^{-3}$	$6.099 \cdot 10^{-3}$
$L = 4$	$4.160 \cdot 10^{-5}$	$1.086 \cdot 10^{-4}$

In order to compare the branching ratio originating from the g.s. $\rightarrow$ g.s. transition with that associated with the transition to all final $(5/2)^+$ states, we define, for convenience, the ratio

$$\lambda \equiv \frac{R_{\text{g.s.}}}{R} = \frac{(94.5)^2 |\mathcal{M}_{(\text{g.s.}\rightarrow\text{g.s.})}|^2}{\sum_f (94.5 - E_x)^2 |\mathcal{M}_{(\text{g.s.}\rightarrow f)}|^2}. \tag{43}$$

This ratio gives the portion of the g.s. $\rightarrow$ g.s. contribution (which is proportional to the matrix element $\mathcal{M}^2_{\text{g.s.}\rightarrow\text{g.s.}}$) into the total rate here computed by the sum over the partial transitions included in our model space. For the g.s. $\to$ g.s. transition $p_e = 94.5$ MeV according to (40). Since $m_e c^2 \ll p_e$ we can consider the approximation $p_e \approx E_e$, which is equivalent to neglecting the electron mass (m_e) in the kinematics of the reaction (9).

According to our calculation the ratio λ takes the value 0.60. This means that the ground state transition exhausts a large portion (60 %) of the sum rule. This result seems to come into contradiction with the predictions found previously by employing closure approximation [18]. The latter tends to overestimate the contribution of the excited states to the total strength.

At this stage we remind that according to closure approximation the contribution of each individual state is effectively taken into account by assuming a mean excitation energy $\bar{E}_x = \langle E_f \rangle - E_{g.s.}$, and using the completeness relation $\sum_f |f\rangle\langle f| = 1$. Therefore

$$\sum_f |\langle f|\Omega|i\rangle|^2 = \langle i|\Omega^+\Omega|i\rangle.$$

The matrix element $\langle i|\Omega^+\Omega|i\rangle$ can be written as a sum of two pieces: a two-body term and a four-body one, that is

$$\langle i|\Omega^+\Omega|i\rangle = \langle i|(\Omega^+\Omega)_{2b}|i\rangle + \langle i|(\Omega^+\Omega)_{4b}|i\rangle. \tag{44}$$

The disagreement that appeared between closure approximation and the present state-by-state calculation can be attributed to the following reasons:

i) Closure approximation takes into account not only the contribution of $0\hbar\omega$ space but also excitations $E \geqslant 0\hbar\omega$, as well as the continuum spectrum. A possible extension of the *s–d* model space from a shell model point of view is quite difficult.

ii) In closure approximation the second term in (44), which includes the four-body forces and which is very complicated, was not taken into account in the previous calculations. Of course, the obvious question arises of how important the contribution of four-body forces are.

iii) On the other hand, the sensitivity of the simple closure approximation with respect to changing the mean excitation energy $\bar{E}_x$ is always problematic.

iv) As we have already mentioned, the present calculation concerns only the $J^\pi = (5/2)^+$ excited states and investigates the contribution of only the Fermi and Gamow–Teller components in the total strength. A more reliable comparison with closure approximation demands the inclusion of all the excited states produced by our *s–d* model space (see [35]).

SUMMARY AND CONCLUSIONS

In the present work we have investigated the exotic neutrinoless muon-to-positron conversion in the presence of nuclei. The appropriate operators have been constructed assuming mixing of massive (Majorana) neutrinos, and the one-pion and two-pion modes have been examined in detail. Since there are no restrictions imposed (as, e.g., in the $0\nu\beta\beta$ decay) for the nuclear target to be used, we have chosen the nucleus ^{27}Al, which is going to be used as a stopping target in the Brookhaven experiment. This nucleus is an *s–d* shell nucleus and for its study we can use the well-tested *s–d* interaction.

From our preliminary results on the reaction $^{27}\mathrm{Al}(\mu^-, e^+)^{27}\mathrm{Na}$, we can conclude the following:

(i) In the light-neutrino mass case, the contribution coming from the Gamow–Teller component of the $\mu^- \rightarrow e^+$ operator dominates the total rate matrix elements.

(ii) The ground-state to ground-state (g.s. $\rightarrow$ g.s.) contribution of the Gamow–Teller transition strength is 49 %.

(iii) In the Fermi case the transition to 2nd excitation state gives the most pronounced contribution (46 %).

(iv) The total strength, resulting from summing over partial transition matrix elements included in our model space, is much smaller than that found previously by using closure approximation.

APPENDIX

According to the gauge models mentioned in section 1, the transition operator Ω can be given in terms of the following components:

$$\Omega_{\mathrm{S}a} = \sum_{i \neq j} \tau_-(i)\tau_-(j)\, \mathrm{e}^{i\mathbf{p}_e\mathbf{r}_i}\, f(r_{ij}) W_{\mathrm{S}a}(ij), \quad a = 1,2,3,4,5, \tag{45}$$

$$\Omega_{\mathrm{A}a} = \sum_{i \neq j} \tau_-(i)\tau_-(j)\, \mathrm{e}^{i\mathbf{p}_e\mathbf{r}_i}\, f(r_{ij}) W_{\mathrm{A}a}(ij), \quad a = 1,2,3,4,5,6,7,8, \tag{46}$$

where

$$W_{\mathrm{S}1}(ij) = 1, \tag{47}$$

$$W_{\mathrm{S}2}(ij) = \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j = -\sqrt{3}\,[\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j]_0^0, \tag{48}$$

$$W_{\mathrm{S}3}(ij) = 3(\boldsymbol{\sigma}_i \cdot \hat{r})(\boldsymbol{\sigma}_j \cdot \hat{r}) - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j = \sqrt{6}\left[\sqrt{4\pi} Y^2(\hat{r}_{ij}) \otimes [\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j]^2\right]_0^0, \tag{49}$$

$$W_{\mathrm{S}4}(ij) = \frac{\hat{r}}{i} = \frac{1}{i\sqrt{3}} \sqrt{4\pi} Y^1(\hat{r}_{ij}), \tag{50}$$

$$W_{\mathrm{S}5}(ij) = \frac{\hat{r}}{i}(\boldsymbol{\sigma}_i \boldsymbol{\sigma}_j) = \frac{1}{i\sqrt{3}}\left[\sqrt{4\pi} Y^1(\hat{r}_{ij}) \otimes (-\sqrt{3})\,[\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j]^0\right]^1, \tag{51}$$

$$W_{\mathrm{A}1}(ij) = i\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j = (-\sqrt{2})\,[\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j]^1, \tag{52}$$

$$W_{\mathrm{A}2}(ij) = \boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j, \tag{53}$$

$$W_{\mathrm{A}3}(ij) = (\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j) \times \frac{\hat{r}}{i} = -\frac{1}{i}\sqrt{\frac{2}{3}}\left[\sqrt{4\pi} Y^1(\hat{r}_{ij}) \otimes (-\sqrt{2})\,[\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j]^1\right]^1, \tag{54}$$

$$W_{A4}(ij) = (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j) \times \hat{r} = \frac{1}{i}\sqrt{\frac{2}{3}} \left[\sqrt{4\pi}Y^1(\hat{r}_{ij}) \otimes (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\right]^1, \tag{55}$$

$$W_{A5}(ij) = (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\frac{\hat{r}}{i} = -\frac{1}{i} \left[\sqrt{4\pi}Y^1(\hat{r}_{ij}) \otimes (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\right]^0_0, \tag{56}$$

$$W_{A6}(ij) = (\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)\frac{\hat{r}}{i} = \frac{1}{i} \left[\sqrt{4\pi}Y^1(\hat{r}_{ij}) \otimes (-\sqrt{2})\left[\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_i\right]^1\right]^0_0, \tag{57}$$

$$W_{A7}(ij) = (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)(i\hat{r}_{ij} \times \hat{R}_{ij}) = \sqrt{\frac{2}{3}} \left[\left[\sqrt{4\pi}Y^1(\hat{r}_{ij}) \otimes \sqrt{4\pi}Y^1(\hat{R}_{ij})\right]^1 \otimes (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_i)\right]^0_0, \tag{58}$$

$$W_{A8}(ij) = \frac{1}{i}\left[(\boldsymbol{\sigma}_i\hat{r})\boldsymbol{\sigma}_j + (\boldsymbol{\sigma}_j\hat{r})\boldsymbol{\sigma}_i - \frac{2}{3}\hat{r}(\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j)\right] = -\frac{2\sqrt{5}}{3i}\left[\sqrt{4\pi}Y^1(\hat{r}_{ij}) \otimes (\sigma_i \otimes \boldsymbol{\sigma}_j)^2\right]^1. \tag{59}$$

Applying the usual multipole decomposition procedure, the operators $\Omega_{\mathrm{S}a}$ and $\Omega_{\mathrm{A}a}$ read

$$\Omega_{\mathrm{S}a} = \sum_L O^{(L,S)J}_{\mathrm{S}a} \cdot \sqrt{4\pi}Y^L(p_e)\delta_{LJ}, \tag{60}$$

$$\Omega_{\mathrm{A}a} = \sum_{LJ} \left[\sqrt{4\pi}Y^L(p_e) \otimes O^{(L,S)J}_{\mathrm{A}a}\right]^1. \tag{61}$$

The operators $O^{(L,S)J}_{\mathrm{S}a}$ and $O^{(L,S)J}_{\mathrm{A}a}$ are given by the following equations:

$$\begin{aligned} O^{(L,S)J}_{\mathrm{S1}} &\equiv \Omega_{\mathrm{F}} = \sum_{l\mathcal{L}} A_{l\mathcal{L}L} \sum_{i<j} \tau_-(i)\tau_-(j)f(r_{ij})j_l(\frac{p_e r_{ij}}{2})j_{\mathcal{L}}(p_e R_{ij}) \times \\ &\times \left[\sqrt{4\pi}Y^l(\hat{r}_{ij}) \otimes \sqrt{4\pi}Y^{\mathcal{L}}(\hat{R}_{ij})\right]^J, \quad S=0, J=L, \end{aligned} \tag{62}$$

where

$$A_{l\mathcal{L}L} = \sqrt{\frac{\hat{l}\hat{\mathcal{L}}}{\hat{L}}}\langle l0\mathcal{L}0|L0\rangle(1+(-1)^l)i^{l+\mathcal{L}}, \tag{63}$$

$$\begin{aligned} O^{(L,S)J}_{\mathrm{S2}} &\equiv \Omega_{\mathrm{GT}} = \sum_{l\mathcal{L}} A_{l\mathcal{L}L} \sum_{i<j} \tau_-(i)\tau_-(j)f(r_{ij})j_l\left(\frac{p_e r_{ij}}{2}\right)j_{\mathcal{L}}(p_e R_{ij}) \times \\ &\times \left[\left[\sqrt{4\pi}Y^l(\hat{r}_{ij}) \otimes \sqrt{4\pi}Y^{\mathcal{L}}(\hat{R}_{ij})\right]^L \otimes (-\sqrt{3})\left[\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j\right]^0\right]^J, \quad S=0, J=L. \end{aligned} \tag{64}$$

Also

$$O_{A1}^{(L,S)J} \equiv \Omega_{A1} = \sum_{l\mathcal{L}} B_{l\mathcal{L}L} \sum_{i<j} \tau_-(i)\tau_-(j) f(r_{ij}) j_l\left(\frac{p_e r_{ij}}{2}\right) j_{\mathcal{L}}(p_e R_{ij}) \times$$
$$\times \left[\left[\sqrt{4\pi}Y^l(\hat{r}_{ij}) \times \sqrt{4\pi}Y^{\mathcal{L}}(\hat{R}_{ij})\right]^L \otimes (-\sqrt{2})\left[\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j\right]^1\right]^J, \quad S=1, J=L, |L\pm 1|, \tag{65}$$

$$O_{A2}^{(L,S)J} \equiv \Omega_{A2} = \sum_{l\mathcal{L}} B_{l\mathcal{L}L} \sum_{i<j} \tau_-(i)\tau_-(j) f(r_{ij}) j_l\left(\frac{p_e r_{ij}}{2}\right) j_{\mathcal{L}}(p_e R_{ij}) \times$$
$$\times \left[\left[\sqrt{4\pi}Y^l(\hat{r}_{ij}) \otimes \sqrt{4\pi}Y^{\mathcal{L}}(\hat{R}_{ij})\right]^L \otimes (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\right]^J, \quad S=1, J=L, |L\pm 1|, \tag{66}$$

$$O_{A3}^{(L,S)J} \equiv \Omega'_{A1} = \sum_{l\mathcal{L}} \sum_{\lambda\Lambda} \Delta_{l\mathcal{L}L}^{\lambda\Lambda} \sum_{i<j} \tau_-(i)\tau_-(j) f'(r_{ij}) j_l\left(\frac{p_e r_{ij}}{2}\right) j_{\mathcal{L}}(p_e R_{ij}) \times$$
$$\times \left[\left[\sqrt{4\pi}Y^\lambda(\hat{r}_{ij}) \otimes \sqrt{4\pi}Y^{\mathcal{L}}(\hat{R}_{ij})\right]^\Lambda \otimes (-\sqrt{2})\left[\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j\right]^1\right]^J,$$
$$S=1, J=L, |L\pm 1|, \lambda=|l\pm 1|, \tag{67}$$

$$O_{A6}^{(L,S)J} = \sum_{l\mathcal{L}} \sum_{\lambda\Lambda} H_{l\mathcal{L}L}^{\lambda\Lambda} \sum_{i<j} \tau_-(i)\tau_-(j) f'(r_{ij}) j_l\left(\frac{p_e r_{ij}}{2}\right) j_{\mathcal{L}}(p_e R_{ij}) \times$$
$$\times \left[\left[\sqrt{4\pi}Y^\lambda(\hat{r}_{ij}) \otimes \sqrt{4\pi}Y^{\mathcal{L}}(\hat{R}_{ij})\right]^\Lambda \otimes (-\sqrt{2})\left[\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_i\right]^1\right]^J,$$
$$S=1, J=L, \lambda=|l\pm 1|, \tag{68}$$

where

$$B_{l\mathcal{L}L} = \sqrt{\frac{\hat{l}\hat{\mathcal{L}}\hat{J}}{3\hat{L}}}(-1)^{J+1}\langle l0\mathcal{L}0|L0\rangle(1-(-1)^l)i^{l+\mathcal{L}}, \tag{69}$$

$$\Delta_{l\mathcal{L}L}^{\lambda\Lambda} = -\overline{B}_{l\mathcal{L}L}(-1)^{l+J+\mathcal{L}}\sqrt{6\hat{l}\hat{L}\hat{\Lambda}}\frac{1}{i}\left\{\begin{array}{ccc} L & 1 & \Lambda \\ 1 & J & 1 \end{array}\right\}\left\{\begin{array}{ccc} 1 & l & \lambda \\ \mathcal{L} & \Lambda & L \end{array}\right\}\langle 10l0|\lambda 0\rangle, \tag{70}$$

$$E_{l\mathcal{L}L}^{\lambda\Lambda} = -\Delta_{l\mathcal{L}L}^{\lambda\Lambda}, \tag{71}$$

$$\overline{B}_{l\mathcal{L}L} = \sqrt{\frac{\hat{l}\hat{\mathcal{L}}\hat{J}}{3\hat{L}}}(-1)^{J+1}\langle l0\mathcal{L}0|L0\rangle(1+(-1)^l)i^{l+\mathcal{L}}, \tag{72}$$

$$Z_{l\mathcal{L}L}^{\lambda\Lambda} = -A_{l\mathcal{L}L}(-1)^{l+J+\mathcal{L}}\sqrt{3\hat{l}\hat{L}\hat{\Lambda}}\frac{1}{i}\left\{\begin{array}{ccc} L & 1 & \Lambda \\ 1 & J & 0 \end{array}\right\}\left\{\begin{array}{ccc} 1 & l & \lambda \\ \mathcal{L} & \Lambda & L \end{array}\right\}\langle 10l0|\lambda 0\rangle, \tag{73}$$

$$H^{\lambda\Lambda}_{l\mathcal{L}L} = -Z^{\lambda\Lambda}_{l\mathcal{L}L}, \tag{74}$$

$$O^{(L,S)J}_{\mathrm{A}7} \equiv \Omega_{\mathrm{A}} = \sum_{l\mathcal{L}}\sum_{l_1 l_2}\sum_{l_3 l_4} \Theta^{Ll\mathcal{L}}_{l_1 l_2 l_3 l_4} \sum_{i<j} \tau_-(i)\tau_-(j) f(r_{ij}) j_l\left(\frac{p_e r_{ij}}{2}\right) j_{\mathcal{L}}(p_e R_{ij}) \times$$
$$\times\sqrt{\frac{2}{3}}\left[\left[\sqrt{4\pi}Y^{l_2}(\hat{r}_{ij}) \otimes \sqrt{4\pi}Y^{l_3}(\hat{R}_{ij})\right]^{l_4} \otimes (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\right]^J, \quad S=1, J=L, \tag{75}$$

where

$$\Theta^{Ll\mathcal{L}}_{l_1 l_2 l_3 l_4} = A_{l\mathcal{L}L} I^{Ll\mathcal{L}}_{l_1 l_2 l_3 l_4}, \tag{76}$$

$$\begin{aligned} I^{Ll\mathcal{L}}_{l_1 l_2 l_3 l_4} &= 3\sqrt{3}(-1)^{l+l_2+1}\hat{l}_1\sqrt{\hat{l}\hat{\mathcal{L}}\hat{l}_4\hat{L}}\langle 10l0|l_2 0\rangle\langle \mathcal{L}010|l_3 0\rangle \left\{\begin{array}{ccc} L & 1 & l_1 \\ 1 & l_4 & 1 \end{array}\right\} \times \\ &\times \left\{\begin{array}{ccc} 1 & l & l_2 \\ \mathcal{L} & l_1 & L \end{array}\right\}\left\{\begin{array}{ccc} l_2 & \mathcal{L} & l_1 \\ 1 & l_4 & l_3 \end{array}\right\}\left\{\begin{array}{ccc} L & 1 & l_4 \\ 1 & L & 0 \end{array}\right\}. \end{aligned} \tag{77}$$

We are now going to discuss the operators appearing in the leptonic R–L interference and in some SUSY mechanisms. Now the relevant operators may have a time component, which is small and, in any case, except for their radial part, is the same with the F and GT discussed above. They also have a space component, which is proportional to $\hat{r}$. They are vectors, which yield a scalar, when combined with the leptonic current. They are of the form

$$\sigma_i(\sigma_j \cdot \hat{r}) + (\sigma_i \cdot \hat{r})\sigma_j - \sigma_i \cdot \sigma_j \quad \text{and} \quad i(\sigma_i - \sigma_j) \times \hat{r}.$$

Thus we get three oprerators $\omega'(k), k = 0, 1, 2$, which can be written as

$$\omega'(k) = \alpha(k)\ [T^k(\text{spin}) \otimes \sqrt{4\pi}Y^1(\hat{r})]^1, \tag{78}$$

$$\alpha(0) = -\frac{1}{3\sqrt{3}}, \quad T^0(\text{spin}) = \sigma_i \cdot \sigma_j, \tag{79}$$

$$\alpha(2) = -\frac{2\sqrt{5}}{3}, \quad T^2(\text{spin}) = [\sigma_i \otimes \sigma_j]^2, \tag{80}$$

$$\alpha(1) = \sqrt{\frac{2}{3}}, \quad T^1(\text{spin}) = \sigma_i - \sigma_j. \tag{81}$$

The above operators are accompanied by the lepton outgoing waves

$$O = (1/2)\exp(i\mathbf{p}_e \cdot \mathbf{R})[\exp(i\mathbf{p}_e \cdot \mathbf{r})/2 + (-1)^{k+1}\exp(-i\mathbf{p}_e \cdot \mathbf{r})]/2. \tag{82}$$

The phase of the second term guarantees that the combined operator is overall symmetric under the exchange of the particles i and j. The last operator can be brought into the form

$$O = \sum_{l\mathcal{L}k\Lambda} \beta(l\mathcal{L}k\Lambda) j_l(p_e r/2) j_{\mathcal{L}}(p_e R)[[\sqrt{4\pi}Y^l(\hat{r}) \otimes \sqrt{4\pi}Y^{\mathcal{L}}(\hat{R})]^{\Lambda} \otimes \sqrt{4\pi}Y^{\Lambda}(\hat{p}_e)]^0, \tag{83}$$

where

$$\beta(l\mathcal{L}k\Lambda) = \frac{1}{2}[1 + (-1)^{l+k+1}]\ i^{l+\mathcal{L}}(-1)^{l+\mathcal{L}}[(2l+1)(2\mathcal{L}+1]^{1/2}\langle l0\mathcal{L}0|\Lambda 0\rangle. \qquad (84)$$

Combining the above factors we obtain

$$\begin{aligned} \Omega'(k) &= \alpha(k) \sum_{l\mathcal{L}k\Lambda} \beta(l\mathcal{L}k\Lambda) \sum_{\lambda,L,J} \gamma(l,\lambda,\mathcal{L},L,k,J,\lambda) j_l(p_e r/2) j_{\mathcal{L}}(p_e R) \times \\ &\times [[\sqrt{4pi}Y^{\lambda}(\hat{r}) \otimes \sqrt{4pi}Y^{\mathcal{L}}(\hat{R})]^L \otimes T^k(\text{spin})]^J \otimes \sqrt{4\pi}Y^{\Lambda}(\hat{p}_e)]^1 \end{aligned} \qquad (85)$$

with

$$\begin{aligned} \gamma(l,\lambda,\mathcal{L},L,k,J,\lambda) &= (-1)^{1+l+\mathcal{L}+J}[(3(2L+1)(2l+1)(2J+1)]^{1/2} \times \\ &\times \langle 10l0|\lambda 0\rangle \begin{Bmatrix} 1 & k & 1 \\ J & \Lambda & L \end{Bmatrix} \begin{Bmatrix} 1 & l & \lambda \\ \mathcal{L} & L & \Lambda \end{Bmatrix}. \end{aligned} \qquad (86)$$

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