Heavy Meson Observables and Dyson-Schwinger Equations

M. A. Ivanov,[1] Yu. L. Kalinovsky,[2] P. Maris[3] and C. D. Roberts[4]

[1] *Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia*

[2] *Laboratory of Computing Techniques and Automation, Joint Institute for Nuclear Research, 141980 Dubna, Russia*

[3] *Center for Nuclear Research, Physics Department, Kent State University, Kent Ohio 44242, USA*

[4] *Physics Division, Bldg. 203, Argonne National Laboratory, Argonne Illinois 60439, USA*

Abstract

Dyson-Schwinger equation (DSE) studies show that the b-quark mass-function is approximately constant, and that this is true to a lesser extent for the c-quark. This observation provides the basis for a study of the leptonic and semileptonic decays of heavy pseudoscalar mesons using a "heavy-quark" limit of the DSEs, which, when exact, reduces the number of independent form factors. Semileptonic decays with light mesons in the final state are also accessible because the DSEs provide a description of light-quark propagation characteristics and light-meson structure. A description of B-meson decays is straightforward, however, the study of decays involving the D-meson indicates that c-quark mass-corrections are quantitatively important.

The Dyson-Schwinger equations provide a nonperturbative, Poincaré invariant, continuum approach to studying quantum field theories: two familiar examples are the gap equation in superconductivity and the Bethe-Salpeter equation describing relativistic 2-body bound states. As a system of coupled integral equations a truncation of the DSEs is necessary to obtain a tractable problem. The simplest truncation scheme is a weak-coupling expansion, which generates every diagram in perturbation theory. Hence, in the intelligent application of DSEs to QCD, there is always a tight constraint on the ultraviolet behaviour. That is crucial in extrapolating into the infrared, in constructing uniformly valid symmetry-preserving truncations, and in developing phenomenological models necessary for anticipating the results of the current generation of hadron physics facilities.

The development of efficacious truncations is not a purely algebraic task, and neither is it always obviously systematic. Nevertheless, it has become clear [1] that truncations which preserve the global symmetries of a theory; for example, chiral symmetry in QCD, are relatively easy to define and implement and, while it is more difficult to preserve local gauge symmetries, much progress has been made

DISCLAIMER

DISCLAIMER

with Abelian theories [2] and more is being learnt about non-Abelian ones. In addition, contemporary phenomenological applications now address a wide range of observables [3], yielding qualitatively robust results and a much-needed intuitive understanding of many observables inaccessible in perturbation theory.

A salient feature of the phenomenological application of DSEs is the significant role played by the necessary momentum-dependent modification of gluon and quark propagators: they are modified in perturbation theory and this modification persists and grows in the nonperturbative domain. For example, in a general covariant gauge the dressed-gluon propagator is characterised by a single scalar function, which we denote $\mathcal{D}(k^2)$. Many studies of the DSE for $D_{\mu\nu}(k)$ show that $\mathcal{D}(k^2)$ is strongly enhanced in the infrared; i.e, its behaviour in the vicinity of $k^2 = 0$ can be represented as a distribution [4], while for $k^2 > 1$-$2\,\mathrm{GeV}^2$ the perturbative result is reliable. With such behaviour manifest in the quark-quark interaction, dynamical chiral symmetry breaking (DCSB) and confinement follow *without* fine-tuning [5].

Both of these phenomena can be addressed through the DSE for the dressed-quark propagator:

$$S(p) := \frac{1}{i\gamma\cdot p + \Sigma(p)} = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)} = i\gamma\cdot p\,\sigma_V(p^2) + \sigma_S(p^2)\,, \qquad (1)$$

where $\Sigma(p)$ is the renormalised dressed-quark self energy, which satisfies

$$\Sigma(p) = (Z_2 - 1)\, i\gamma\cdot p + Z_4\, m^\zeta + \, Z_1 \int_q^\Lambda \, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(q,p), \qquad (2)$$

with $\Gamma^a_\nu(q;p)$ the dressed-quark-gluon vertex, m^ζ the current-quark mass, Z_1, Z_2 and Z_4 renormalisation constants, and ζ the renormalisation point. $\int_q^\Lambda := \int^\Lambda d^4q/(2\pi)^4$ represents mnemonically a *translationally-invariant* regularisation of the integral, with Λ the regularisation mass-scale. With the infrared-enhanced interaction introduced in Ref. [6] and current-quark masses corresponding to

$$\begin{array}{cccc} m_{u/d}^{1\,\mathrm{GeV}} & m_s^{1\,\mathrm{GeV}} & m_c^{1\,\mathrm{GeV}} & m_b^{1\,\mathrm{GeV}} \\ 6.6\,\mathrm{MeV} & 140\,\mathrm{MeV} & 1.0\,\mathrm{GeV} & 3.4\,\mathrm{GeV} \end{array} \qquad (3)$$

one obtains [7] the dressed-quark mass function depicted in Fig. 1. It is clear that for light quarks (u, d and s) there are two distinct domains: perturbative and nonperturbative. In the perturbative domain the magnitude of $M(p^2)$ is governed by the the current-quark mass, while for $p^2 < 1\,\mathrm{GeV}^2$ the mass-function rises sharply. This is the nonperturbative domain where the magnitude of $M(p^2)$ is determined by the DCSB mechanism; i.e., the enhancement in the dressed-gluon propagator.

For a given flavour, the ratio $\mathcal{L}_f := M_f^E/m_f^\zeta$ is a single, quantitative measure of the importance of the DCSB mechanism in modifying that quark's propagation characteristics. As illustrated in Eq. (4),

flavour	u/d	s	c	b	t
$\frac{M^E}{m^{\zeta\sim 20\,\mathrm{GeV}}}$	150	10	2.3	1.4	$\to 1$

(4)

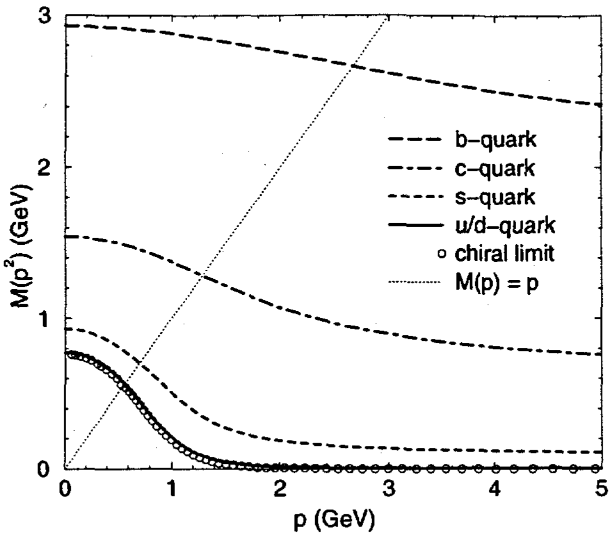

Figure 1: $M(p^2) := B(p^2)/A(p^2)$ obtained in solving the quark DSE. The solution of $M^2(p^2) = p^2$ defines M^E, the Euclidean constituent-quark mass.

this ratio provides a natural classification of quarks as either light or heavy. For light-quarks $\mathcal{L}_f$ is characteristically 10-100 while for heavy-quarks it is only 1-2. The values of $\mathcal{L}_f$ signal the existence of a characteristic DCSB mass-scale: M_χ. At $p^2 > 0$ the propagation characteristics of a flavour with $m_f^\zeta < M_\chi$ are altered significantly by the DCSB mechanism, while for flavours with $m_f^\zeta \gg M_\chi$ it is irrelevant, and explicit chiral symmetry breaking dominates. It is apparent that $M_\chi \sim 0.2\,\mathrm{GeV} \sim \Lambda_{\rm QCD}$.

This forms the basis for a simplification of the study of heavy-meson observables [8] that we summarise herein. It motivates an exploration of the fidelity of the approximation

$$S_{c/b}(p) = \frac{1}{i\gamma\cdot p + \hat{M}_{c/b}}\,, \tag{5}$$

where $\hat{M}_{c/b} \sim M^E_{c/b}$, so that with $p_{1\mu} := m_{H_1}\, v_\mu := (\hat{M}_{f_Q} + E)\, v_\mu$, the heavy-quark propagator is

$$S_{c/b}(k+p_1) = \frac{1}{2}\,\frac{1 - i\gamma\cdot v}{k\cdot v - E} + \mathrm{O}\left(\frac{|k|}{\hat{M}_{c/b}}, \frac{E}{\hat{M}_{c/b}}\right)\,. \tag{6}$$

(v_μ is the heavy meson velocity, $v^2 = -1$, and $E > 0$ is the difference between the heavy-meson mass and the effective-mass of the heavy-quark.) Many simplifications follow from neglecting the $1/\hat{M}$-corrections; e.g., it reduces the number of independent form factors required to describe heavy-meson $\to$ heavy-meson decays, relating them to a minimal number of so-called "universal" form factors, which is a characteristic feature of "heavy-quark" symmetry [9]. It is likely that the magnitude of M_b^E makes Eq. (6) quantitatively reliable, however, in employing the same reduction for the c-quark, one may expect quantitatively important corrections.

The light quark propagators are not limited in this way. They retain their full momentum dependence, which is characterised efficaciously in the parametrisation [10]

$$\bar{\sigma}_S^f(x) = 2\bar{m}_f \mathcal{F}(2(x+\bar{m}_f^2)) + \mathcal{F}(b_1 x)\mathcal{F}(b_3 x)\left(b_0^f + b_2^f \mathcal{F}(\epsilon x)\right), \quad (7)$$

$$\bar{\sigma}_V^f(x) = \frac{2(x+\bar{m}_f^2) - 1 + e^{-2(x+\bar{m}_f^2)}}{2(x+\bar{m}_f^2)^2}, \quad (8)$$

where: $f = u, s$ (isospin symmetry is assumed); $\mathcal{F}(y) := (1-\mathrm{e}^{-y})/y$; $x = p^2/(2D)$; $\bar{m}_f = m_f/\sqrt{2D}$; and

$$\bar{\sigma}_S^f(x) := \sqrt{2D}\,\sigma_S^f(p^2), \qquad \bar{\sigma}_V^f(x) := 2D\,\sigma_V^f(p^2), \quad (9)$$

with D a mass scale. This algebraic form combines the effects of confinement and dynamical chiral symmetry breaking with free-particle (asymptotically-free) behaviour at large, spacelike-p^2. The parameters: $\bar{m}_f$, $b^f_{0\ldots3}$, in Eqs. (7) and (8) take the values

	$\bar{m}_f$	b_0^f	b_1^f	b_2^f	b_3^f
$u:$	0.00897	0.131	2.90	0.603	0.185
$s:$	0.224	0.105	$\underline{2.90}$	0.740	$\underline{0.185}$

, (10)

which were determined in a least-squares fit to a range of light-hadron observables. The values of $b^s_{1,3}$ are underlined to indicate that the constraints $b^s_{1,3} = b^u_{1,3}$ were imposed [10]. The scale parameter $D = 0.160\,\mathrm{GeV}^2$.

The heavy-quark expansion introduced above can be employed in the analysis of semileptonic pseudoscalar $\to$ pseudoscalar decays: $P_{H_1}(p_1) \to P_{H_2}(p_2)\,\ell\,\nu$, where P_{H_1} represents either a B or D meson with momentum p_1 ($p_1^2 = -m_{H_1}^2$) and P_{H_2} can be a D, K or π meson with momentum p_2 ($p_2^2 = -m_{H_2}^2$). (Light $\to$ light transitions are discussed in Ref. [11].) The invariant amplitude describing the decay is

$$A(P_{H_1} \to P_{H_2}\ell\nu) = \frac{G_F}{\sqrt{2}}\,V_{qQ}\,\bar{\ell}\gamma_\mu(1-\gamma_5)\nu\,M_\mu^{P_{H_1}P_{H_2}}(p_1,p_2), \quad (11)$$

where G_F is the Fermi weak-decay constant, V_{qQ} is the appropriate element of the Cabibbo-Kobayashi-Maskawa matrix (q denotes a light-quark and Q a heavy-quark) and the hadronic current is

$$M_\mu^{P_{H_1}P_{H_2}}(p_1,p_2) := \langle P_{H_2}(p_2)|\bar{q}\gamma_\mu Q|P_{H_1}(p_1)\rangle = f_+(t)(p_1+p_2)_\mu + f_-(t)q_\mu, \quad (12)$$

with $t := -q^2$. The form factors, $f_\pm(t)$, contain all the information about strong interaction effects in these processes and their accurate estimation is essential to the extraction of V_{qQ} from a measurement of a semileptonic decay rate. In impulse approximation

$$M_\mu^{P_{H_1}P_{H_2}}(p_1,p_2) = \quad (13)$$
$$\frac{N_c}{16\pi^4}\int d^4k\,\mathrm{tr}\left[\bar{\Gamma}_{H_2}(k;-p_2)S_q(k+p_2)i\gamma_\mu S_Q(k+p_1)\Gamma_{H_1}(k;p_1)S_{q'}(k)\right].$$

Hitherto unspecified in Eq. (13) is $\Gamma_{H_1}(k;p_1)$, the Bethe-Salpeter amplitude for the H_1-meson. It can be obtained by solving the Bethe-Salpeter equation in a truncation consistent with that employed in the quark DSE. However, since we have parametrised that solution, we follow Ref. [10] and do the same for this amplitude; i.e., for the π- and K-mesons we assume $\Gamma_{\pi/K}(k;P) = i\gamma_5\,\mathcal{E}(k^2)$ and employ the algebraic parametrisation [10]:

$$\mathcal{E}(k^2) = \frac{\sqrt{2}}{f_H}\,\frac{C_0\,\mathrm{e}^{-k^2/[2D]} + \sigma_S(k^2)|_{m_f=0}}{\sigma_V(k^2)|_{m_f=0}}\,, \qquad (14)$$

which in concert with Eqs. (7) and (8) provides an efficacious algebraic representation of $\chi_{\pi/K}(k;P) := S(q+P/2)\,\Gamma_{\pi/K}(k;P)\,S(q-P/2)$. $C_0 = 0.214\,\mathrm{GeV}$ is chosen to yield a calculated value $f_\pi = 0.131$. $f_K = 0.160\,\mathrm{GeV}$.

For a heavy-meson, Bethe-Salpeter equation studies [12] suggest the *Ansatz*

$$\Gamma_{H_{1f}}(k;p_1) = \gamma_5\left(1 + \frac{1}{2}i\gamma\cdot v\right)\frac{1}{\mathcal{N}_{H_{1f}}}\,\varphi(k^2)\,, \qquad (15)$$

where, using Eq. (6), the canonical normalisation condition is

$$\mathcal{N}_{H_{1f}}^2 = \frac{1}{m_{H_{1f}}}\,\frac{N_c}{32\pi^2}\int_0^\infty du\,\varphi(z)^2\,\left(\sigma_S^f(z) + \sqrt{u}\,\sigma_V^f(z)\right) := \frac{1}{m_{H_{1f}}\kappa_f^2}\,, \qquad (16)$$

with $z = u - 2E\sqrt{u}$ and f labelling the light-quark flavour. In a solution of the Bethe-Salpeter equation the form of $\varphi(k^2)$ is completely determined. However, here it characterises our Ansatz and we choose

$$\varphi(k^2) = \exp\left(-k^2/\Lambda^2\right)\,, \qquad (17)$$

where Λ is a free parameter. As long as $\varphi(k^2)$ is a non-negative, non-increasing, convex up function of k^2, calculated results are insensitive to its detailed form. The leptonic decay constant in the heavy-quark limit is straightforward to determine once the Bethe-Salpeter amplitude is known:

$$f_{H_1} = \frac{\kappa_f}{\sqrt{m_{H_1}}}\frac{N_c}{8\pi^2}\int_0^\infty du\,(\sqrt{u} - E)\,\varphi(z)\,\left[\sigma_S^f(z) + \frac{1}{2}\sqrt{u}\,\sigma_V^f(z)\right]\,, \qquad (18)$$

from which it is clear that

$$f_{H_f}\sqrt{m_{H_f}} = \text{const.} \qquad (19)$$

From Eqs. (6), (15), (16) and (19), and the pseudoscalar meson mass formula [6]:

$$f_H\,m_H^2 \;=\; \mathcal{M}_H^\zeta\,r_H^\zeta\,,\;\; \mathcal{M}_H := \mathrm{tr}_{\rm flavour}\left[M_{(\zeta)}\left\{T^H,\left(T^H\right)^{\rm t}\right\}\right]\,, \qquad (20)$$

$$ir_H^\zeta \;=\; Z_4\int^\Lambda \frac{d^4q}{(2\pi)^4}\,\frac{1}{2}\mathrm{tr}\left[\left(T^H\right)^{\rm t}\gamma_5\mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-)\right]\,, \qquad (21)$$

where $M_{(\zeta)} = \text{diag}(m_u^\zeta, m_d^\zeta, m_s^\zeta, \ldots)$ and T^H is a flavour matrix identifying the channel under consideration, it also follows [3] that

$$m_{H_f} \propto \hat{m}_Q \tag{22}$$

in the heavy-quark limit, where $\hat{m}_Q$ is the renormalisation point invariant current-quark mass. The linear trajectory becomes apparent for $m_H \geq m_K$ [3, 7]. In contrast, for small current-quark masses, Eq. (20) yields what is commonly known as the Gell-Mann–Oakes–Renner relation. In Eq. (20) one has a single, exact formula that provides a unified description of light- and heavy-meson masses.

Using Eqs. (6) and (15) one finds [13] from Eqs. (12) and (13) that the $B_f \to D_f$ decay is particularly simple to study in the heavy-quark limit. It is described by one form factor:

$$f_\pm(t) = \frac{1}{2}\frac{m_{D_f} \pm m_{B_f}}{\sqrt{m_{D_f} m_{B_f}}}\,\xi_f(w)\,, \tag{23}$$

$$\xi_f(w) = \kappa_f^2 \frac{N_c}{32\pi^2}\int_0^1 d\tau\,\frac{1}{W}\int_0^\infty du\,\varphi(z_W)^2\left[\sigma_S^f(z_W) + \sqrt{\frac{u}{W}}\sigma_V^f(z_W)\right], \tag{24}$$

with $W = 1 + 2\tau(1-\tau)(w-1)$, $z_W = u - 2E\sqrt{u/W}$ and

$$w = \frac{m_{B_f}^2 + m_{D_f}^2 - t}{2 m_{B_f} m_{D_f}} = v_{B_f}\cdot v_{D_f}\,. \tag{25}$$

The minimum physical value of w is $w_{\min} = 1$, which corresponds to maximum momentum transfer with the final state meson at rest; the maximum value is $w_{\max} \simeq (m_{B_f}^2 + m_{D_f}^2)/(2m_{B_f} m_{D_f}) = 1.6$, which corresponds to maximum recoil of the final state meson with the charged lepton at rest. The canonical normalisation of the Bethe-Salpeter amplitude, Eq. (16), automatically ensures that

$$\xi_f(w=1) = 1\,. \tag{26}$$

Equation (23) illustrates a general result: in the heavy-quark limit, the semileptonic decays of heavy mesons are described by a single, universal function: $\xi_f(w)$.

The analysis of heavy $\to$ light decays is more difficult because, as remarked above, the current-quark mass of the u- and s-quarks $m_{u/s} \leq M_\chi \sim O(\Lambda_{\rm QCD})$. Hence the momentum-dependent modification of the dressed-quark propagator cannot be ignored, and the description of these decays requires a good understanding of light-quark propagation characteristics and the internal structure of light-mesons. The form factor that determines the width is

$$f_+^{H_1 H_2}(t) = \kappa_{q'}\frac{\sqrt{2}}{f_{H_2}}\frac{N_c}{32\pi^2}\,F_{q'}(t; E, m_{H_1}, m_{H_2})\,, \tag{27}$$

where

$$F_{q'}(t; E, m_{H_1}, m_{H_2}) = \frac{4}{\pi}\int_{-1}^1 \frac{d\gamma}{\sqrt{1-\gamma^2}}\int_0^1 d\nu\int_0^\infty u^2 du\,\varphi(z_1)\,\mathcal{E}(z_1)\,W_{q'}(\gamma,\nu,u)\,, \tag{28}$$

	DATA/ESTIMATES	$f_B = 0.170\,\mathrm{GeV}$
(E,Λ) (GeV)		(0.442,1.408)
Σ^2/N		0.48
$f_+^{B\pi}(14.9\,\mathrm{GeV}^2)$	0.82 ± 0.17 [14]	$0.84^\dagger$
$f_+^{B\pi}(17.9\,\mathrm{GeV}^2)$	1.19 ± 0.28 [14]	$1.02^\dagger$
$f_+^{B\pi}(20.9\,\mathrm{GeV}^2)$	1.89 ± 0.53 [14]	$1.30^\dagger$
$\mathrm{Br}(B^0 \to \pi^- \ell^+ \nu)$	$[1.8 \pm 0.4 \pm 0.3 \pm 0.2] \times 10^{-4}$ [15]	$2.0 \times 10^{-4\dagger}$
$f_+^{B\pi}(0)$	$0.18 \to 0.49$ [16]	0.46
$f_+^{DK}(0)$	0.74 ± 0.03 [17]	0.62
ρ^2	$0.91 \pm 0.15 \pm 0.06$ $1.53 \pm 0.36 \pm 0.14$ [18]	0.87
f_{B_s} (GeV)	0.195 ± 0.035 [19]	0.184
f_{B_s}/f_B	1.14 ± 0.08 [19]	1.083
f_D (GeV)	0.200 ± 0.030 [19]	0.285
f_{D_s} (GeV)	0.220 ± 0.030 [19]	0.304
f_{D_s}/f_D	1.10 ± 0.06 [19]	1.066

Table 1: Calculated results cf. data (experimental or lattice simulations) when we require $f_B = 0.170\,\mathrm{GeV}$, which is the central value estimated in Ref. [19]. Quantities marked by † are used to constrain the parameters (E, Λ) by minimising $\Sigma^2 := \sum_{i=1}^{N} ([y_i^{\rm calc} - y_i^{\rm data}]/\sigma(y)_i^{\rm data})^2$, where N is the number of data items used. NB: 1) the values of f_D and f_{D_s} are obtained via Eq. (19) from f_B and f_{B_s}, respectively, using $m_B = 5.27$, $m_{B_s} = 5.375$, $m_D = 1.87$ and $m_{D_s} = 1.97\,\mathrm{GeV}$; 2) the experimental determination of ρ^2 is sensitive to the form of the fitting function, e.g., see Ref. [18]; 3) an analysis of four experimental measurements of $D_s \to \mu\nu$ decays yields $f_{D_s} = 0.241 \pm 0.21 \pm 0.30\,\mathrm{GeV}$ [20].

with $W_{q'}(\gamma, \nu, u)$ depending on the light-quark propagator and its derivatives [8].

All that is necessary for the calculation of the mesonic semileptonic heavy $\to$ heavy and heavy $\to$ light transition form factors, and heavy-meson leptonic decay constants is now specified. There are two free parameters: the binding energy, E, introduced after Eq. (5) and the width, Λ, of the heavy meson Bethe-Salpeter amplitude, introduced in Eq. (17). The dressed light-quark propagators and light-meson Bethe-Salpeter amplitudes were completely fixed in the application of this framework to the study of π- and K-meson properties. The primary goal of this study is to determine whether, with these two parameters, a description and correlation of existing heavy-meson data is possible using the DSE framework. Some key results are presented in Table 1, which also describes how the parameters (E, Λ) were fixed.

The calculated form of $\xi(w)$ is depicted in Fig. 2. It yields a value for $\rho^2 :=$

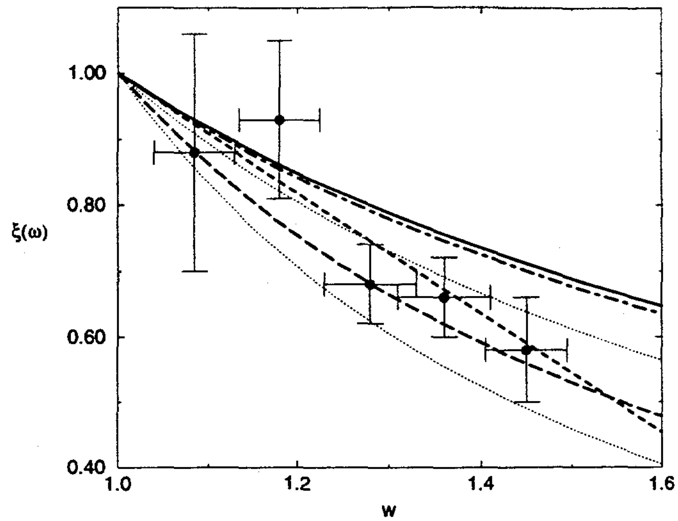

Figure 2: Calculated form of $\xi(w)$ cf. recent experimental analyses. The solid line was obtained assuming only that the b-quark is heavy, the dash-dot line assumed the same of the c-quark [8]. Experiment: data points - Ref. [21]; short-dashed line - linear fit from Ref. [18], $\xi(w) = 1 - \rho^2\,(w-1)$, $\rho^2 = 0.91 \pm 0.15 \pm 0.16$; long-dashed line - nonlinear fit from Ref. [18], $\xi(w) = [2/(w+1)]\, \exp\left[(1-2\rho^2)\,(w-1)/(w+1)\right]$, $\rho^2 = 1.53 \pm 0.36 \pm 0.14$. The two light, dotted lines are this nonlinear fit evaluated with the extreme values of ρ^2: upper line, $\rho^2 = 1.17$ and lower line, $\rho^2 = 1.89$.

$-\xi'(w=1) = 0.87 - 0.92$,[1] close to that obtained with a linear fitting form [18], however, $\xi(w)$ has significant curvature and deviates quickly from that fit. The curvature is, in fact, very well matched to that of the nonlinear fit [18], however, the value of ρ^2 reported in that case is very different from the calculated value. The derivation of the formula for $\xi(w)$ assumes that the heavy-quark limit is valid not only for the b-quark but also for the c-quark. Therefore these results suggest that the latter assumption is only accurate to approximately 20%; i.e., $1/\hat{M}_c$-corrections are quantitatively important.

$f_+^{B\pi}(t)$ is depicted in in Fig. 3. A good *interpolation* of the result is provided by

$$f_+^{B\pi}(t) = \frac{0.458}{1 - t/m_{\rm mon}^2}\,,\; m_{\rm mon} = 5.67\,{\rm GeV}\,. \tag{29}$$

This value of $m_{\rm mon}$ can be compared with that obtained in a fit to lattice data: [14] $m_{\rm mon} = 5.6 \pm 0.3$.

The calculated form of $f_+^{DK}(t)$ is depicted in Fig. 4. The t-dependence is also well-approximated by a monopole fit. The calculated value of $f_+^{DK}(0) = 0.62$ is approximately 15% less than the experimental value [17]. That is also a gauge of the size of $1/\hat{M}_c$-corrections, which are expected to reduce the value of the D- and D_s-meson leptonic decay constants calculated in the heavy-quark limit: $f_D = 285\,{\rm MeV}$,

[1]In this framework the minimum possible value for ρ^2 is $1/3$ [13].

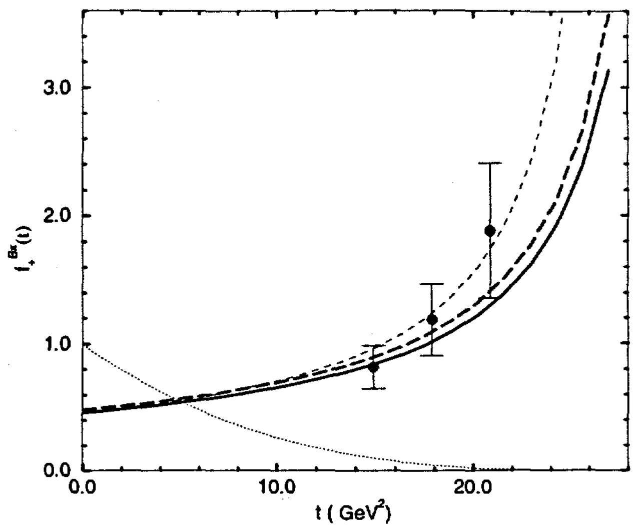

Figure 3: Calculated form of $f_+^{B\pi}(t)$. The solid line was obtained assuming only that the b-quark is heavy, the dashed line assumed the same of the c-quark [8]. The data were obtained in lattice simulation [14] and the light, short-dashed line is a vector dominance, monopole model: $f_+(t) = 0.46/(1 - t/m_{B^*}^2)$, $m_{B^*} = 5.325\,\mathrm{GeV}$. The light, dotted line is the phase space factor $|f_+^{B\pi}(0)|^2 \left[(t_+ - t)(t_- - t)\right]^{3/2} / (\pi m_B)^3$ that appears in the expression for the width, which illustrates that the $B \to \pi e \nu$ branching ratio is determined primarily by the small-t behaviour $f_+^{B\pi}(t)$.

$f_{D_s} = 298\,\mathrm{MeV}$. A 15% reduction yields $f_D = 0.24\,\mathrm{GeV}$ and $f_{D_s} = 0.26\,\mathrm{GeV}$, values which are consistent with lattice estimates [19] and the latter with experiment [20].

It must be noted that Ref. [8] explicitly *did not* assume vector meson dominance. The calculated results reflect only the importance and influence of the dressed-quark and -gluon substructure of the heavy mesons. That substructure is manifest in the dressed propagators and bound state amplitudes, which fully determine the value of every calculated quantity. That simple-pole *Ansätze* provide efficacious interpolations of the calculated results on the accessible kinematic domain is not surprising, given that the form factor must rise slowly away from its value at $t = 0$ and the heavy meson mass provides a dominant intrinsic scale, which is only modified slightly by the scale in the light-quark propagators and meson bound state amplitudes.

This presentation illustrates the phenomenological application of a heavy-quark limit of the DSEs that is based on the result that the mass function of heavy-quarks evolves slowly with momentum. Heavy-mesons are seen to be little different from light-mesons: they are bound states of finite extent with dressed-quark constituents. The results summarised here indicate that the heavy-quark limit can be used to develop a quantitatively reliable description of B-meson observables. However, it is inadequate for D-meson observables, where corrections of 15-20% can be expected. A significant feature of the DSE approach is that it provides a single framework for the correlation of heavy $\to$ heavy and heavy $\to$ light transitions *and* for their

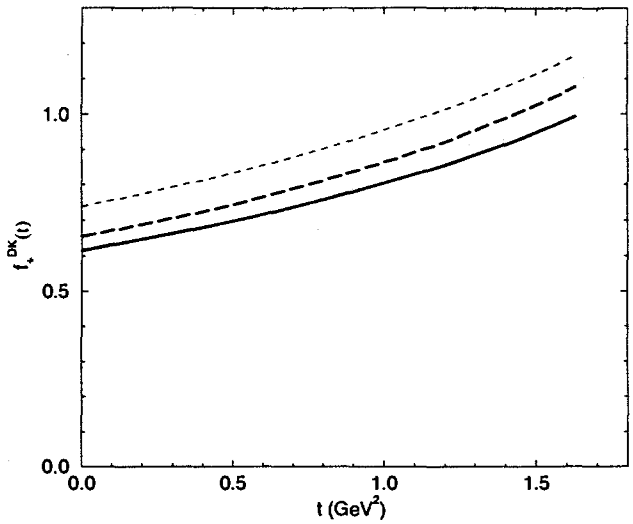

Figure 4: Calculated form of $f_+^{DK}(t)$: the solid line was obtained assuming only that the b-quark is heavy, the dashed line assumed the same of the c-quark [8]. The light, short-dashed line is a vector dominance, monopole model: $f_+(q^2) = 0.74/(1 - q^2/m_{D_s^*}^2)$, $m_{D_s^*} = 2.11\,\mathrm{GeV}$.

correlation with light meson observables, which are dominated by effects such as confinement and DCSB.

This work was supported in part by the US Department of Energy, Nuclear Physics Division, under contract no. W-31-109-ENG-38.

References

[1] A. Bender, C. D. Roberts and L. v. Smekal, Phys. Lett. B **380**, 7 (1996); C. D. Roberts in *Quark Confinement and the Hadron Spectrum II*, edited by N. Brambilla and G. M. Prosperi (World Scientific, Singapore, 1997).

[2] A. Bashir, A. Kizilersu and M.R. Pennington, Phys. Rev. D **57**, 1242 (1998); and references therein.

[3] C. D. Roberts, "Nonperturbative QCD with Modern Tools", nucl-th/9807026.

[4] N. Brown and M. R. Pennington, Phys. Rev. D **39**, 2723 (1989); M. R. Pennington, "Calculating hadronic properties in strong QCD", e-print hep-ph/9611242.

[5] F. T. Hawes, P. Maris and C. D. Roberts, "Infrared Behaviour of Propagators and Vertices", nucl-th/9807056, to appear in Phys. Lett. B.

[6] P. Maris and C. D. Roberts, Phys. Rev. C **56**, 3369 (1997).

[7] P. Maris and C. D. Roberts, "Differences between Heavy and Light Quarks", in *Rostock 1997, Progress in heavy quark physics*, edited by M. Beyer, T. Mannel and H. Schröder; nucl-th/9710062.

[8] M. A. Ivanov, Yu. L. Kalinovsky, P. Maris and C. D. Roberts, Phys. Rev. C **57**, 1991 (1998).

[9] N. Isgur and M. B. Wise, Phys. Lett. B **237**, 527 (1990).

[10] C. J. Burden, C. D. Roberts and M. J. Thomson, Phys. Lett. B **371**, 163 (1996).

[11] Yu. Kalinovsky, K. L. Mitchell and C. D. Roberts, Phys. Lett. B **399**, 22 (1997).

[12] C. J. Burden *et al.*, Phys. Rev. C **55**, 2649 (1997).

[13] M. A. Ivanov, Yu. L. Kalinovsky, P. Maris and C. D. Roberts, Phys. Lett. B **416**, 29 (1998).

[14] UKQCD Coll. (D. R. Burford *et al.*), Nucl. Phys. B **447**, 425 (1995).

[15] CLEO Coll. (J. P. Alexander *et al.*), Phys. Rev. Lett. **77**, 5000 (1996).

[16] L. Lellouch, Nucl. Phys. B **479**, 353 (1996).

[17] Particle Data Group (R. M. Barnett *et al.*), Phys. Rev. D **54**, 1 (1996).

[18] CLEO Coll. (J.E. Duboscq *et al.*), Phys. Rev. Lett. **76**, 3899 (1996).

[19] J. M. Flynn and C. T. Sachrajda, "Heavy Quark Physics From Lattice QCD", e-print hep-lat/9710057.

[20] J. Richman, in the proceedings of the *28th International Conference on High Energy Physics*, Warsaw, Poland, 25-31 July 1996, ed. Z. Ajduk and A. K. Wroblewski (World Scientific, Singapore, 1997) 143.

[21] ARGUS Collaboration, Z. Phys. C **57**, 249 (1993).