DEVELOPMENT OF A NEUTRON TRANSPORT CODE FOR MANY-GROUP TWO-DIMENSIONAL HETEROGENEOUS CALCULATIONS BY THE METHOD OF CHARACTERISTICS

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ABSTRACT

The method of characteristics (MOC) is gaining increased popularity in the reactor physics community all over the world because it gives a new degree of freedom in nuclear reactor analysis. The MARIKO code solves the neutron transport equation by the MOC in two-dimensional real geometry. The domain of solution can be a rectangle or right hexagon with periodic boundary conditions on the outer boundary. Any reasonable symmetry inside the domain can be fully accounted for. The geometry is described in three levels - macro-cells, cells, and regions. The macro-cells and cells can be any polygon. The outer boundary of a region can be any combination of straight line and circular arc segments. Any level of embedded regions is allowed. Procedures for automatic geometry description of hexagonal fuel assemblies and reflector macro-cells have been developed. The initial ray tracing procedure is performed for the full rectangular or hexagonal domain, but only azimuthal angles in the smallest symmetry interval are tracked.

The MOC integration procedure is computationally too heavy to be used for the self-scattering, up-scattering, and fission-source iterations. The method of transmission probabilities (TP) is combined with the MOC for all these. The iteration strategy includes solution of the TP-equations once before and once after the MOC integration in each group in order to accelerate the convergence of the self-scattering source. In addition a multi-group version of the TP method is applied once before and once after the pass over all thermal groups in order to accelerate the convergence of the up-scattering source. After a full outer iteration with the MOC integration one or more additional outer iterations can be performed without the MOC integration, i.e. with the escape and transmission probabilities calculated in the last full outer iteration. Only 6 full outer iterations are needed for convergence in case of many-group calculation of a 30° sector of a WWER-440 assembly and 9-12 iterations for small core-reflector problems. In case of large core-reflector problems with 10000-16000 regions the number of outer iterations required for convergence is 16-26. But the convergence rate in case of full core calculations is very slow still.

Introduction

The method of characteristics (MOC) became the standard transport method in WIMSE[1, 2] and in CASMO-4[3] in 1993. Since then many new transport codes based on MOC have been developed[4, 5, 6]. The method of characteristics has great advantages compared to the collision probability method, because it is applicable for large problems and solves the neutron transport equation for the generic angular neutron flux, from which any reactor physics quantity can be calculated. Compared to the S_N methods MOC has the advantage of being capable to model the real geometry of the fuel lattices without any homogenization.

Like the S_N methods, MOC solves the neutron transport equation in each group in selected discrete directions, but unlike the S_N method it follows strictly the neutron rays through the problem region, thus using the exact solution of the transport equation in its characteristics form. The problem region is covered with sufficient number of parallel tracks for each selected direction, so that the spatial dependence of the angular flux will be adequately accounted for. Any geometry can be treated, provided adequate ray tracing routines are available. The only approximation is in the representation of the spatial and angular dependence of the group neutron source. The MOC can treat linearly anisotropic scattering without any special complications. The accuracy of the solution depends on the number of selected directions, the track spacing, and the detail in spatial representation of the neutron source, i.e. on the power of the available computer.

The theory of MOC is very simple, but its implementation is not. This paper presents the author's experience in developing the neutron transport code MARIKO[7, 8], based on MOC. In the next section the basic equations of MOC and the method of transmission probabilities, which are combined in MARIKO, are presented. Then some key topics of the geometry modeling are discussed. In the next section the generation of periodic tracks, originally used in CACTUS[9, 10], for both rectangular and hexagonal domains is outlined. Some key topics of the initial tracking and the ray tracing are discussed in the next two sections. The iteration strategy is outlined next. The results of some test calculations of a WWER-440 fuel assembly, showing the sensitivity of the results on various parameters, are presented also.

Basic equations

A neutron direction, $\vec{\Omega}_{mn}$, is defined by the *azimuthal angle* (the angle between the projection of $\vec{\Omega}_{mn}$ on the (x,y) plane and the x axis), φ_m, and the polar *level* (the angle between $\vec{\Omega}_{mn}$ and the (x,y) plane), θ_n. The same azimuthal angles are used in all polar levels, because the ray tracing is the same for all polar levels.

The static neutron transport equation for direction $\vec{\Omega}_{mn}$ in case of linearly anisotropic scattering is the following[11]:

$$\vec{\Omega}_{mn}.\nabla\Phi_{gmn}(\vec{r}) + \Sigma_g(\vec{r})\Phi_{gmn}(\vec{r}) = \\ = \lambda^{-1}\chi_g(\vec{r})\sum_h \nu\Sigma^f_h(\vec{r})\phi_h(\vec{r}) + \sum_h \Sigma^{s0}_{h\to g}(\vec{r})\phi_h(\vec{r}) + 3\vec{\Omega}_{mn}.\sum_h \Sigma^{s1}_{h\to g}(\vec{r})\vec{J}_h(\vec{r}) \ , \quad (1)$$

where:

Φ_{gmn} is 4π times the angular flux in group g and direction $\vec{\Omega}_{mn}$;

ϕ_h and $\vec{J}_h$ are the scalar flux and current in group h;

Σ_g is the total cross section for group g;

λ is the static eigenvalue;

χ_g is the fission spectrum for group g;

$\nu\Sigma_h^f$ is the macroscopic neutron production cross section in group h;

$\Sigma_{h\to g}^{s0}$ and $\Sigma_{h\to g}^{s1}$ are the zeroth and first terms in the expansion of the scattering cross section in a series of Legendre polynomials.

The neutron source in group g, region i, and direction $\vec{\Omega}_{mn}$, which corresponds to the RHS of Eq. 1, is presented in the following form:

$$Q_{gimn}(x,y) = \bar{Q}_{gi} + 3\Omega_{xmn}\bar{Q}^{a}_{xgi} + 3\Omega_{ymn}\bar{Q}^{a}_{ygi} + \bar{Q}'_{xgi}(x-x_i) + \bar{Q}'_{ygi}(y-y_i),$$

where (x_i, y_i) is the centroid point of region i and

$$\begin{aligned}
\bar{Q}_{gi} &= \lambda^{-1}\chi_{gi}\sum_h \nu\Sigma^f_{h,i}\bar{\phi}_{hi} + \sum_h \Sigma^{s0}_{h\to g,i}\bar{\phi}_{hi},\\
\bar{Q}^{a}_{\alpha gi} &= \sum_h \Sigma^{s1}_{h\to g,i}\bar{J}_{\alpha hi},\quad \alpha = x,y,\\
\bar{Q}'_{\alpha gi} &= \lambda^{-1}\chi_{gi}\sum_h \nu\Sigma^f_{h,i}\bar{\phi}'_{\alpha hi} + \sum_h \Sigma^{s0}_{h\to g,i}\bar{\phi}'_{\alpha hi},\quad \alpha = x,y.
\end{aligned}$$

The (x,y)-projection of a neutron ray is called a *track*. An auxiliary Cartesian (u,v) coordinate system, rotated towards (x,y) to an angle of φ_m, is introduced, so that all tracks in azimuthal angle φ_m are parallel to the u-axis. Each track is divided into track *segments* by the region boundaries. The source along segment s in region i will be also a linear function:

$$Q_{mns}(u) = \bar{Q}_{mns} + \bar{Q}'_{ui}(u-u_s),$$

where u_s is the u-coordinate of the segment's mid-point and $\bar{Q}_{mns}$ is the segment-averaged source. The group index in this equation and below is omitted. With this source the transport equation is written in its characteristics form

$$\cos\theta_n \frac{d\Phi_{mns}(u)}{du} + \Sigma_i\Phi_{mns}(u) = \bar{Q}_{mns} + \bar{Q}'_{ui}(u-u_s).$$

This equation is solved for the out-flux at the segment's end-point, Φ^{out}_{mns}, provided the in-flux at the segment's start point, Φ^{in}_{mns}, is known:

$$\Phi^{out}_{mns} = \Phi^{in}_{mns}[1 - E_1(\tau_{mns})] + \frac{\bar{Q}_{mns}}{\Sigma_i}E_1(\tau_{mns}) + \frac{\bar{Q}'_{ui}\cos\theta_n}{2\Sigma_i^2}E_3(\tau_{mns}), \qquad (2)$$

where $\tau_{mns} = \Sigma_i L_{ms}/\cos\theta_n$ is the optical segment length, with L_{ms} the segment length. The E_i functions are defined as follows:

$$\begin{aligned}
E_1(\tau) &= 1 - e^{-\tau},\\
E_3(\tau) &= 2\left[\tau - \left(1 - e^{-\tau}\right)\right] - \tau\left(1 - e^{-\tau}\right).
\end{aligned}$$

Eq. 2 is applied to calculate the out-fluxes for each segment along the periodic track. The RHS of Eq. 2 has clear physical meaning. The first term is the transmission probability for the angular flux between the start and end points of the segment. If the start point

is on edge l and the end point is on edge k of region i, then the first term, multiplied by $\cos\theta_n\delta_m$ (δ_m is the the track separation), is the contribution from this segment to the transmission probability, T_{ilk}, for the in-coming neutrons on edge l of region i to reach edge k without collision. The next two terms, also multiplied by $\cos\theta_n\delta_m$, can be recognized as contribution to the escape probability, R_{ik}, for the source neutrons in region i to escape through edge k without collision.

The out-going partial currents, j_{ik}^+, the region to edge escape probabilities, R_{ik}, and the edge to edge transmission probabilities, T_{ilk}, are calculated by the following expressions:

$$\begin{aligned}
j_{ik}^+ &= \sum_{mn} W_{mn}\cos\theta_n\delta_m \sum_{s\in ik}\Phi_{mns}^{\text{out}},\\
R_{ik} &= \sum_{mn} W_{mn}\cos\theta_n\delta_m \sum_{s\in ik}\left[\frac{\tilde{Q}_{mns}}{\Sigma_i}E_1(\tau_{mns}) + \frac{\tilde{Q}'_{ui}\cos\theta_n}{2\Sigma_i^2}E_3(\tau_{mns})\right]/\bar{Q}_i,\\
T_{ilk} &= \sum_{mn} W_{mn}\cos\theta_n\delta_m \sum_{s\in ilk}[1-E_1(\tau_{mns})]\Phi_{mns}^{\text{in}}/j_{il}^-,
\end{aligned}$$

where W_{mn} is the integration weight of direction $\vec{\Omega}_{mn}$, $s\in ik$ means summation over all segments in the current direction in region i ending on edge k, and $s\in ilk$ means summation over all segments in the current direction in region i starting on edge l and ending on edge k The region-averaged flux, $\bar{\Phi}_{imn}$, and the region-averaged x- and y-components of the current are calculated by

$$\begin{aligned}
\Sigma_i\bar{\phi}_i &= \bar{Q}_i + \sum_{mn} W_{mn}\cos\theta_n\Delta\Phi_{imn},\\
\Sigma_i\bar{J}_{\alpha i} &= \bar{Q}_{\alpha i}^{\text{a}} + \sum_{mn} W_{\alpha mn}\Omega_{\alpha mn}\cos\theta_n\Delta\Phi_{imn},\ \alpha = x,y,
\end{aligned}$$

where

$$\Delta\Phi_{imn} = A_i^{-1}\delta_m\sum_{s\in i}(\Phi_{mns}^{\text{in}} - \Phi_{mns}^{\text{out}}),$$

$s\in i$ means all segments in the current direction in region i, A_i is the area of region i. The calculation of W_{mn} is discussed in [7].

Since the linear source is only an approximation, not directly related to the neutron balance, the second equation of the P_1-approximation to the transport equation is used to calculate the region-averaged x- and y-derivatives of the scalar flux:

$$\bar{\phi}'_{\alpha gi} = -3\Sigma_{1gi}\bar{J}_{\alpha gi} + 3\sum_h \Sigma_{h\to g,i}^{s1}\bar{J}_{\alpha hi},\ \ \alpha = x,y,$$

where Σ_1 is the current-weighted total cross section.

For the method of transmission probabilities the neutron balance equation for region i can be written as

$$\Sigma_i\bar{\phi}_i A_i = \bar{Q}_i A_i + \sum_{k\in i} j_{ik}^- - \sum_{k\in i} j_{ik}^+,$$

and the equations for the out-going partial currents are

$$j_{ik}^+ = R_{ik}\bar{Q}_i + \sum_{l\in i} T_{ilk}j_{il}^-.$$

These equations are transformed into equivalent ones with the self-scattering terms excluded.

Geometry description

The MARIKO code solves the neutron transport equation in a two-dimensional *domain*, which can be either a rectangle or a right hexagon, with the most general periodic (translational) boundary conditions. Any reasonable symmetry inside the domain is fully accounted for. The user input, most of the calculations, and all the output are for the smallest sector of symmetry. The ray tracing is performed in the full domain, but only azimuthal angles in the smallest symmetry interval are traced. For example, if a WWER-440 assembly with 30° symmetry is calculated, then only azimuthal angles in the interval $(0, 30°)$ are traced. This is equivalent to tracing the 30° sector in 12 times more azimuthal angles in the whole angular interval, which result in reflecting the ray on all three sides of the triangle. An option with free boundary conditions has been included recently with the outer domain boundary being any convex polygon.

When tracing the entire domain with periodic boundary conditions the ray does not change its azimuthal angle on the domain boundary. Thus the initial tracking procedure is simple and fast. A more essential advantage is that only two numbers (the region number and the segment length) per track segment are sufficient for the MOC-integration procedure.

There are three levels of geometry structures—macro-cells, cells, and regions. The domain is divided into macro-cells, they are divided into cells, and the cells are divided into regions. The outer boundary of a macro-cell or cell can be any polygon, while the outer boundary of a region can be practically any combination of straight line and circular arc segments, called edges. The outer boundaries of macro-cells and cells are described by the (x, y)-coordinates of their vertexes. The outer boundary of a region is described by the (x, y)-coordinates of its vertexes and the curvature radiuses of its edges, negative for none-convex edges and zero for straight line edges. Any level of embedded regions is allowed and they are described in the same way, except that the number of the nearest enclosing region must be specified. Thus the geometry description is general enough.

The geometry description in MARIKO is not meant for manual input, but for coding in Fortran. Because a fuel assembly consists of many identical fuel cells, geometry cell types are defined first in local coordinate systems, then they are put over the macro-cells by specifying the global (x, y)-coordinates, where the centre of the geometry cell type's local coordinate system is placed. Thus each geometry structure is described independently from the others. This approach proved to be fruitful and modules of subroutines for description of the WWER-440 reactor have been developed. A fuel assembly may consist of any number of adjacent 30° sectors. The macro-cells in the reflector are similar to the fuel macro-cells, but the hexagonal cells can be divided in 6 triangles only. A separate model has been developed for the absorber macro-cell, because finer spatial disretization is required. Given the standard integer rectangular coordinates of a macro-cell in the reflector, the code automatically overlays the core shroud, basket, and shaft. The user supplied input data are very simple. Lower level geometry input is also available.

Periodic tracks

A periodic track (Ref. [9]) starts at a point on the domain outer boundary in an azimuthal angle and after a number of transfers by the boundary conditions returns to the starting point. Thus a periodic track consists of many small tracks (usual sense), which are all in the same azimuthal angle in case of periodic boundary conditions. The periodic tracks

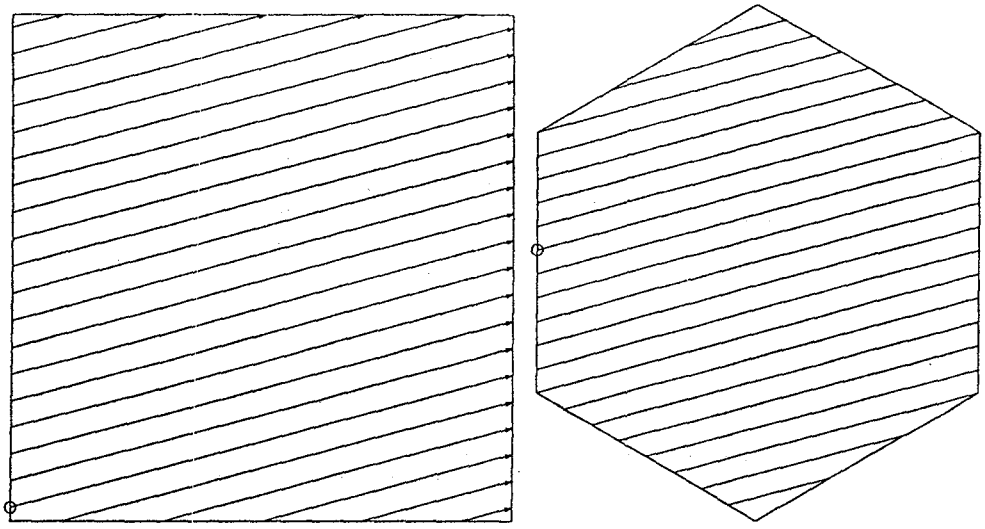

Figure 1: Perodic tracks for rectangular and hexagonal domain with periodic boundary conditions

cannot be drawn in any azimuthal angle. In case of a rectangular domain with width A and height B the following relations must be strictly hold:

$$\begin{aligned} N_A\delta/\sin\varphi &= A, \\ N_B\delta/\cos\varphi &= B, \end{aligned}$$

where N_A and N_B are the number of times the periodic track intersects the A-sides and B-sides, φ is the azimuthal angle, and δ is the track separation. The integers N_A and N_B must be relatively prime numbers, otherwise the periodic track will consist of several repeated periodic tracks. Given initial values for δ and φ, a simple algorithm finds the closest values, which satisfy the above equations. If δ is small compared to A and B, which is the case in calculating a fuel assembly, the difference between the initial and final values of δ and φ is very small. Such a periodic track is shown on Fig. 1 with $N_A = 5$ and $N_B = 19$.

Periodic tracks can be constructed for a hexagonal domain as well. The most simple way is to represent the hexagonal array by an array of rectangles (two rectangles per hexagon). Then the algorithm for rectangular geometry is applied with some additional constrains ($N_A + N_B$ must be even, the track separation for the rectangular array must be twice smaller than for the hexagonal one, etc.) A periodic track in case of periodic boundary conditions for a hexagonal domain is shown on Fig. 1.

The periodic tracks have essential advantages for the method of characteristics compared to the simple equidistant tracks. At first the neutron ray is followed strictly after transfer from the boundary conditions without the need to interpolate out-going angular fluxes. Another advantage is that only one starting angular flux is needed per traced direction. But the most significant advantage is that in practice there is no need for angular flux iterations.

Initial tracking

The initial tracking is aimed at preparing information for the real ray tracing so that the latter will be as fast as possible, because the initial tracking is performed only once and the ray tracing is performed many times. Initial tracking is performed for all azimuthal angles in the smallest symmetry interval. Because a periodic track is in a single azimuthal angle, it is convenient to rotate the domain so that the tracks are horizontal in the new (u, v) coordinate system. Each track is divided by the region boundaries into (track) segments. The first quantity to be determined is the segment length. It is sufficient for the flat source approximation, but for the linear source approximation the u-coordinate of the segment's mid point is also needed. In MARIKO only the u-coordinate of the starting point of the first segment on a track is saved, and the segment lengths are used to calculate the required u_s following the track from left to right. The v-coordinate of the segments mid-points is the same for all segments on a track and it is saved per track as well. Because the number of tracks is many times less than the number of segments, this gives significant memory savings.

Another quantity to be prepared by the initial tracking is the region number by segment number. In MARIKO a single integer number is used to save both the global region number (in the entire domain) and the edge-couple number local to the intersected region. The global region number is multiplied by 256 and the local edge-couple number is added. Thus the total number of regions cannot exceed $2^{24} - 1$ and the number of edges of a region cannot exceed 16. Both margins seem reasonable. In integrating the transmission probabilities the edge-couple number in the smallest sector of symmetry is needed and it is calculated by one addition, using an auxiliary array by global region number, which is the base edge-couple number in the smallest sector of symmetry minus the global region number times 256. Thus two 4-byte words per segment and another two per track are sufficient to save all data for the ray tracing.

The starting angular flux

The starting angular flux is needed to start the integration along the periodic track, but it is unknown. An initial value with unknown error is available from the previous sweep in the same group and neutron direction or from the initial guess. In order to be precise and to do the MOC-integration with the true angular flux, the following procedure is available in MARIKO. Starting from the periodic track's starting point with the available initial starting angular flux, an optical length of τ_{max} is passed calculating only the angular flux without integration, where τ_{max} is an input quantity. If $\tau_{max} = 10$, it is obvious that the error in the starting angular flux will be completely "forgotten" after passing 10 mean free paths. The integration starts from this point on, continues to the end of the periodic track, and then the initial τ_{max} part of it is passed again, this time with integration. On reaching the end of the periodic track, the new value of the starting angular flux is saved.

Since the total optical length of the periodic track is much longer than τ_{max} (except in single fuel cell calculations), the cost of this additional procedure is negligible. After this procedure had been implemented, a study for the optimum value of τ_{max} was performed. It was found that in case of assembly calculations, setting τ_{max} equal to 0 does not change neither the final results nor the convergence behavior of the whole iteration process. The explanation is simple—because the error in the starting flux is small and dies out in a distance much smaller than the total length of the periodic track, the impact of the error

on the integrated results is negligible. Therefore, the integration may start from the very beginning of the periodic track.

Iteration strategy

The MOC in its simplest version is used to calculate only the scalar flux by region, which leads to slow convergence because of the self-scattering and up-scattering even for single cell problems. But having the generic angular neutron flux, it is possible to calculate the partial currents trough region edges and region to edge escape probabilities as well as edge to edge transmission probabilities. In the very first version of MARIKO the simple mesh re-balance method (MRM) has been tested for acceleration of the self-scattering and up-scattering iterations. It does not require escape and transmission probabilities, but its efficiency is not good even in case of single cell problems with MOX fuel and fine spatial discretization. That is why the method of transmission probabilities (MTP) was adopted. It should be noted that this version of the MTP is exact, because the escape and transmission probabilities are calculated each time the MOC integration for a neutron group is performed. Of course, these probabilities depend on the current angular flux distribution, especially in the fast energy range, but when the solution has converged the solution of the MTP is fully consistent with that of the MOC. In order to get full consistency double precision must be used in calculating the final coefficients, although they can be stored in single precision.

The MTP is used in two different ways in each outer iteration (with fixed fission source). For a particular neutron group it is applied once before and once after the MOC-integration. The first time it is used with the old coefficients from the last pass through the same group (the MRM is used in the very first pass) and aims at improving the scalar flux distribution for the current group in accordance with the new in-source from fission and in-scattering. Then the MOC-integration is performed and new scalar fluxes, currents, partial currents, and escape and transmission probabilities are calculated for the current group. The MTP is applied again to improve the scalar flux distribution further. This procedure is applied for each group, and it is sufficient to account for the self-scattering effects.

For the up-scattering effects a many-group version of the MTP has been developed. It solves the equations simultaneously for all thermal groups. It is applied once before the thermal groups are to be passed, but after the last fast group has been passed, and once after they have been passed. This turned out to be the only way to account efficiently for the up-scattering effects in case of core-reflector problems, when the coupling through the scattering is probably stronger than the spatial coupling. No additional pass through the lowest energy groups is necessary.

With this iteration strategy the convergence rate in all energy ranges is nearly the same. In some cases it is advantageous to repeat the iteration in each one of the highest energy groups. However, the estimates of the dominance ratio for the power iterations are unstable. In small-size problems the convergence rate of the outer iterations is very high. Depending on the convergence criteria, 5-6 outer iterations are sufficient for many group calculations of a 30° sector of a WWER-440 fuel assembly. For the WWER-440 absorber problem 8-11 outer iterations are sufficient. For small core-reflector problems 9-12 outer iterations are sufficient as well. Medium size problems with 10000-16000 region require 16-26 outer iterations. But for full core problems no solution for acceleration of the outer iterations has been found. One possibility is to use the MTP for additional

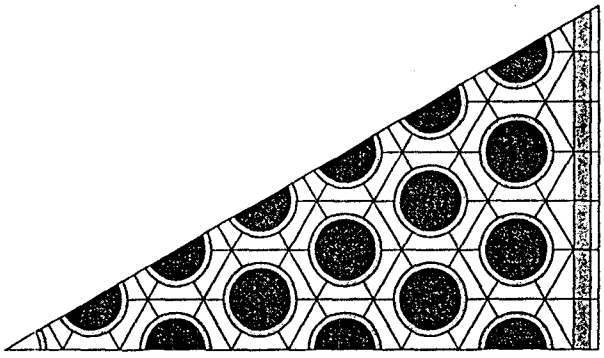

Figure 2: Geometry model of a 30° sector of a WWER-440 assembly

outer iterations without the MOC-integration. It gives positive results during the early stage of the outer iteration process, but is inefficient when the errors become small. One reason for this could be the fact, that the MTP does not account for the changes in the current, which determines the effects of the linear source and P_1-scattering. In any case a more powerful PC is needed to study this problem.

Test calculations

Results of a sensitivity study of the dependence of k_∞, the pin-wise power peaking factor, and the CPU time (for a 350 MHz Pentium II PC) on the method of spatial approximation of the source (flat or linear source), the cross sections, the number of azimuthal angles, the number of polar levels, and the track separation for a 30°-sector of a WWER-440 assembly are presented in Table 1. The geometry model is presented on Fig. 2. The reference case is with linear source, P_1-scattering, 5 azimuthal angles in the interval (0,30°), the optimal two level quadrature set from Ref. [12], and track separation of 0.1 cm. The effect of the linear source method on k_∞ is only 33 pcm, because there are no strong gradients. The effect of the P_1-scattering on k_∞ is also small, but the power peaking factor changes by 2%. In this case 4 azimuthal angles are sufficient, but 5 azimuthal angles are optimal. The error in k_∞ with 2 polar levels is only 9 pcm. The results with different track separations show that 0.1 cm is a reasonable choice.

In case of small core-reflector problems with dominant leakage the effect of the P_1-scattering is nearly 1% in k_∞. 5 azimuthal angles are sufficient, but the error with 2 polar levels is about 0.2% and 3 polar levels has to be used.

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Table 1: Sensitivity studies for a 30° sector of a WWER-440 fuel assembly

Method	Cross sections	Azimuthal angles	Polar levels	Track separation [cm]	k_∞	Peaking factor	CPU [s]
LSA	Σ^{s1}	5	2	0.100	1.20769	1.123	19
FSA	Σ^{s1}	5	2	0.100	+.00033	-.005	15
LSA	Σ^{s0}	5	2	0.100	-.00066	+.020	21
LSA	Σ^{s1}	3	2	0.100	-.00030	-.008	11
LSA	Σ^{s1}	4	2	0.100	-.00011	+.003	14
LSA	Σ^{s1}	9	2	0.100	+.00000	+.001	27
LSA	Σ^{s1}	5	3	0.100	+.00014	-.001	23
LSA	Σ^{s1}	5	4	0.100	+.00009	-.001	28
LSA	Σ^{s1}	5	2	0.400	+.00003	+.017	7
LSA	Σ^{s1}	5	2	0.200	-.00001	-.004	12
LSA	Σ^{s1}	5	2	0.050	+.00001	+.002	28
LSA	Σ^{s1}	5	2	0.020	+.00001	+.001	64

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