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ANALYTICAL SOLUTION FOR BEAM WITH TIME-DEPENDENT BOUNDARY CONDITIONS VERSUS RESPONSE SPECTRUM

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ABSTRACT

This paper studies the responses of a uniform simple beam for which the supports are subjected to time-dependent conditions. Analytical solution in terms of series was presented for two cases: (1) Two supports of a simple beam are subjected to a harmonic motion, and (2) One of the two supports is stationary while the other is subjected to a harmonic motion. The results of the analytical solution were investigated and compared with the results of conventional response spectrum method using the beam finite element model. One of the applications of the results presented in this paper can be used to assess the adequacy and accuracy of the engineering approaches such as response spectra methods. It has been found that, when the excitation frequency equals the fundamental frequency of the beam, the results from response spectrum method are in good agreement with the exact calculation. The effects of initial conditions on the responses are also examined. It seems that the non-zero initial velocity has pronounced effects on the displacement time histories but it has no effect on the maximum accelerations.

1. INTRODUCTION

Frequently there is a need to analyze problems of transverse vibration of beams subjected to time-dependent support excitations. One of the typical examples of this type of problem is a beam-like structure, such as piping system, subjected to earthquake ground motion. Generally speaking, in the nuclear power plant design, calculation of responses of a piping system or equipment subjected to support excitations due to dynamic loads may fall into this category of problem.

The methods commonly used for the analysis of this class of problem are the single response spectrum method [1, 2], the multiple response spectrum method [3, 4, 5], and the time history method [6, 7]. These methods are approximate and convenient engineering approaches for which computer programs have been developed. All these methods need development of a discrete meshing model (or a lumped mass model) and numerical schemes for solution. In the development of a discrete meshing model, a question that generally arises is how many lumped masses or nodes or modes should be considered to calculate the responses adequately or reliably. The answer to this question is that it depends on the input, the responses, the problem and the solution. The paradox is that the solution is not available, because it is what we are looking for. Thus, there is a need to find some basic problems in this category where analytical methods can be applied in order to find analytical solution by which the results can be compared with those obtained by engineering approaches listed above.

To our knowledge, the analytical solution for this type of problem seems to be simple but it is not readily available in any standard textbooks on structural dynamics. The closest one that is in the classical textbook on engineering vibration, for example, is in [10]. It presents an example for which the middle point of a simple beam is performing a given motion. The case of a cantilever beam for which the free end is subjected to a prescribed motion can also be found in [10]. For a simple beam whose supports subjected to a harmonic motion, some results were presented in [11]. It is one of the objectives of this paper to find the solution and to show the results of a simple beam for which one of the two supports is stationary while the other is subjected to a harmonic motion.

In Section 2 of this paper, the solution of a uniform beam subjected to a time-dependent boundary condition is presented. The procedures for finding the analytical solution for a typical problem are also demonstrated. In Section 3, numerical results are obtained for a beam subjected to harmonic support excitations with various frequencies. In Section 4, results from the previous section are compared with those obtained by the conventional engineering approach - response spectrum method.

In sections 3 and 4 it is shown how the analytical solution results reveal some interesting behaviors of response of a continuous system. It seems apparent that the complexity of response of a continuous system cannot always be produced by the conventional engineering approach based on simplification and dynamic systems with finite degrees of freedom. This shall not be interpreted as criticizing the inappropriateness of the conventional engineering approach that has its merit in convenience in design. Frequently, engineers are challenged by the questions regarding the discrepancies between the anticipated actual behavior and the calculated results based on engineering calculations that involve many assumptions and approximations. An inside understanding may help to resolve the discrepancies that are encountered in engineering calculations and analyses. One of the applications of the results presented in this paper can be used to assess the adequacy and accuracy of the engineering approaches such as response spectra methods.

2. FORMULATION AND SOLUTION

The equation of motion for transverse vibrations of a beam neglecting shear deformation and rotary inertia is given in basic theory [1] as:

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}+C_s I\frac{\partial^3 y}{\partial x^2 \partial t}\right)+\beta\frac{\partial y}{\partial t}+\rho A\frac{\partial^2 y}{\partial t^2}=p(x,t) \qquad (1)$$

In Eq. (1), t is the time, x the coordinate measured along the beam axis, E the modulus of elasticity, A the cross-sectional area of the beam, I the moment of inertia of the beam, ρ the mass density of the beam, C_s the coefficient of viscosity of the beam material, β the coefficient of viscous resistance to transverse motion of the beam, $y(x, t)$ the transverse deflection of the beam axis and $p(x, t)$ the external load.

For a beam supported at x=0 and x=ℓ, various possible time-dependent boundary conditions may have the following form:

$$y(0,t)=f_1(t) \qquad y(l,t)=f_5(t) \qquad (2)$$

$$\frac{\partial y}{\partial x}\Big|_{x=0}=f_2(t) \qquad \frac{\partial y}{\partial x}\Big|_{x=l}=f_6(t)$$

(3)

$$M(0,t) = -EI\frac{\partial^2 y}{\partial x^2}\Big|_{x=0} = f_3(t) \qquad M(l,t) = -EI\frac{\partial^2 y}{\partial x^2}\Big|_{x=l} = f_7(t) \tag{4}$$

$$V(0,t) = -EI\frac{\partial^3 y}{\partial x^3}\Big|_{x=0} = f_4(t) \qquad V(l,t) = -EI\frac{\partial^3 y}{\partial x^3}\Big|_{x=l} = f_8(t) \tag{5}$$

Any two of the conditions at $x=0$ and any two of the conditions at $x=1$ furnish four conditions. When a beam is subjected to support excitation without external load, Eq. (1) is a homogeneous differential equation. Since the boundary conditions are time-dependent, the separation of variable technique, normally used for solving the partial differential equation, will encounter difficulties. However, by changing the variables such that the boundary conditions, in terms of the new variable, become homogeneous, the equation of motion becomes non-homogeneous for which the general solution consists of a homogeneous part and a particular part. The procedures are similar to the ones developed in Ref. [8].

Consider a beam with hinged ends subjected to the same support excitation at both ends given by:

$$y(0,t)=y(l,t)=f(t) \tag{6}$$

For hinged ends, at x=0 and x=ℓ we have:

$$\frac{\partial^2 y(0,t)}{\partial x^2} = \frac{\partial^2 y(l,t)}{\partial x^2} = 0 \tag{7}$$

Initial conditions are:

$$y(x,0) = 0, \qquad \dot{y}(x,0) = v_0. \tag{8}$$

Introduce a new variable u(x, t) such that: $\quad y(x,t) = f(t)+u(x, t)$

where $$u(x,t) = \sum_{n=1}^{\infty} q_n(t) Y_n(x) \tag{9}$$

To satisfy the boundary conditions (6) and (7), we select: $$Y_n(x) = \sin(\lambda_n x) \tag{10}$$

Where: $$\lambda_n = \frac{n\pi}{l} \tag{11}$$

For metal, the coefficient of viscosity C_s is small and, in this study, assumed to be zero. After substitution of Eqs. (9) and (10), the equation of motion, becomes:

$$\sum_{n=1}^{\infty}[q_n(t)\lambda_n^4 + \frac{\beta}{EI}\dot{q}_n(t) + \frac{\rho A}{EI}\ddot{q}_n(t)]Y_n(x) = \frac{-\rho \cdot A}{EI}\ddot{f}(t) - \frac{\beta}{EI}\dot{f}(t) \tag{12}$$

Application of orthogonal conditions to Eq. (12) gives:

$$\ddot{q}_n(t) + 2\omega_n \varsigma \dot{q}_n(t) + \omega_n^2 q_n(t) = A_n(t) \tag{13}$$

where $$\omega_n^2 = \frac{EI\lambda_n^4}{\rho A}, \tag{14}$$

$$\varsigma_n = \frac{\beta}{2\rho A \omega_n}, \tag{15}$$

$$A_n(t) = -\frac{4}{n\pi}[\ddot{f}(t) + 2\omega_n \varsigma_n \dot{f}(t)] \tag{16}$$

When $f(t)$ is a given function, then $A_n(t)$ can be found by the above expression. Now the problem becomes equivalent to a SDOF system with natural frequency ω_n and having damping coefficient β subjected to a forcing function $A_n(t)$ given by (16). The solution is:

$$q_n(t) = \frac{1}{\omega_d}\int_0^t A_n(\tau)\cdot e^{-\varsigma_n\omega_n(t-\tau)}\cdot \sin[\,\omega_d(t-\tau)]\cdot d\tau \tag{17}$$

where $\omega_d = \omega_n\sqrt{1-\varsigma^2}$

After $q_n(t)$ is obtained from Eq. (17), then:

$$y(x,t) = f(t) + \sum_{n=odd} q_n(t)\cdot \sin(\frac{n\pi x}{l}) \tag{18}$$

The same procedure with slight modifications is also applicable to the case for which the excitations at the two supports are different time-dependent functions.

3. RESULTS OF HARMONIC SUPPORT EXCITATION

3.1 Two supports of a simple beam subjected to same motion

A special case with both supports subjected to a harmonic motion given by:

$$f(t) = A_{spt}\cdot \sin(\omega_f t) \tag{19}$$

where A_{spt} is the amplitude and ω_f is the frequency of the motion, is chosen as an example. For simplification, A_{spt} is assumed to be 1. After substitution, Eq. (13) becomes:

$$\ddot{q}_n(t) + 2\omega_n\varsigma\dot{q}_n(t) + \omega_n^2 q_n(t) = \frac{4\omega_f^2}{n\pi}\sin(\omega_f t) - \frac{8\varsigma_n\omega_n\omega_f}{n\pi}\cos(\omega_f t) \tag{20}$$

for which the solution is:

$$\begin{aligned} q_n(t) = {} & C_1\{\cos(\omega_f t) - e^{-\varsigma_n\omega_n t}\cdot[\cos(\omega_d t) + \frac{\varsigma\omega_n}{\omega_d}\sin(\omega_d t)]\} \\ & + C_2[\sin(\omega_f t) - e^{-\varsigma_n\omega_n t}\cdot \sin(\omega_d t)] \end{aligned} \tag{21}$$

It satisfies the initial conditions: $q_n(0)=\dot{q}_n(0) = 0$. In Eq. (21). C_1 and C_2 are determined by Equation (20). Thus, we have

$$C_1 = \frac{-8\varsigma_n\omega_n^3\omega_f}{n\pi D} \tag{22}$$

and

$$C_2 = \frac{4\omega_f^2}{n\pi D}[(1-4\varsigma_n^4)\omega_n^2 - \omega_f^2] \tag{23}$$

where

$$D = (\omega_n^2 - \omega_f^2)^2 + 4(\varsigma_n\omega_n\omega_f)^2 \tag{24}$$

The initial condition $q(0)=0$ corresponding to $y(x,0)=0$, this means that the beam is initially not deformed; the initial $\dot{q}_n(0)=0$ corresponding to $\dot{y}(x,0) = \dot{f}(0)$, this implies that the beam is subjected to a velocity $A_{spt}{*}\omega_f$ at t=0sec. Eq. (20) is similar to a damped single degree freedom oscillator subjected to a harmonic forcing function. A resonance occurs when $\omega_f = \omega_n$ for n=1, 3, 5,... To study the responses, we consider a beam as shown in Figure 1 for which the first natural frequency is f_1 =1.405 Hz. The time history of the displacement for the middle point of the beam with $\omega_f = \omega_1 = 2\pi f_1$ is shown in Figure 2. The displacement time histories at the same point of the beam for the $\omega_f = \sqrt{3}$, 4 and 9 times ω_1 are shown in Figures 3, 4, and 5, respectively. The damping value in terms of percent of critical damping is $\xi = 0.02$. When the frequency of the excitation motion equals the first natural frequency, the steady state response at the center of the beam is, from Eqs. (21), (22), (23), and (24), approximately given by:

$$y_{\max} = \frac{2}{\pi * \varsigma} \tag{25}$$

The envelope of the peaks in Fig. 2 approaches this value. For $\xi = 0.02$, the maximum deflection at the center of the beam can reach 31.8 times the amplitude of the support motion. The resonance is clearly demonstrated in Figure 2. It takes about 20 cycles of support excitations to reach the amplitude that is 30 times the amplitude of the excitation motion.

Properties of Beam: $A = 50\ \text{cm}^2$ $\quad \ell = 500\ \text{cm}$ $\quad \rho = 0.784\ \text{kg/cm}^3$
$I = 1000\ \text{cm}^4$ $\quad E = 2.\text{x}10^6\ \text{kgf/cm}^2$

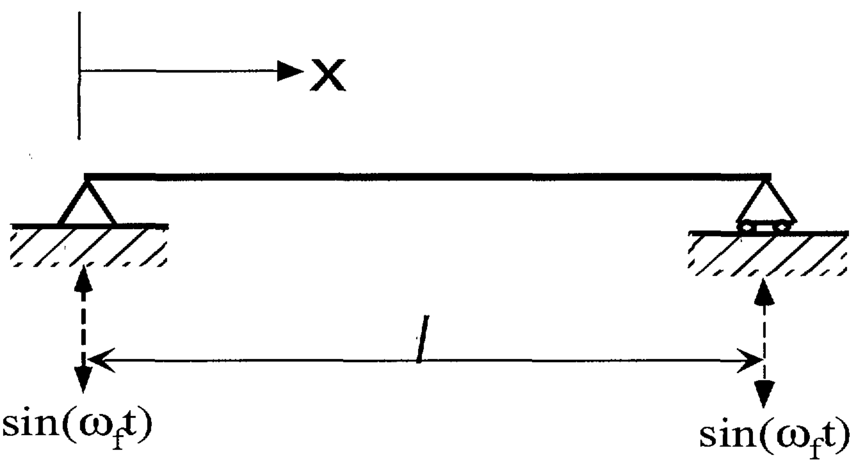

Figure 1: Both Supports of a Simple Beam Subjected to a Harmonic Motion

When the frequency of the excitation is not the same as the fundamental frequency of the beam, the maximum responses are reached at the beginning while the envelopes are gradually decreasing. When the frequency of the excitation coincides with higher natural frequencies of the beam, the steady state envelope values are higher than those for which the excitation frequencies do not coincide with any of the natural frequencies of the beam. In this set of calculations, ξ is assumed to be constant for all frequencies. This implies that the damping coefficient β increases with the frequencies. On the other hand, if β is considered to be constant, then ξ is inversely proportional to ω_n. For $\beta = 0.02$ with the same set of excitation

$$y_{\max} = \frac{2\omega_n \rho A}{\pi * \beta} \tag{26}$$

For $\beta = 0.02$, y_{max} is 22.48 times the amplitude of the support motion. When the frequency of the excitation is not the same as the fundamental frequency of the beam, the behavior of the response is quite similar to that for ξ equal to a constant. When the frequency of the excitation coincides with the natural frequency of a higher mode, the amplification is higher than the case for which ξ is considered as a constant.

The damping coefficient β may depend on frequency in other forms than those discussed above. In Ref. [9] the nature of some important damping mechanisms is discussed and an indication is given of how the damping depends on the amplitude and frequency of the cyclic motion.

The deflection curves at various times for $\omega_f = \omega_1$ are shown in Figure 6. Similar curves for $\omega_f = \sqrt{3}$, 4 and 9 times ω_1 are shown in Figures 7, 8 and 9. The maximum amplitude of the deflection curve for the resonance case (i.e., Figure 6) is larger than those of non-resonance cases (i.e., Figures 7 and 8). It is clear in Figure 6, 7 and 8, that the deflection curves are predominately made of the first mode of the beam. When the frequency of the support motion is in resonance with the third mode of the beam, the contribution of the third mode of the beam is clearly seen in Figure 9, when t is about 3.6 sec.

3.2 One of the two supports of a simple beam subjected to a harmonic motion

Properties of Beam: $A = 50$ cm^2 $\ell = 500$ cm $\rho = 0.784$ kg/cm^3
$I = 1000$ cm^4 $E = 2.\text{x}10^6$ kgf/cm^2

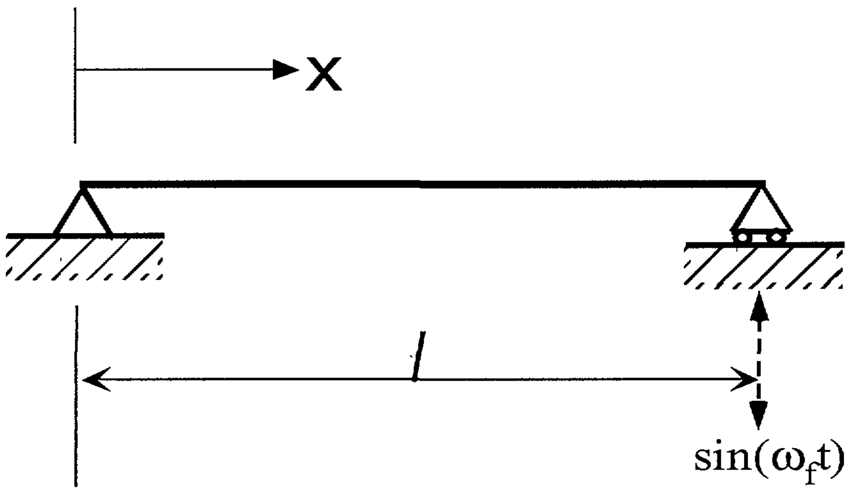

Figure 10: Right Support of a Simple Beam Subjected to a Harmonic Motion

We consider the same simply supported beam presented in 3.1 except that the support at $x=0$ is not moving but the one at $x=l$ (see Figure 10) is subjected to a harmonic motion defined by:

$$f(t) = \sin(\omega_f t) \tag{27}$$

where ω_f is the frequency of the harmonic motion. For the present case, the boundary conditions are:

$$y(0,t) = 0 \tag{28}$$

$$y(l,t) = f(t) \tag{29}$$

Introducing a new variable u(x, t) such that

$$y(x,t) = f(t)\cdot\frac{x}{l} + u(x,t) \tag{30}$$

where

$$u(x,t) = \sum_{n=1}^{\infty} q_n(t) Y_n(x) \tag{31}$$

It can be shown that the boundary conditions (28) and (29) are satisfied with the same $Y_n(x)$ as the one defined by Eq. (10).

The equation of motion, after substitution of Eqs. (30) and (31), becomes:

$$\sum_{n=1}^{\infty}[q_n(t)\lambda_n^4+\frac{\beta}{EI}\dot{q}_n(t)+\frac{\rho A}{EI}\ddot{q}_n(t)]Y_n(x)=\frac{-\rho A}{EI}\cdot\frac{x}{l}\ddot{f}(t)-\frac{\beta}{EI}\cdot\frac{x}{l}\dot{f}(t) \quad (33)$$

It has the same form as Eq. (13) after applying orthogonal conditions, except the right hand side of the equal sign is replaced with

$$B_n(t)=(-1)^n\frac{2}{n\pi}[\ddot{f}(t)+2\omega_n\varsigma_n\dot{f}(t)] \quad (34)$$

In terms of given *f(t)*, the solution to Equation (33) is:

$$q_n(t)=C'_1\{\cos(\omega_f t)-e^{-\varsigma_n\omega_n t}\cdot[\cos(\omega_d t)+\frac{\varsigma\omega_n}{\omega_d}\sin(\omega_d t)]\} + C'_2[\sin(\omega_f t)-e^{-\varsigma_n\omega_n t}\cdot\sin(\omega_d t)] \quad (35)$$

The initial conditions $q_n(0)=\dot{q}_n(0)=0$, are identically satisfied. In Eq. (35), C'_1 and C'_2 are determined by Eq. (33). Thus, we have

$$C'_1=\frac{(-1)^n 4\varsigma_n\omega_n^3\omega_f}{n\pi D} \quad (36)$$

and

$$C'_2=\frac{(-1)^{n+1}2\omega_f^2}{n\pi D}[(1-4\varsigma_n^4)\omega_n^2-\omega_f^2] \quad (37)$$

where D is the same as Eq. (24) and $n=1,2,3,\ldots\ldots$

The time history of the displacement for the middle point of the beam with $\omega_f = \omega_1$, is shown in Figure 11. The time histories for the same point of the beam for $\omega_f = \sqrt{3}$, 4, and 9 times ω_1 are shown in Figures 12, 13, and 14, respectively. The resonance of this case is similar to the resonance of the case in 3.1 as shown in Figure 2 except the magnitude of the amplitude is about 50% of the response of the beam excited by the same harmonic motion on both supports.

The deflection curves at various times for $\omega_f = \omega_1$ are shown in Figure 15. Similar curves for $\omega_f = \sqrt{3}$, 4 and 9 times ω_1 are shown in Figures 16, 17 and 18. Unlike the previous case for which the deflection curves are symmetric and consist of mode shapes with odd integers, the deflection curves for the current case are made of mode shapes with both odd and even integers. In Figure 17, the second mode is seen when time equal to about 3.6sec. When the frequency of the support motion is in resonance with the third mode of the beam, the contribution of the third mode of the beam is seen in Figure 18, when t is about 7.7sec.

3.3 Effects due to Initial Conditions

In order to understand the effects on the response due to initial conditions, the same example used 3.1 is considered except the support motion is defined by the following time dependent functions:

$$F(t) = A_{spt}\cdot\frac{t}{T_f}\sin(\varpi_f t), \quad \text{when } t \le T_f$$

And

$$F(t) = A_{spt}\cdot\sin(\varpi_f t), \quad \text{when } t > T_f.$$

In the above expressions, T_f (=$2\pi/\omega_f$) is the period of the harmonic motion of the support. With support motion defined above, when t=0, $\dot{F}(0)=0$, the initial condition $\dot{q}(0)=0$, corresponding to $\dot{y}(x,0)=0$, means that the beam is initially at rest. This condition is more realistic in the practical situations.

Following similar procedures as those given in 3.1, the responses in terms of series solution can be found. The time history of the displacement for the middle point of the beam with $\omega_f = \omega_1$, the resonance case, for this example (zero initial conditions) is plotted as solid curve in Figure 19 together with the time history, shown as dashed line, of the displacement for the same point of the beam calculated in 3.1 (initial velocity of the beam not zero). As expected, these two curves are almost identical except in the beginning of the time histories. For a non-resonance case, when $\omega_f = 4*\omega_1$, the corresponding two time histories are plotted in Figure 20. The effects of initial conditions on the displacement time histories are clearly shown. For various excitations with f_f = 1.405Hz, 10Hz, 12.5Hz and 35.6Hz, the maximum accelerations for the zero initial conditions and those for the non-zero initial conditions, y(x,0) =0, $\dot{y}(x,0) = A_{spt} \cdot \varpi_f$, are practically the same as shown in Table 7. It seems that the initial conditions affect more on the displacement time histories than on the responses in terms of maximum accelerations.

4. COMPARISON OF RESULTS OBTAINED BY RESPONSE SPECTRUM METHOD

The same beam as shown in Fig. 1 is modeled by 10 equal elements. Elementary Euler beam elements were used in the model. A response spectrum method using uniform support motion adopted in a finite element program, was applied to find the responses of the beam. To apply the response spectrum method, the input response spectrum for the support excitation defined by $sin(\omega_f t)$ is calculated according to the following relation:

$$S_d = \sqrt{\frac{1+(2\varsigma\omega_f/\omega_n)^2}{[1-(\omega_f/\omega_n)^2]^2+(2\varsigma\omega_f/\omega_n)^2}} \quad (38)$$

This spectrum in terms of base displacement defined by $sin(\omega_f t)$ is based on steady state. The use of steady state response spectrum should produce upper bound of the responses.

The first nine frequencies, participation factors, modal displacement and spectra values calculated by the finite element program, are summarized in Table 1. For excitation frequency equal to the first natural frequency of the beam, the maximum displacement and moment at center of the beam and the maximum shear force at either end of the beam are summarized in Table 2. These results are in good agreement with those calculated by the exact method shown in Section 3. As expected, this good agreement is because the response spectrum method being used here produces the steady state responses of the beam. For excitation frequency that is different from the first natural frequency of the beam, for example, f_f = 10 Hz, the maximum displacement and moment at the center of the beam and the maximum shear force at either end of the beam are summarized in Table 3. The results calculated by the response spectrum method are lower than the steady state responses calculated by exact solution derived in Section 3 by about 5%, 20% and 40% for maximum displacement, moment and shear, respectively. For other excitation frequencies, the results calculated by these two methods are compared in Tables 4 and 5. When excitation frequency coincides with the natural frequency of the third mode of the beam, the response spectrum method predicts the results lower by about 14%, 2% and 18% for the maximum displacement, moment and shear, respectively. When excitation frequency coincides with the 5th natural frequency of the beam, the response spectrum method predicts the results lower by about 78%, 15% and 80% for the maximum displacement, moment and shear, respectively.

For various excitation frequencies (f_f = 1.405 Hz, 10 Hz, 12.5 Hz and 35.37 Hz), the maximum accelerations calculated by the relations derived in Section 3 are compared with those obtained by response spectrum method using finite element program. The comparison is shown in Table 6.

For the case that two supports of the simple beam are subjected to the same harmonic motion, the comparison indicates that, when the excitation frequency equals the fundamental frequency of the beam, the maximum acceleration from the exact calculation corresponding to the steady state is in good agreement with the maximum value obtained by the response spectrum method using SRSS rule for combining the contributions from various modes. This good agreement is expected, because the response spectrum method produces the steady state responses of the beam. SRSS) does not recognize this. For excitation frequency not equal to any of the natural frequencies of the beam, the maximum acceleration for transient may be higher than that of the steady state according to the exact solution. For example, when excitation frequency is 10 Hz, which does not coincide with any of the natural frequencies of the beam, the maximum acceleration calculated by the response spectrum method over-predicts the steady state result by about 32%. For excitation frequency equal to the natural frequency of the third mode of the beam, the maximum acceleration of the steady state value based on the exact solution shown in 3.1 is about 9% above the steady state maximum acceleration calculated by the response spectrum method using SRSS rule. For excitation frequency equal to the natural frequency of the fifth mode of the beam, the maximum acceleration for steady state is about the same as that for the transient state according to the exact solution. However, the maximum acceleration calculated by the response spectrum method under-predicts the maximum value obtained from the exact calculation by about 24%. The discrepancies found above may be caused by the coarseness of the discrete meshing of the model. The use of high order finite elements may show improvement.

For the case that one of the two supports of the simple beam is subjected to a harmonic motion, the comparison indicates that the maximum responses from the exact calculation are lower than the corresponding maximum values obtained by the response spectrum method based on uniform support motion and using SRSS rule for combining the contributions from various modes.

5. CONCLUSION

Analytical solution for a beam subjected to harmonic support excitations is developed. The approach is straightforward after the time-dependent boundary condition problem is converted to a conventional boundary value problem by introducing of new variables or functions. It is applicable for beams subjected to other forms of excitation than harmonic motion.

The results for a sample problem considered herein are derived in analytical form without approximation by which the results obtained by other engineering approximate approach can be compared to test for adequacy and accuracy.

The results for various damping relations are discussed. By investigating how the responses vary with damping and frequencies tests can be performed to produce data by which the measured responses can be compared with the calculated results to determine the frequency dependent damping coefficients.

An examination of the time histories of the responses presented in Section 3 for various excitation frequencies indicates that the conventional engineering approach such as response spectrum method can produce only limited information such as steady state responses. When the excitation frequency equals the fundamental frequency of the beam, the results from response spectrum method are in good agreement with the exact calculation. However, when the support excitation frequencies are not the same as the first natural frequency of the beam, the results from the response spectrum method can either over-predict or under-predict the responses calculated by the exact solution. We shall not categorically accept the idea that the response spectrum method always produces conservative results. With the results obtained by exact solution such as those discussed here, the responses calculated by engineering approaches, for example, response spectrum method or the direct integration method can be compared to get better understanding in order to judge whether the responses are real ones or simply numerical inaccuracy caused by numerical calculations.

The effects of initial conditions on the responses are also examined. It seems that the non-zero initial velocity has pronounced effects on the displacement time histories but it has no effect on the maximum accelerations.

When the two supports of a beam are subjected to two different time-dependent functions, the approach presented herein can be used to assess the adequacy and accuracy of the approaches to calculate responses of structural systems subjected to multiple support response spectra input [4, 5]. The superposition of the results in 3.1 and 3.2 can produce the responses of a beam for which two supports are subjected to two different time-dependent functions.

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Table 1
Dynamic Characteristics for Beam Shown in Figure 1
Calculated by Response Spectrum Method

Mode Num	Freq. (Hz)	Displace. at Center of Beam	Modal Participation Factor Left Sup.	Modal Participation Factor Right Sup.
1	1.405	0.3162	-1.9966	-1.9966
2	5.619	0	0.9733	-0.9733
3	12.636	0.3162	0.6206	0.6206
4	22.424	0	-0.435	0.435
5	34.869	0.3160	0.3162	0.3162
6	49.64	0	-0.2298	0.2298
7	65.887	0.3160	0.1611	0.1611
8	81.74	0	0.1028	-0.1028
9	93.94	0.3162	-0.05	-0.05

Table 2
Maximum Displacement, Moment and Shear
$f_f = f_1 = 1.405$ Hz

Mode Num	Displace Resp. Spec. Value at Support	Max. Disp. at Center of Beam (cm)	Max. Mom. (cm-kgf)	Max. Shear (kgf)
1	-99.848	31.57	2.51×10^6	1.55×10^4
2	0	0	0	0
3	0.2376	-0.0751	5.74×10^4	932
4	0	0	0	0
5	0.12105	0.0383	9.19×10^4	1837
6	0	0	0	0
7	0.0617	-0.0195	-105×10^5	1700
8	0	0	0	0
9	-0.019	0.00606	5.4×10^4	334
SRSS Combined		31.57	2.52×10^6	1.58×10^4
Max. Values from Exact Solution		(1) 31.83	(1) 2.51×10^5	(1) 1.58×10^4

NOTE (1): The maximum values are reached when t approaches infinite.

Table 3
Maximum Displacement, Moment and Shear
$f_f = 10$ Hz

Mode Num	Displace Resp. Spec. Value at Support	Max. Disp. at Center of Beam (cm)	Max. Mom. (cm-kgf)	Max. Shear (kgf)
1	-4.106	1.2985	1.034×10^5	638.9
2	0	0	0	0
3	2.088	-0.6604	5.049×10^5	8170
4	0	0	0	0
5	0.057	0.018	-4.33×10^4	860.5
6	0	0	0	0
7	0.00783	-0.0025	1.227×10^4	216.3
8	0	0	0	0
9	-0.00123	0.0004	3.083×10^3	21.5
SRSS Combined		1.4569	5.049×10^5	8170
Exact	Transient (0-1 sec)	9.5	12.7×10^5	1.6×10^4
	Steady State	1.53	5.96×10^5	1.14×10^4

Table 4
Maximum Displacement, Moment and Shear
$f_f = 9f_1 = 12.65$ Hz

Mode Num	Displace Resp Spec. Value at Support	Max. Disp. at Center of Beam (cm)	Max. Mom. (cm-kgf)	Max. Shear (kgf)
1	-4.049	1.2804	1.019×10^5	630
2	0	0	0	0
3	30.811	-9.743	5.45×10^5	1.205×10^4
4	0	0	0	0
5	0.0978	0.0309	7.424×10^4	1485
6	0	0	0	0
7	0.01262	-0.004	2.154×10^4	348.5
8	0	0	0	0
9	-0.00193	0.00061	5447	33.07
SRSS Combined		9.8272	7.45×10^6	1.205×10^5
Exact	Transient (0-1 sec)	15.6	4.6×10^6	7.4×10^4
	Steady State	11.2	7.6×10^6	1.42×10^5

Table 5
Maximum Displacement, Moment and Shear
f_f=25 f_1 =33.2Hz

Mode Num		Displace Resp Spec. Value at Support	Max. Disp. at Center of Beam (cm)	Max. Mom. (cm-kgf)	Max. Shear (kgf)
1		-4.0	1.2648	$1.0x10^5$	622.3
2		0	0	0	0
3		1.4282	0.4516	$-3.45x10^5$	5587
4		0	0	0	0
5		14.466	4.5745	$1.098x10^7$	$2.196x10^4$
6		0	0	0	0
7		0.1285	-0.0406	$-2.193x10^3$	$3.548x10^3$
8		0	0	0	0
9		-0.0166	-0.00525	$4.688x10^7$	289.7
SRSS Combined			4.7678	$1.099x10^7$	$2.197x10^5$
Exact	Transient (0-1 sec)		32.63	$1.44x10^7$	$3.9x10^5$
	Steady State		8.5	$1.27x10^7$	$3.95x10^5$

Table 6
Comparison of Accelerations (in g's) for Various Frequencies of Support Motion

		$F_f = f_1$ = 1.405Hz	F_f = 10Hz	$F_f = 9f_1$ = 12Hz	$F_f = 25f_1$ =35.2Hz
Max. Value SRSS Combined		2.51	5.06	61.9	231
Two supports of a simple beam subjected to harmonic motion					
Exact	Trans (0-1s)	0.35	7.1	53	310
	Steady State	2.52	4.0	68.2	316
One support of a simple beam subjected to harmonic motion					
Exact	Trans (0-1s)	0.15	3.61	26.8	155
	Steady State	1.25	1.98	34	158

Table 7
Comparison of Max. Accelerations (in g's) of a simple beam with zero initial velocity and those of a beam with non zero initial velocity for Various Base Motion Frequencies

		$F_f = f_1$ = 1.405Hz	F_f = 10Hz	$F_f = 9f_1$ = 12Hz	$F_f = 25f_1$ =35.2Hz
Two supports of a simple beam subjected to harmonic motion with non-zero initial velocity					
Exact	Trans (0-1s)	0.35	7.1	53	310
	Steady State	2.52	4.0	68.2	316
Two supports of a simple beam subjected to harmonic motion with zero initial velocity					
Exact	Trans (0-1s)	0.18	7.1	53	310
	Steady State	2.52	3.96	68.1	316

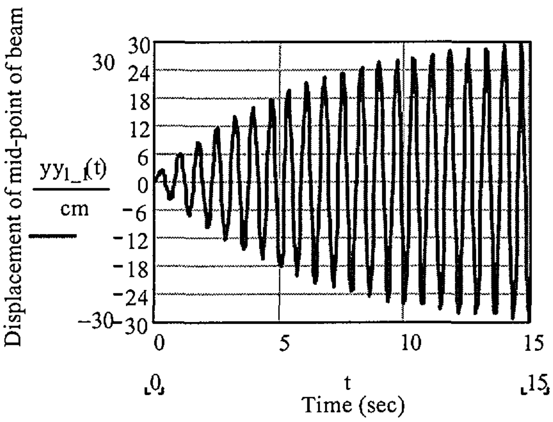

Figure 2: Displacement of mid-point of beam with Two supports excited by a harmonic motion f_f= 1.405Hz (1st natural frequency of beam, a resonance case)

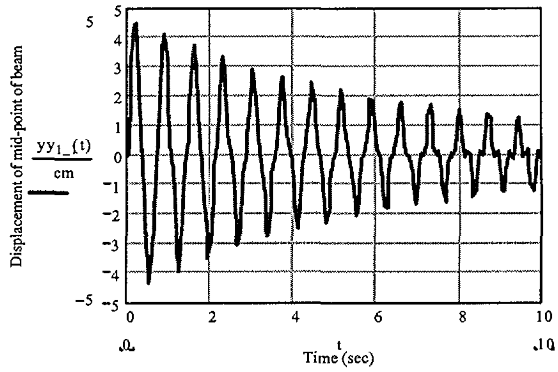

Figure 3: Displacement of mid-point of beam with Two supports excited by a harmonic motion f_f =4.15Hz (Not a resonance case)

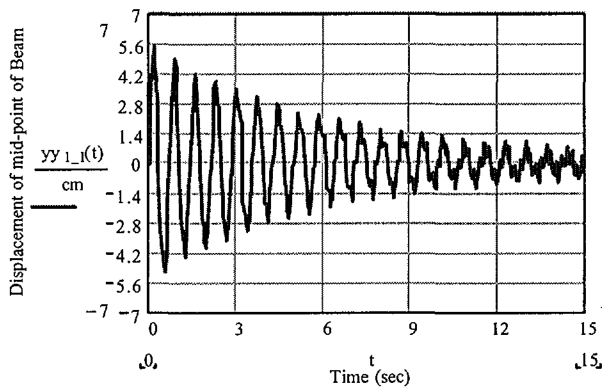

Figure 4: Displacement of mid-point of beam with Two supports excited by a harmonic motion f_f =4*1.405Hz (Not a resonance case)

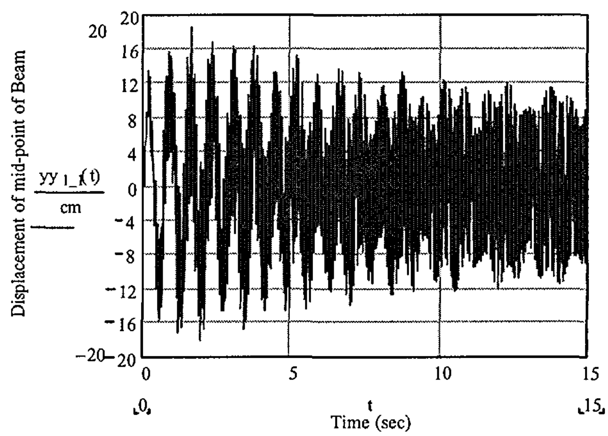

Figure 5: Displacement of mid-point of beam with Two supports excited by a harmonic motion f_f =9*1.405Hz (3rd natural frequency of beam, a resonance case)

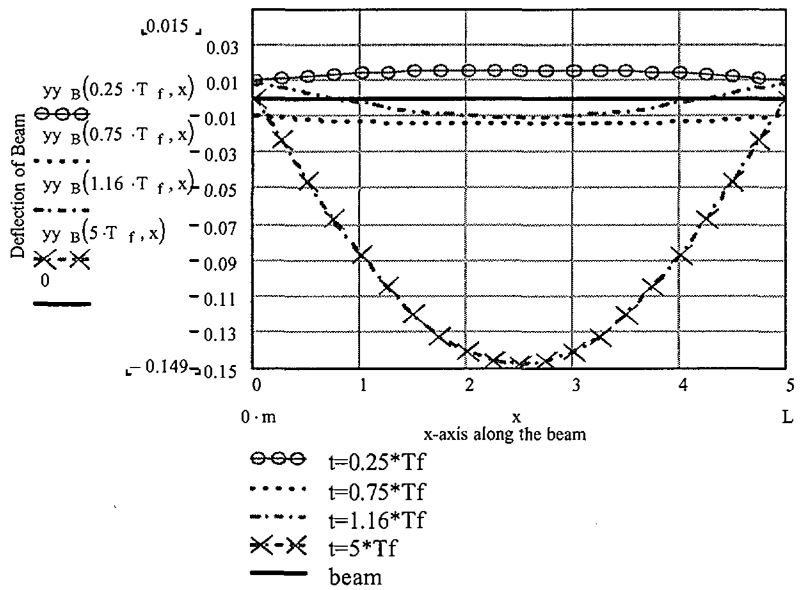

Figure 6: Deflection of Beam with Two supports subjected to A Harmonic Motion with f_f = 1.405Hz (1st natural frequency of beam, a resonance case)

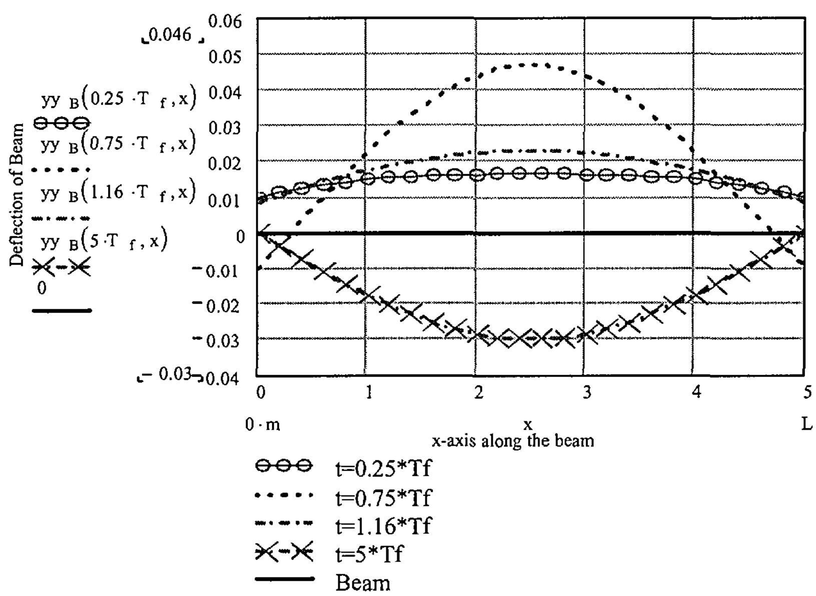

Figure 7: Deflection of Beam with Two supports subjected to A Harmonic Motion with f_f = 4.215Hz (Not a resonance)

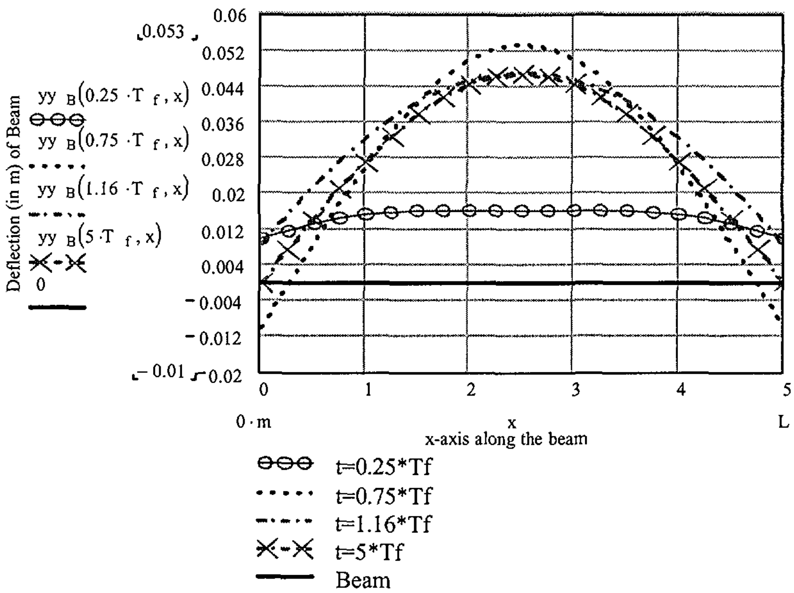

Figure 8: Deflection of Beam with two supports subjected to a Harmonic Motion with $f_f = 4*1.405Hz$ (Not a resonance case)

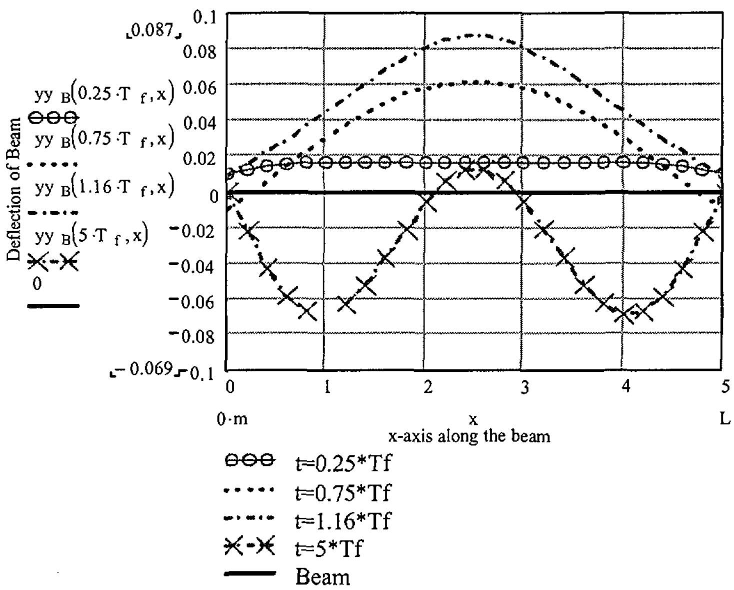

Figure 9: Deflection of Beam with two supports subjected toa Harmonic Motion with $f_f=$ 9*1.405Hz (3rd natural frequency of beam a resonance at higher mode of beam)

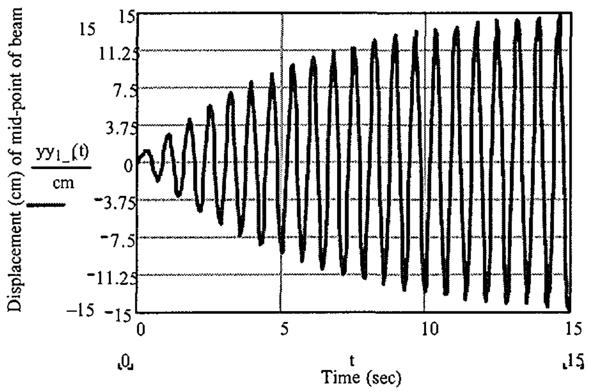

Figure 11: Displacement of mid-point of beam with one support being stationary while the other one excited by a harmonic motion $f_f = 1.405$Hz (1st natural frequency of beam, a resonance case)

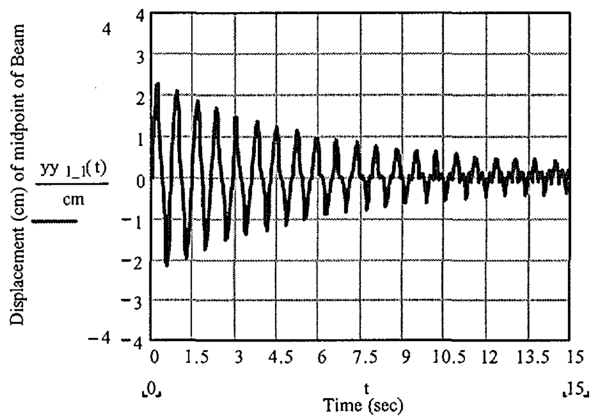

Figure 12: Displacement of mid-point of beam with one support being stationary while the other one excited by a harmonic motion $f_f = 4.215$Hz (Not a resonance case)

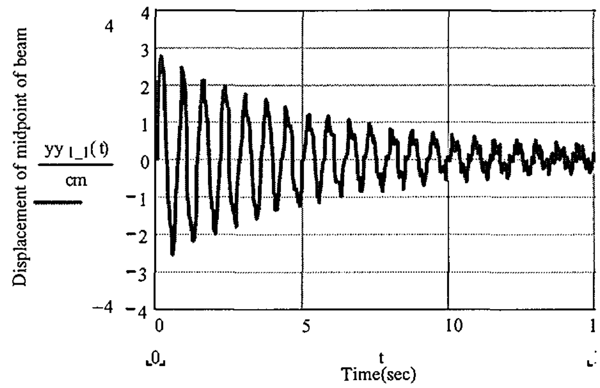

Figure 13: Displacement of mid-point of beam with one support being stationary while the other one excited by a harmonic motion $f_f = 4*1.405$Hz (Not a resonance case)

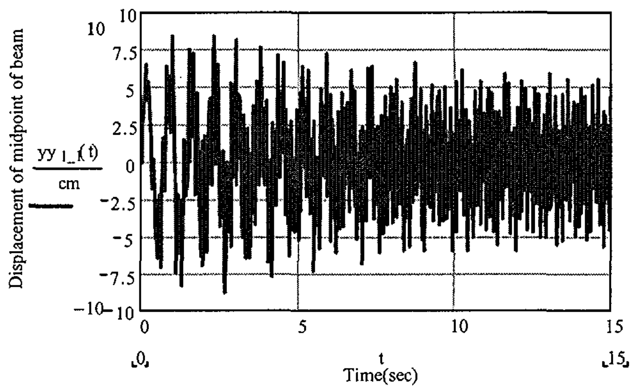

Figure 14: Displacement of mid-point of beam with one support being stationary while the other one excited by a Harmonic motion $f_f = 9*1.405$Hz (3rd natural frequency of beam, resonance at higher mode of beam)

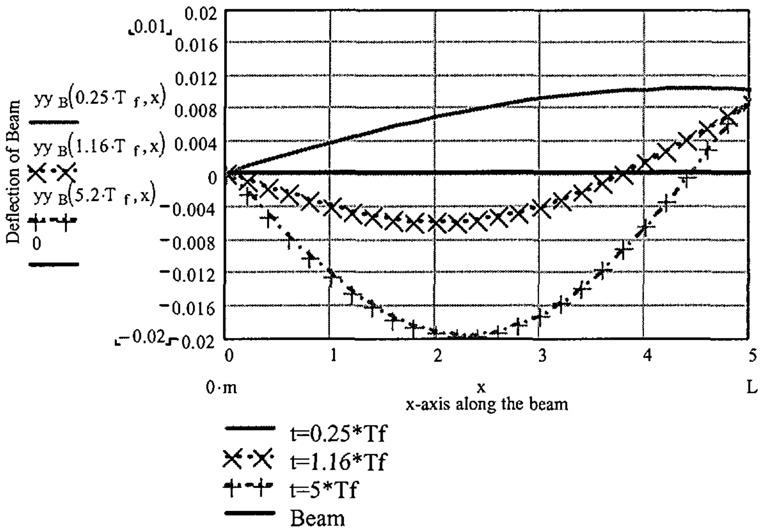

Figure 15: Deflection of Beam with <u>one support</u> being stationary while the other one excited by a harmonic motion

f_f = 1.405Hz (1st natural frequency of beam, a resonance case)

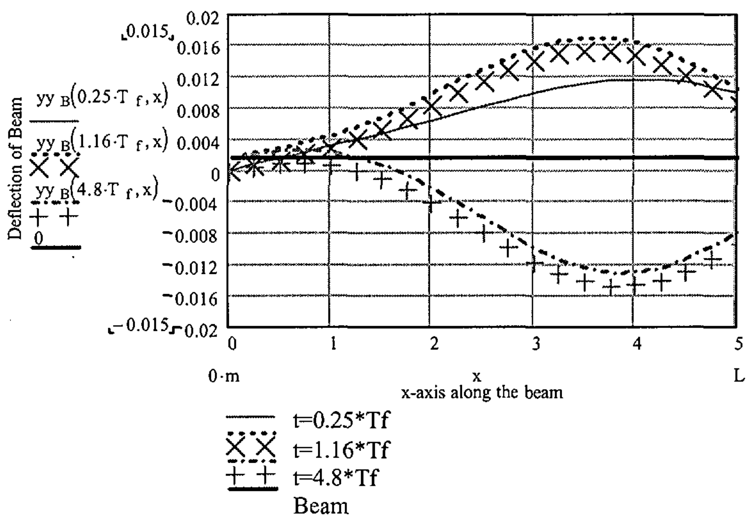

Figure 16: Deflection of Beam with <u>one support</u> being stationary while the other one excited by a harmonic motion

f_f = 4.215Hz (Not resonance case)

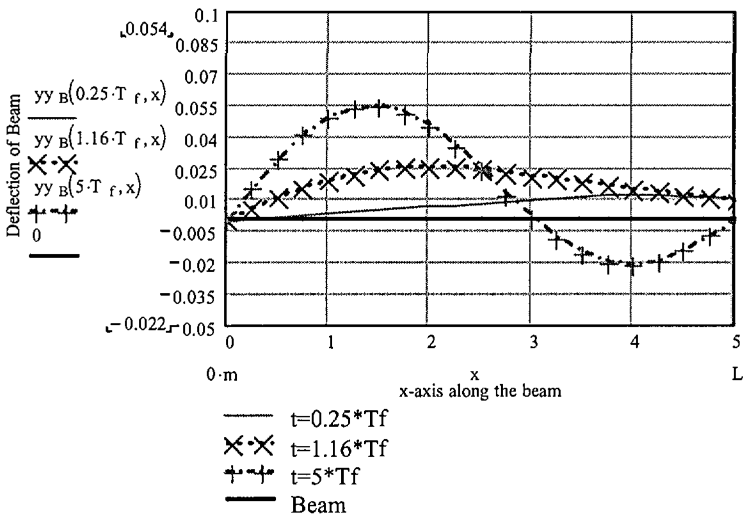

Figure 17: Deflection of Beam with one support being stationary while the other one excited by a harmonic motion $f_f = 4*1.405$Hz (2nd natural frequency of beam, resonance at higher mode of beam)

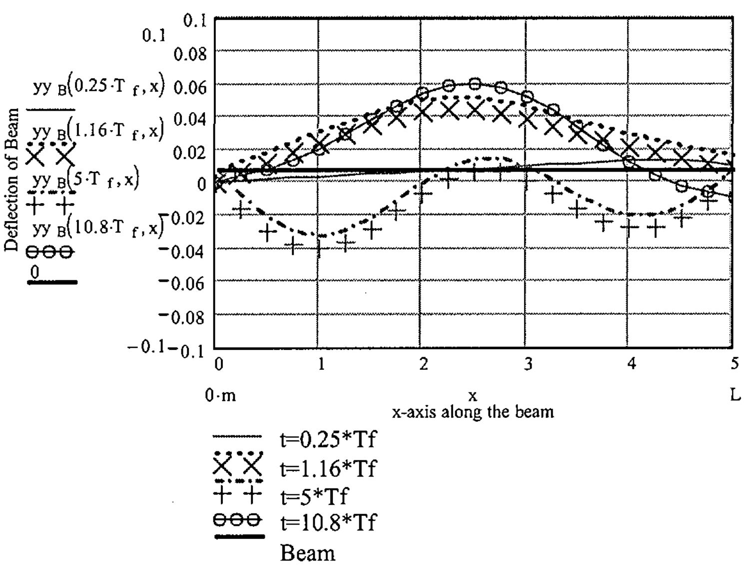

Figure 18: Deflection of Beam with one support being stationary while the other one excited by a Harmonic motion $f_f = 9*1.405$Hz (3rd natural frequency of beam, resonance at higher mode of beam)

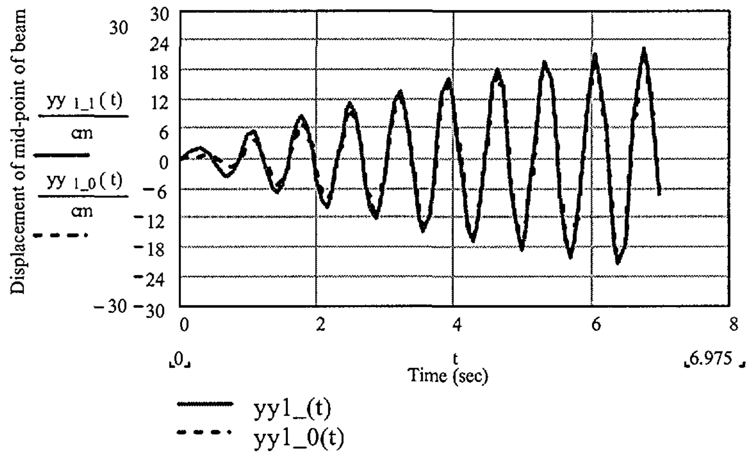

Figure 19: Comparison of two time histories, one with zero initial velocity and the other with non-zero initial velocity, of displacement for the center point of a beam subjected to the same harmonic motion (F_f=F_1=1.405Hz) on two supports

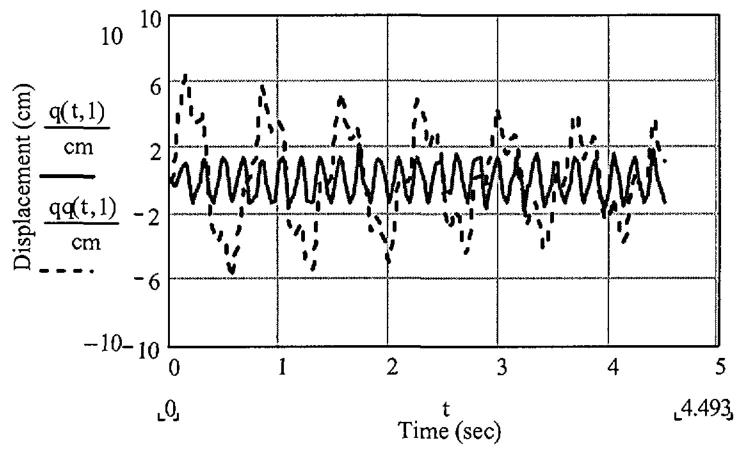

Figure 20: Comparison of two displacement-time histories, one with zero initial condition and the other with non-zero initial condition, of a simple beam subjected to harmonic motion F_f=5.6Hz