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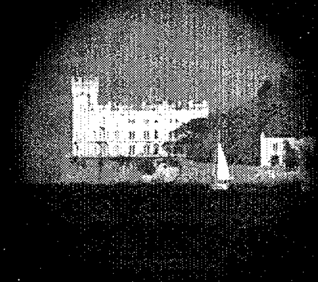

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BACKSCATTER FACTOR AND ABSORPTION RATIO OF FIBROUS ZIRCONIA MEDIA IN THE VISIBLE

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Abstract

Fibrous thermal insulations are widely used to conserve energy in ambient to high temperature applications including buildings, solar collectors, heat exchangers, furnaces and thermal protection systems of reusable launch vehicles. It has long been recognised that zirconia has the lowest thermal conductivity of commercial refractories. The thermal conductivity of a zirconia fibrous medium is strongly dependent of its bulk density; high bulk densities of zirconia fibers provide the most effective insulation at high temperatures. Lee's theory for radiative transfer through fibrous media is used in this paper. The two-flux model is applied to determine the backward and forward parameters of a medium of zirconia fibers oriented in parallel planes. Theoretical calculations of the backscatter factor and absorption ratio of this medium are carried out in the visible spectrum for different size parameters of the fibers and for three different temperatures. Our results show that the backscatter factor of zirconia fibrous insulations is maximum, and therefore the heat transfer by the fibrous medium is the lowest, for a size parameter of 0.45 for all the temperatures studied. We also observed that the backscatter factor decreases with increasing temperature.

INTRODUCTION

Fibrous thermal insulations are widely used to conserve energy in ambient to high temperature applications including buildings, solar collectors, heat exchangers, furnaces and thermal protection systems of reusable launch vehicles [1-3]. So better understanding of the heat transfer characteristics of fibrous insulations is essential where thermal energy could be transferred by radiation, conduction or convection. Radiative transfer usually accounts for more than 30 % of the heat transport in these systems [9, 10] and even becomes the predominant mode of energy transfer when temperature increases from moderate to high [4,5]. Reducing the radiative heat transfer through the fibrous insulation can thus have a major impact on its insulating efficiency and production cost. Various approaches for radiative transfer modelling in fibrous insulations have been developed and can be found in different papers [4-11]. The review paper of Baillis and Sacadura [12] may also be consulted.

Among these radiative models, the simplest and most used is certainly the two–flux method [4-6, 9-12]. This model is based on the assumption of an isotropic scattering of radiation by the material in the forward and backward hemispheres.

The two-flux model developed by Lee and summarised in [4 - 6] that takes into consideration the effect of fibers orientation and distribution in the medium is applied in the present investigation. The fibers in the material are considered to be infinitely long cylinders and randomly oriented in parallel planes. The spectral extinction and scattering efficiencies for a single fiber are determined using electromagnetic scattering theory as can be found in standard textbooks [13-15]. These efficiencies which are functions of both the complex refractive index and geometric dimensions of the fibers can then be integrated over the size and orientation distributions of the fibers to give the extinction and scattering coefficients of the fibrous medium. The radiative coefficients obtained are inserted in the model to evaluate the backscatter factor and absorption ratio of a fibrous zirconia medium, at three different temperatures: 300°C, 500°C, and 850°C and for wavelengths between 0.3 and 0.75 μm.

2. MATERIALS AND METHODS

2.1. Materials

A zirconia fibrous medium is studied. Zirconia has a melting point around 2700 °C and can be used as an insulating material [17,18]. It has long been recognised that zirconia has the lowest thermal conductivity of commercial refractories. The thermal conductivity of a zirconia fibrous medium is strongly dependent of its bulk density; high bulk densities of zirconia fibers provide the most effective insulation at high temperatures [18]. Ghanashyam et al. [17] evaluated the index of refraction n and the extinction index k of zirconia for three temperatures: 300°C, 500°C, and 850°C in the visible spectrum, as indicated in table 1 below.

Table 1: Optical properties of zirconia [17]

Temp. (°C)	λ (μm)	n	$k.10^3$	Temp. (°C)	λ (μm)	n	$k.10^3$	Temp. (°C)	λ (μm)	n	$k.10^3$
	0.30	1.744	12.187		0.30	1.962	12.187		0.30	2.000	10.000
	0.35	1.750	7.5000		0.35	1.894	6.2500		0.35	1.962	5.0000
	0.40	1.712	5.0000		0.40	1.870	3.7500		0.40	1.950	3.1250
	0.45	1.720	4.0625		0.45	1.856	3.1250		0.45	1.925	2.5000
850	0.50	1.712	4.0625	500	0.50	1.865	2.8125	300	0.50	1.906	2.1875
	0.55	1.725	2.8125		0.55	1.870	2.5000		0.55	1.896	1.8750
	0.60	1.725	3.1250		0.60	1.862	1.8750		0.60	1.892	1.2500
	0.65	1.725	3.4375		0.65	1.862	1.8750		0.65	1.894	0.6500
	0.70	1.725	3.7500		0.70	1.862	2.1875		0.70	1.894	0.3250
	0.75	1.725	4.6875		0.75	1.862	2.5000		0.75	1.894	0.1562

The fibers, which can be 1 - 2 millimetres long and 1 - 10μm in diameter, are idealised as infinitely long cylinders because their length is far greater than the incident thermal radiation. We also consider in this study that the fibers are randomly oriented in planes parallel to the boundaries of the medium with a bulk density of about 0.40 g/cm^3. The extinction and the scattering efficiencies of fibers modelled as infinitely long circular cylinders are well established from electromagnetic scattering theory as can be found in [14,15]. They read

$$Q_{e\lambda}(m\alpha,\phi) = (\lambda / r\pi)\mathrm{Re}\{T(\theta=0)\} \quad (1)$$

$$Q_{s\lambda}(m\alpha,\phi) = (\lambda / 2r\pi^2)\int_0^{2\pi} i(\theta,\phi)d\theta \quad (2)$$

where $i(\theta,\phi)$ is the angular distribution of the radiation scattered by a tilted cylinder, as defined by Kerker in [14] and Re{T (θ)} is the real part of the amplitude function of the scattered radiation. Numerical computations of these efficiencies have been performed using a code similar to the one described by Yousif and Boutros in [16].

These efficiencies and the number density of fibers combine to give the extinction and scattering coefficients of the fibrous medium. For monosize and randomly oriented fibers in parallel planes, with a number density N, these coefficients are written as

$$K_\lambda(\xi) = \frac{rN}{\pi}\int_0^{2\pi} Q_{e\lambda}(m\alpha,\phi)\delta\left(\xi_f - \frac{\pi}{2}\right)d\omega_f \quad (3)$$

$$\sigma_{s\lambda}(\xi) = \frac{rN}{\pi}\int_0^{2\pi} Q_{s\lambda}(m\alpha,\phi)\delta\left(\xi_f - \frac{\pi}{2}\right)d\omega_f \quad (4)$$

and the absorption coefficient follows as

$$\sigma_{a\lambda}(\xi) = K_\lambda(\xi) - \sigma_{s\lambda}(\xi) \quad (5)$$

2.2. Methods

The following analysis strictly follows the model developed by Lee [4-6]. The radiation field within the medium is given by the equation of transfer [13] with angle notation shown in figure 1:

$$\mu\frac{dI_\lambda(\xi,\omega,y)}{dy}=\sigma_{a\lambda}I_{b\lambda}(y)-K_\lambda(\xi)I_\lambda(\xi,\omega,y)+\frac{1}{4\pi}\int_{\Omega'}<\sigma_{s\lambda}p(\Omega'-\Omega)>I_\lambda(\Omega')d\Omega' \quad (6)$$

where $\mu=\cos(\xi)$

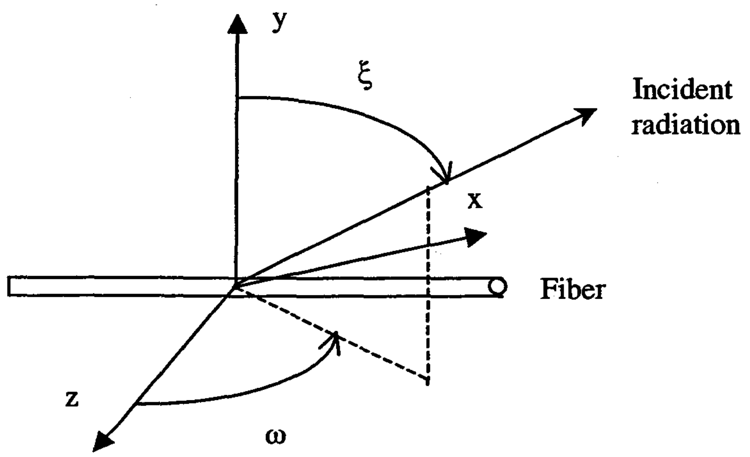

Fig. 1 : Angle notation.

In the particular case of fibers randomly oriented in planes, the product of the scattering coefficient by the phase function is defined as

$$<\sigma_{s\lambda}p>(\xi,\phi)=\frac{N\lambda}{2\pi^3}\int_0^{2\pi}i(\theta,\phi)\delta\left(\xi_f-\frac{\pi}{2}\right)d\omega_f \quad (7)$$

The two – flux model is then written as

$$\mu\frac{dI_\lambda^+}{dy}=-K_\lambda(\mu)I_\lambda^++\sigma_{a\lambda}I_{b\lambda}+F_\lambda(\mu)I_\lambda^++B_\lambda(\mu)I_\lambda^-,\qquad 0<\mu\le1 \quad (8a)$$

$$\mu\frac{dI_\lambda^-}{dy}=-K_\lambda(\mu)I_\lambda^-+\sigma_{a\lambda}I_{b\lambda}+F_\lambda(\mu)I_\lambda^-+B_\lambda(\mu)I_\lambda^+,\qquad -1\le\mu<0 \quad (8b)$$

where the spectral forward $F_\lambda(\mu)$ and backward $B_\lambda(\mu)$ parameters are defined as

$$F_\lambda(\mu)=\frac{1}{4\pi}\int_\Omega<\sigma_{s\lambda}\cdot p_\lambda>d\Omega \quad (9a)$$

$$B_\lambda(\mu)=\frac{1}{4\pi}\int_{\cup}<\sigma_{s\lambda}\cdot p_\lambda>d\Omega \tag{9b}$$

These expressions can be expanded to give

$$F_\lambda(\mu)=\frac{N\lambda}{2\pi^3}\int_0^{2\pi}\int_{\theta_c+\pi}^{\theta_c+2\pi} i(\theta,\phi)\delta\left(\xi_f-\frac{\pi}{2}\right)d\theta d\omega_f \tag{10a}$$

$$B_\lambda(\mu)=\frac{N\lambda}{2\pi^3}\int_0^{2\pi}\int_{\theta_c}^{\theta_c+\pi} i(\theta,\phi)\delta\left(\xi_f-\frac{\pi}{2}\right)d\theta d\omega_f \tag{10b}$$

With the critical angle of observation θ_c given by the relationship

$$\cos\theta_c=\frac{\sin\xi\sin(\omega-\omega_f)}{\left[1-\sin^2\xi\cos^2(\omega-\omega_f)\right]^{1/2}} \tag{11}$$

The conservation equation satisfied by the spectral forward $F_\lambda(\mu)$ and backward $B_\lambda(\mu)$ parameters is

$$\sigma_{s\lambda}(\mu)=F_\lambda(\mu)+B_\lambda(\mu) \tag{12}$$

The average spectral backscatter factor and absorption ratio are obtained after dividing by the effective extinction coefficient as

$$\overline{B}_\lambda=\frac{1}{\overline{K}_\lambda}\int_0^1 B_\lambda(\mu)d\mu \tag{13a}$$

$$\overline{\beta}_\lambda=\frac{1}{\overline{K}_\lambda}\int_0^1 \sigma_{a\lambda}(\mu)d\mu \tag{13b}$$

with

$$\overline{K}_\lambda=\int_0^1 K_\lambda(\mu)d\mu \tag{13c}$$

The net radiative flux in the fibrous insulation can then be obtained by solving equations (8a) and (8b) as

$$q=\int_0^\infty \frac{e_{1\lambda}-e_{2\lambda}}{\left(\frac{1}{\varepsilon_{1\lambda}}+\frac{1}{\varepsilon_{2\lambda}}-1\right)+\left(\overline{\beta}_\lambda-2\overline{B}_\lambda\right)\overline{K}_\lambda L}d\lambda \tag{14}$$

where $e_{1\lambda}$ and $e_{2\lambda}$ are the Planck's emissive powers of the two boundaries with known characteristics $(\varepsilon_{1\lambda},T_1)$ and $(\varepsilon_{2\lambda},T_2<T_1)$; L is the thickness of the fibrous medium.

3. RESULTS AND DISCUSSION

It has long been recognised that zirconia has the lowest thermal conductivity of commercial refractories. The thermal conductivity of zirconia fibrous media is strongly dependent of its bulk density. Measurements performed by the ZIRCAR Company [18] clearly demonstrate that high bulk densities of zirconia fibers provide the most effective insulation at high temperatures (fig. 2)

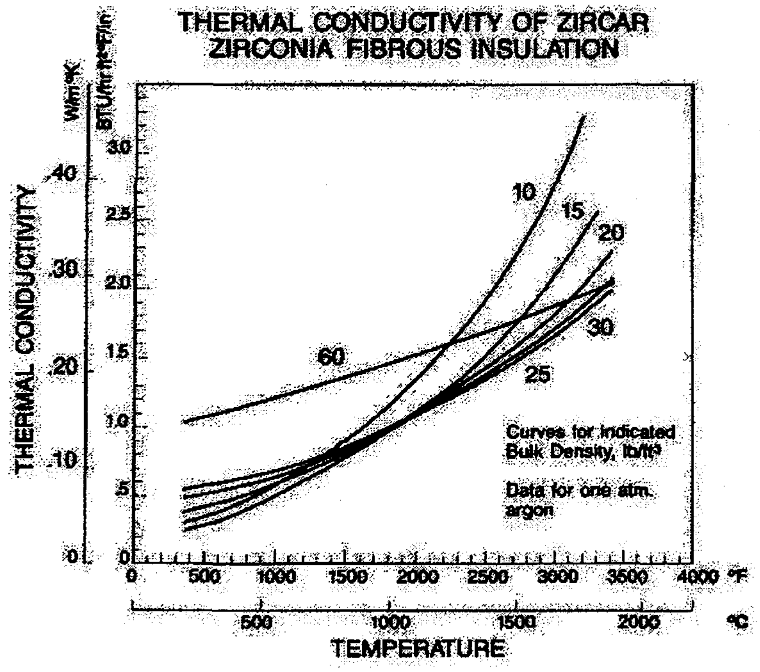

Fig. 2: Thermal conductivity of zirconia fibrous insulations [18]

We present in this section the effect of the fiber size parameter, fiber temperature, and the wavelength of the incident radiation on the variations of the backscatter factor and absorption ratio of the fibrous medium. These two radiative parameters are the most pertinent for characterising the efficiency of a thermal insulation. The backscatter factor represents all the radiation scattered into a hemisphere due to radiation traversing in all directions in the opposite hemisphere. It has been defined in equation (13a) as the integral of the backscatter parameter over a hemisphere divided par the effective extinction coefficient. On the opposite side the absorption ratio as defined in equation (13b) magnifies the amount of radiation travelling in the forward direction.

In figures 3 to 8 we present the effect of the wavelength λ of the incident radiation on the variations of the backscatter factor $\overline{B}_\lambda$ and absorption ratio $\overline{\beta}_\lambda$ of the medium as functions of the fibers size parameter $\alpha=2\pi r/\lambda$ and for different average temperatures T of the medium. As was done by Jeandel et al. in [10], T is taken to be the mean value $(T_1+T_2)/2$ of the two boundaries temperatures.

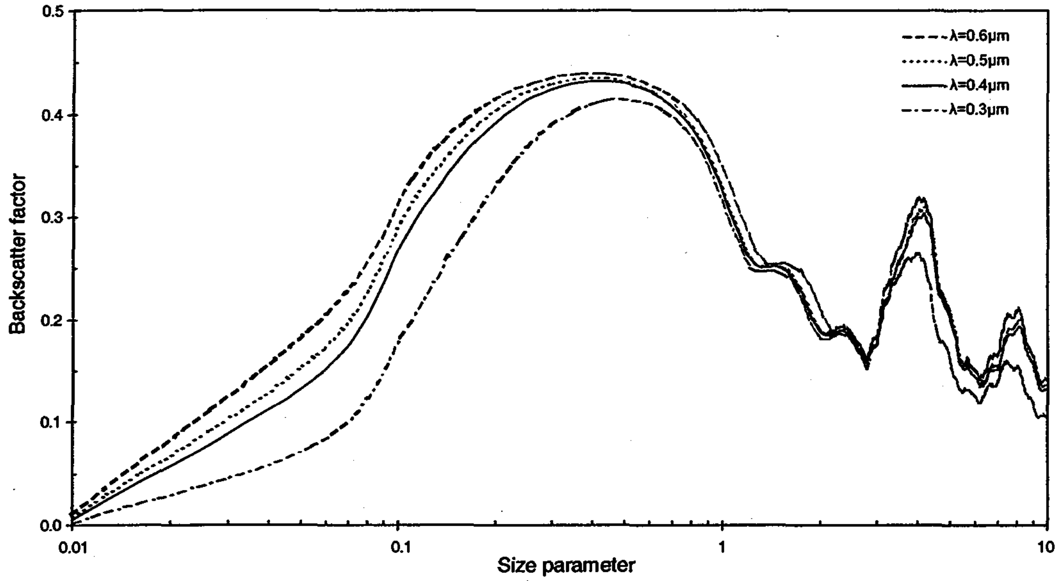

Fig. 3 : Backscatter factor vs. size parameter; T=850°C

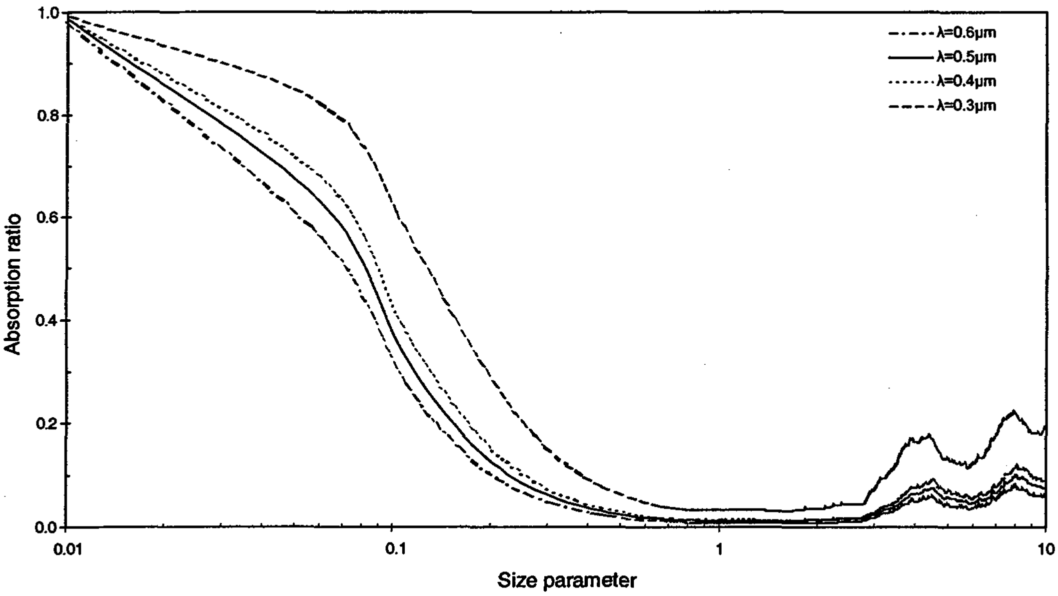

Fig. 4 : Absorption ratio vs. size parameter; T=850°C

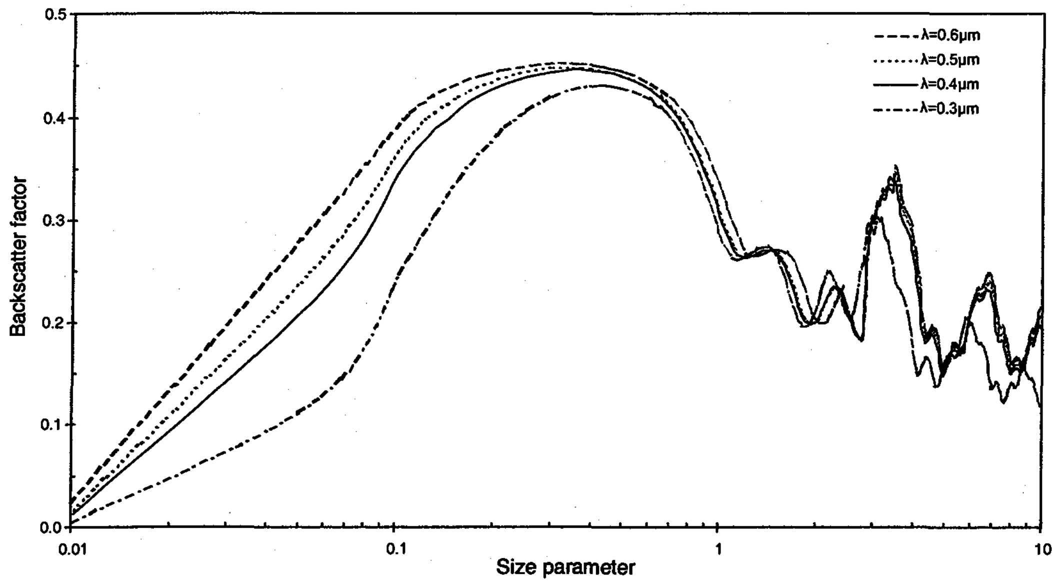

Fig. 5 : Backscatter factor vs. size parameter; T=500°C

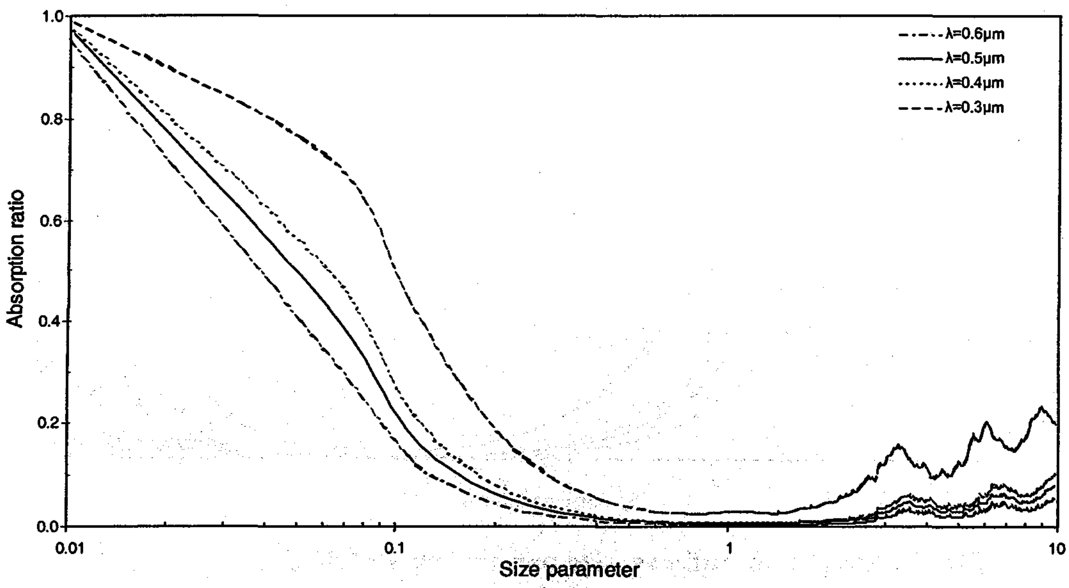

Fig. 6 : Absorption ratio vs. size parameter; T=500°C

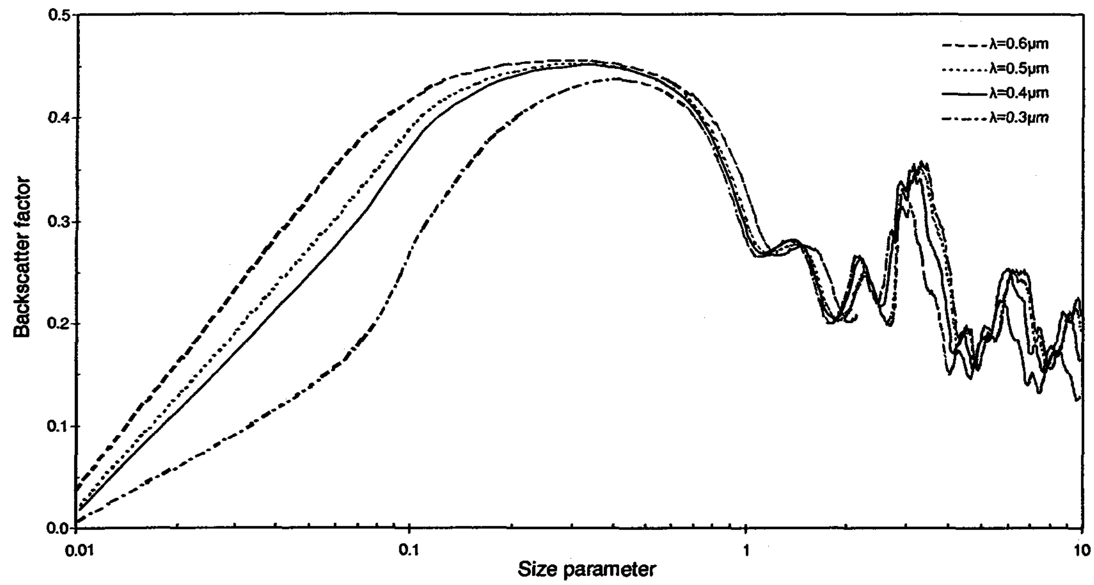

Fig. 7 : Backscatter factor vs. size parameter; T=300°C

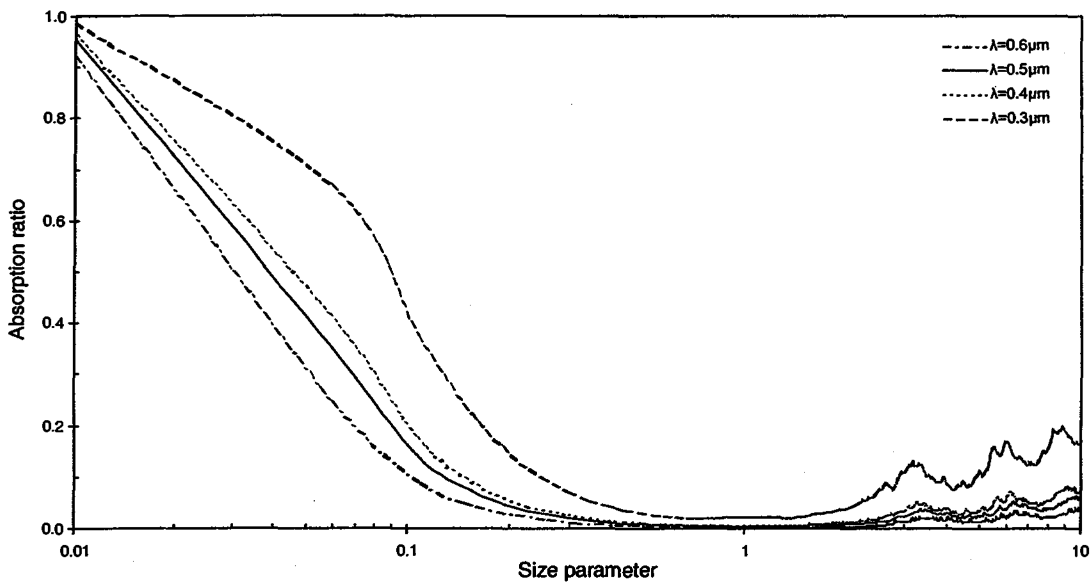

Fig. 8 : Absorption ratio vs. size parameter; T=300°C

Figures 9 to 12 show the effect of the temperature of the fibers on the variations of the backscatter factor and absorption ratio as functions of the fiber size parameter in figures 9 and 10 and as functions of the incident radiation wavelength in figures 11 and 12.

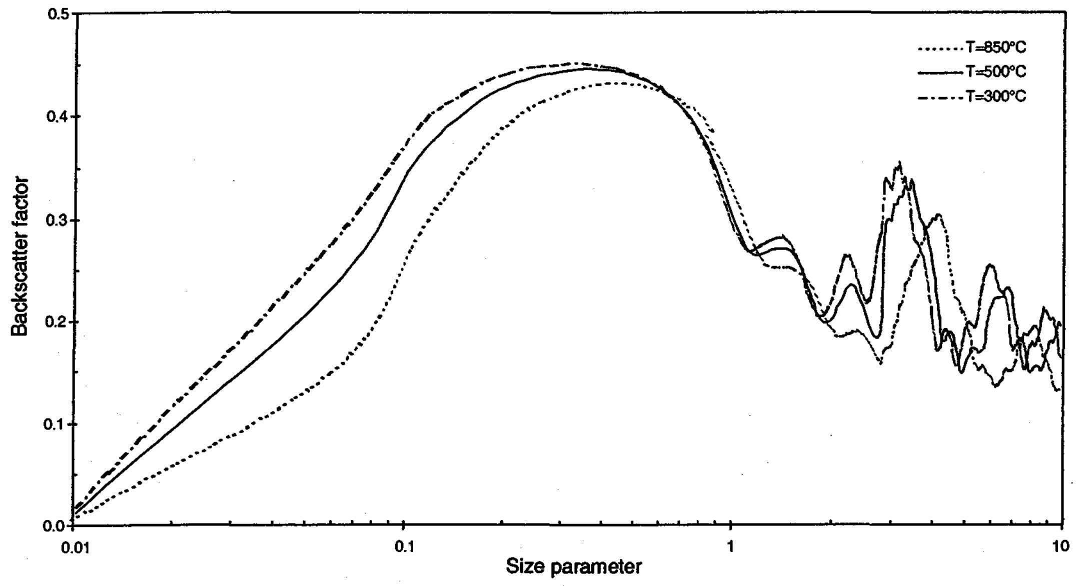

Fig. 9 : Backscatter factor vs. size parameter; λ=0.4μm

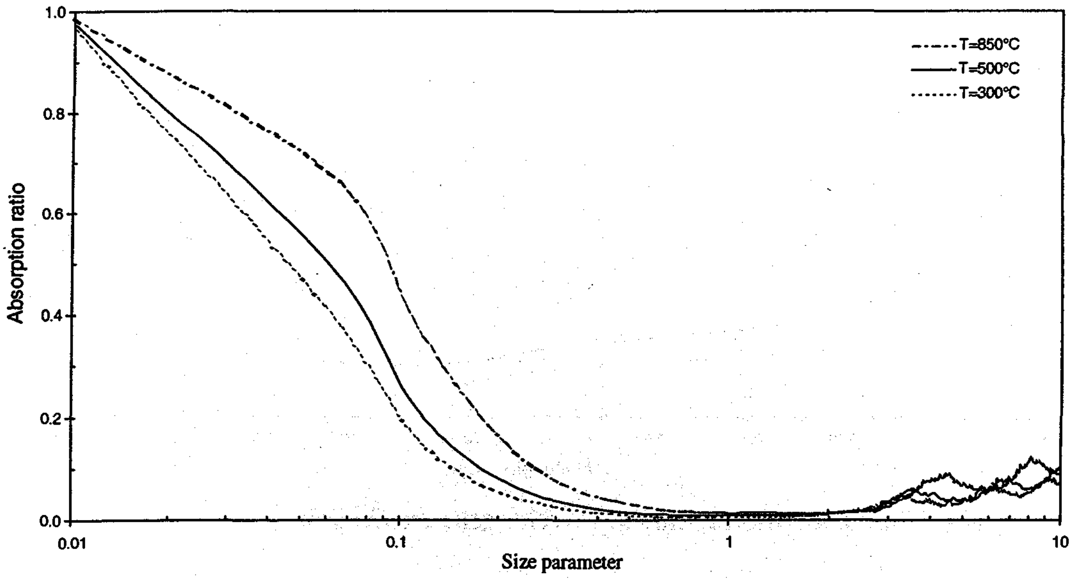

Fig. 10 : Absorption ratio vs. size parameter; λ=0.4μm

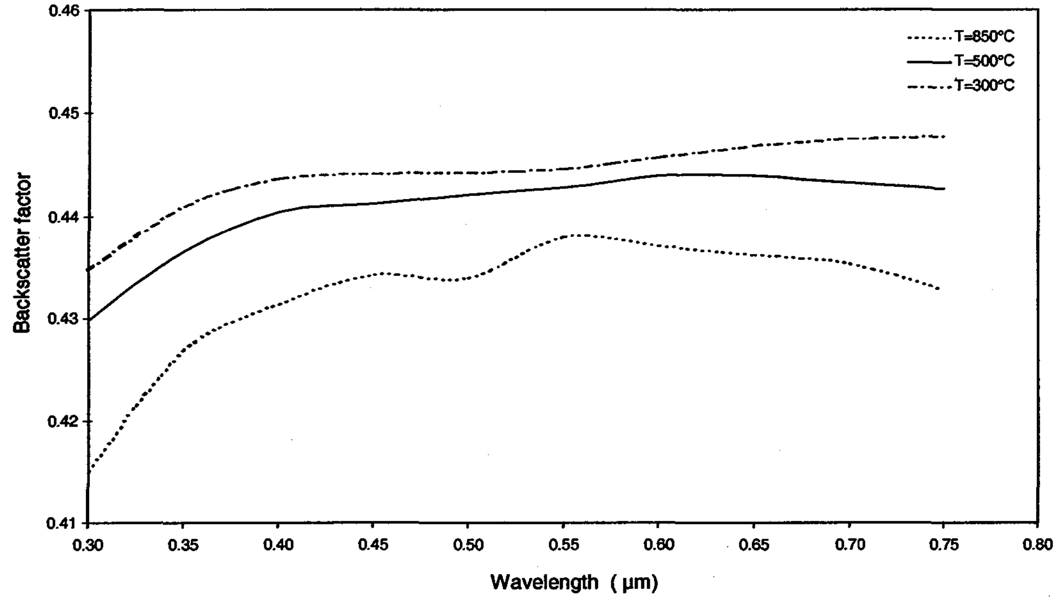

Fig. 11 : Backscatter factor vs. wavelength; 2πr/λ=0.5

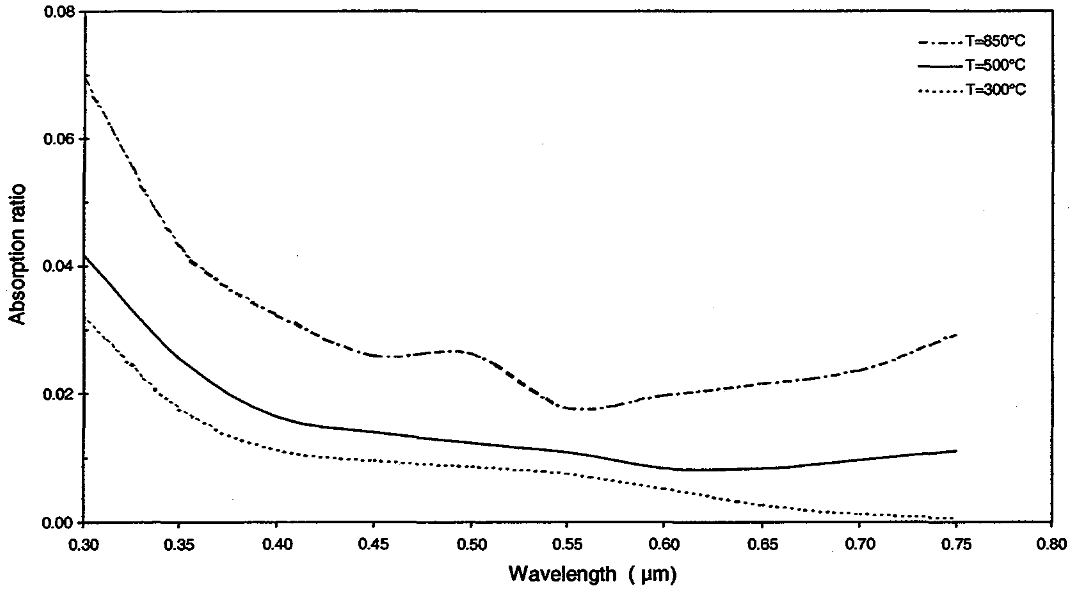

Fig. 12 : Absorption ratio vs. wavelength; 2πr/λ=0.5

From this study we can draw the following conclusions:

- The backscatter factor increases with the size parameter of the fibers (figs. 3,5,7,9) from a minimum value near zero for very thin fibers reaching a maximum around α=0.45, which corresponds to the lowest heat transfer by the fibrous medium. The absorption ratio of the fibrous medium shows exactly an opposite behaviour (4,6,8,10). We also observe that the backscatter factor of zirconia fibrous media slightly increases when the wavelength of the radiation striking the medium also increases for all the temperatures studied; at the same time the absorption ratio decreases. This can simply be explained by the decreasing behaviour of the extinction index k with an almost constant value of the refractive index n of the zirconia fibers from λ=0.3 μm to 0.75 μm.

- The backscatter factor of zirconia fibrous media decreases (fig. 11) and the absorption ratio also increases (fig. 12) with increasing temperatures. This is due to the increase of the refractive index n that consequently results in an increase of the scattering coefficient of the medium. Such a behaviour has also been observed by Lee [5].

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