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Abstract. A supersymmetry scheme is proposed for nuclear cluster systems. The bosonic sector of the superalgebra describes the relative motion of the clusters, while its fermionic sector is associated with their internal structure. An example of core+α configurations is discussed in which the core is a p-shell nucleus and the underlying superalgebra is U(4|12). The α-cluster states of the nuclei ^{20}Ne and ^{19}F are analyzed and correlations between their spectra, electric quadrupole transitions, and one-nucleon transfer reactions are interpreted in terms of U(4|12) supersymmetry.

PACS. 21.60.Fw Models based on group theory – 21.60.Gx Cluster models – 11.30.Pb Supersymmetry

Supersymmetric theories which give a unified treatment of bosonic and fermionic degrees of freedom have been formulated in several branches of physics. Arguably the most successful applications have been found in nuclear physics, and among them supersymmetry based on the interacting boson-fermion model [1,2] is the most widely known. In this group-theoretical approach the algebras associated with the bosonic and the fermionic components of the nuclear systems are unified in a superalgebra (or graded Lie algebra) giving rise to correlations between the spectroscopic properties of neighbouring even-even, odd-even and odd-odd nuclei. The validity of these predictions has been confirmed for several examples [2,3].

Other supersymmetric theories in nuclear physics relate "deep" and "shallow" real [4] and complex [5] potentials describing the interaction of composite nuclear systems, attempt to describe identical superdeformed bands in quartets of neighbouring nuclei [6] and correlate parameters of one-pion-exchange potentials [7].

Here we propose yet another application of supersymmetry related to clustering, where the relative motion of the clusters is described by boson (oscillator quantum) excitations, while the internal structure of the clusters is interpreted in terms of fermionic degrees of freedom. An algebraic approach of this type has been formulated previously in terms of the semimicroscopic algebraic cluster model (SACM) [8], according to which the clusters are treated in terms of Elliott's SU(3) model [9], while the relative motion of the clusters is described in terms of the SU(3) basis of the vibron model [10].

The SACM basis is a symmetry-dictated truncation of the complete SU(3) shell-model basis of the whole nucleus and, therefore, is free from Pauli-forbidden states and from spurious centre-of-mass excitations. At the same time it also reflects the cluster character of states. A number of cluster systems (mainly in the sd-shell region) were analyzed in terms of the SU(3) limit of the SACM [11,12] and the (approximate) validity of this symmetry scheme was confirmed. The dependence of the model parameters on the mass number was also studied, and it was shown that the parameters of the SACM Hamiltonian for core+α-type cluster systems vary smoothly in the $A = 16$ to 20 region [13]. Similar systematics were also found for the parameters of the electric quadrupole transition operator [14].

Our attempt to give a unified treatment of cluster systems in a more general symmetry-based framework is also motivated by the fact that some cluster bands of neighbouring nuclei (*e.g.* in ^{20}Ne, ^{19}F, ^{18}F and ^{18}O) are sometimes interpreted as each other's correspondents (see *e.g.* [15] and references therein). In the simplest case we consider core+α-type cluster systems in which the core cluster is a p-shell nucleus. The bosonic sector is identified with the relative motion of the clusters, while the fermions are defined as *holes* on the p shell. The zero-fermion case then corresponds to a closed-shell core (^{16}O). The addition of fermions corresponds to a decrease in mass of the nucleus, while the addition of bosons is equivalent to an increase in the number of relative excitation quanta. A physical argument in support of this type of supersymmetry is that the typical energy of the fermionic and bosonic excitations is in the same range for these nuclei: The shell excitation quanta are $\hbar\omega \simeq 13$ MeV in this region and the typical nucleon separation energies are also of this order for most nuclei close to the valley of stability.

The boson creation and annihilation operators $b^{\dagger(l,0)l}_{m_l}$ and $\tilde{b}^{(0,l)l}_{m_l}$ are those of the vibron model [10] (*i.e.* π and σ bosons for $l = 1$ and 0), where the superscript indicates the SU(3) tensor character (λ, μ). The 16 generators of $U_B(4)$ are constructed as SU(3)-coupled [16] bilinear products of these operators:

$$B^{(\lambda,\mu)L}_{M_L}(l,l') = [b^{\dagger(l,0)} \times \tilde{b}^{(0,l')}]^{(\lambda,\mu)L}_{M_L} . \quad (1)$$

Of these, the eight $B^{(1,1)L}_{M_L}$ operators (with $L = 1$ and 2) generate $SU_B(3)$. In what follows, the subscripts B and F denote quantities in the bosonic and fermionic sectors.

The fermion operators $a^{\dagger(0,1)lst}_{m_l m_s m_t}$ and $\tilde{a}^{(1,0)lst}_{m_l m_s m_t}$, which create and annihilate a hole on the p shell, have SU(3) character $(\lambda, \mu) = (0,1)$ and (1,0), respectively, and carry orbital angular momentum $l = 1$, spin $s = 1/2$ and isospin $t = 1/2$. They describe $3\times2\times2 = 12$ single-particle states and the bilinear products

$$A^{(\lambda,\mu)LST}_{M_L M_S M_T} = [a^{\dagger(0,1)\frac{1}{2}\frac{1}{2}} \times \tilde{a}^{(1,0)\frac{1}{2}\frac{1}{2}}]^{(\lambda,\mu)LST}_{M_L M_S M_T} \quad (2)$$

generate $U_F(12)$. For $S = T = 0$ and $(\lambda, \mu) = (1,1)$ one gets the 8 generators of the orbital $SU_F(3)$ algebra. For $(\lambda, \mu) = (0,0)$ the 16 generators of Wigner's $U^{ST}_F(4)$ supermultiplet algebra are obtained.

To embed the bosonic and fermionic algebras in a superalgebra, one has to define generators which create a fermion and annihilate a boson, or *vice versa.* They can be constructed as

$$D^{(\lambda,\mu)Lst}_{M_L m_s m_t}(l') = [a^{\dagger(0,1)st} \times \tilde{b}^{(0,l')}]^{(\lambda,\mu)Lst}_{M_L m_s m_t} . \quad (3)$$

Note that the spin-isospin character of the operators (3) is determined by that of the fermion operators $a^{\dagger(0,1)lst}_{m_l m_s m_t}$. The inverse of the operators (3) (which create a boson and annihilate a fermion) can be constructed similarly.

The relevant classification is

$$\begin{aligned} &U(4|12) \supset U_B(4) \times U_F(12) \\ &\supset SU_B(3) \times SU_F(3) \times U^{ST}_F(4) \\ &\supset SU(3) \times SU^S_F(2) \times SU^T_F(2) \\ &\supset SO(3) \times SU^S_F(2) \times SU^T_F(2) \supset \mathrm{Spin}(3) \times U^T_F(1) \ , (4) \end{aligned}$$

which is the group structure of the SACM for core+α cluster systems [13], embedded in U(4|12). The associated quantum numbers are also those of the SACM [13], extended with the fermion and total particle numbers N_F and $\mathcal{N} = N_B + N_F$, which label the representations of $U_F(12)$ and U(4|12). The remaining algebras and the associated quantum numbers play the same role as in the SACM [13]: the $U_B(4)$, $SU_B(3)$, and $SU_F(3)$ representation labels $N_B = n_\pi + n_\sigma$, n_π and (λ_F, μ_F) denote the total boson number, the dipole boson number, and Elliott's SU(3) labels of the core. The SU(3), $U^{ST}_F(4)$, $SU^S_F(2)$, $SU^T_F(2)$, SO(3), Spin(3) and $U^T_F(1)$ labels (λ,μ), $[f_1, f_2, f_3, f_4]$, S, T, L, J and M_T stand for the quantum numbers labelling the unified nucleus in terms of the Elliott's LS-coupled SU(3) shell model. As in the SACM, the SU(3) representations (λ, μ) are obtained from the SU(3) multiplication $(n_\pi, 0) \times (\lambda_F, \mu_F)$, keeping only those contained in the fully antisymmetric SU(3) model space of the unified nucleus. The number of the dipole bosons n_π, *i.e.* the number of harmonic oscillator quanta in the relative motion of the clusters, also determines the respective shell $(n\hbar\omega)$ of the unified nucleus via $n = n_\pi - n_\pi^{\min}$: states with $n_\pi < n_\pi^{\min}$ are excluded due to the Pauli blocking between the nucleons of the two clusters [13] (*i.e.* the Wildermuth condition). For ^{20}Ne=^{16}O+α, $n_\pi^{\min}$ is 8 which corresponds to raising the four nucleons of the α particle to the empty sd shell. For ^{19}F=^{15}N+α, there is a hole on the p shell, so in this case $n_\pi^{\min} = 7$. Keeping only the essential quantum numbers, the basis states can be labelled as

$$|\mathcal{N} N_B n_\pi, (\lambda_F, \mu_F); (\lambda, \mu)\chi LSJM_J TM_T\rangle \ . \quad (5)$$

The total number of particles $\mathcal{N}$ is chosen by taking into account the physical relevance of N_F and N_B, the number of fermions and bosons. N_F should be at least the maximal number of holes on the p-shell, 12. With $\mathcal{N} = 12$, the maximal number of π bosons is also 12 ($N_B = n_\pi + n_\sigma$).

Table 1. Parameters (in MeV) of the Hamiltonian (6) obtained for the ^{16}O+α and the ^{15}N+α systems from separate (columns 1 and 2) and joint (column 3) fits. Boldface numbers indicate parameters that were not fitted.

	^{16}O+α	^{15}N+α	^{16}O+α and ^{15}N+α
γ_B	**13.185**	**13.185**	**13.185**
θ_B	−0.3573	−0.8622	−1.0621
δ_B	−0.4611	−0.7210	−0.6565
δ	—	0.1784	0.1729
β	0.1441	0.1911	0.1576
ξ	—	0.0220	0.0373
ξ'	—	0.8168	0.6638

In what follows the spectroscopic information on the α-cluster states of ^{20}Ne and ^{19}F are analyzed in terms of U(4|12). To generate energy spectra, a Hamiltonian is constructed from the Casimir invariants of the algebras in (4). This Hamiltonian is similar to the one used previously in the SACM to systematically fit the energy spectra of several neighbouring sd-shell nuclei in terms of a core+α configuration [13] but here *constant* (mass-independent) parameters instead of smoothly varying ones are taken:

$$\begin{aligned} H = &\gamma_B n_\pi + \theta_B(-1)^{n_\pi} + \delta_B C^{(2)}(SU_B(3)) \\ &+ \delta C^{(2)}(SU(3)) + \beta L\cdot L + (\xi + (-1)^{n_\pi}\xi')L\cdot S \ , (6) \end{aligned}$$

where $C^{(2)}(..)$ denotes the Casimir invariant of the relevant SU(3) algebra with eigenvalue $\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu$. Such Hamiltonians can be related to effective nucleon-nucleon interactions [17] and potential models [18].

Table 1 displays the parameters fitted to 25 and 26 α-cluster states of ^{20}Ne and ^{19}F. Fits are performed to the two nuclei separately, and also jointly. First the parameters describing the in-band rotational structure of the bands (β, ξ, ξ') are determined and next those that fix the bandhead energies. The γ_B parameter is not fitted but kept at the value of an oscillator constant appropriate for $A = 20$ nuclei [13]. A new term $\theta_B(-1)^{n_\pi}$ is considered here to account for the relative position of the positive-

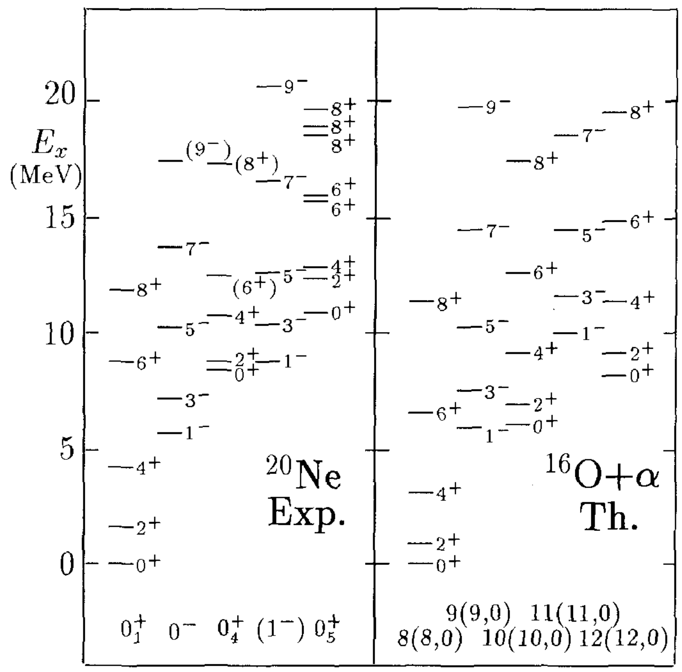

Fig. 1. Energies of α-cluster states in ^{20}Ne. The model spectrum (right panel) has been generated by using the parameters determined from the joint fit of the ^{16}O+α and the ^{15}N+α systems. The quantum numbers $n_\pi(\lambda,\mu)$ which identify the cluster bands are also displayed.

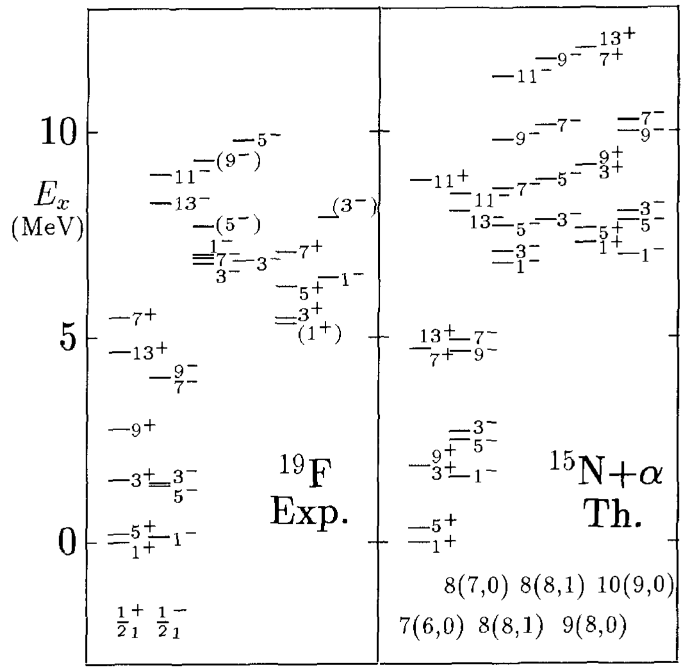

Fig. 2. Same as fig. 1 for ^{19}F states, now labelled with $2J^\pi$.

and negative-parity bandheads. (Parity-dependent interactions are also used in microscopic [19] and phenomenologic [20] cluster models describing the ^{15}N+α system.) Table 1 demonstrates that the separate and joint fits result in fairly similar parameter sets. (We note that the δ_B value found for the ^{16}O+α system must be compared to $\delta_B+\delta$ obtained for ^{15}N+α [13].)

The spectra of ^{16}O+α and ^{15}N+α obtained from the joint fit are shown in figs. 1 and 2, respectively. (Only α-cluster states of these nuclei are shown: states with a different nature typically appear above $E_x \simeq 5$ MeV in both cases.) In assigning the states to α-cluster bands, standard compilations [21] are used as well as other works that suggest further band assignments based on experimental arguments [19,22]. The set of natural-parity states usually assigned to a $K^\pi=1^-$ band in ^{20}Ne [21] is included and tentatively identified with model states belonging to the $n_\pi=11$ band. We also note that there are several candidates for the $J^\pi=6^+$ and 8^+ states of the 0_5^+ α-cluster band [22], which we associate with model states with $n_\pi=12$. As a measure for the validity of the supersymmetry, we calculate $(\sum_i |E_{Th.i}-E_{Ex.i}|)/\sum_i E_{Ex.i}=0.13$, which is comparable to the value 0.14 obtained for ^{190}Os and ^{191}Ir assuming U(6|4) supersymmetry [2].

Similarly to the energy spectrum, separate and joint fits were performed for the electric quadrupole transition rates for the α-cluster states of ^{20}Ne and ^{19}F [19,21]. This required the calculation of the matrix elements of

$$T^{(E2)} = q_B Q_B^{(2)} + q_F Q_F^{(2)} \qquad (7)$$

where $Q_B^{(2)}$ and $Q_F^{(2)}$ are generators of SU$_B$(3) and SU$_F$(3), respectively. This operator can connect states only within the same shell. The results are shown in table 2. For ^{20}Ne the ^{16}O core is inert, has no contribution to the $E2$ transition and, consequently, only q_B is fitted, resulting in $q_B=2.8504$ e fm^2. A separate fit to the E2 transitions in ^{19}F gives $q_B=3.5490$ e fm^2 and $q_F=14.5593$ e fm^2, while a simultaneous fit of the two nuclei yields $q_B=2.8619$ e fm^2 and $q_F=7.6527$ e fm^2. Although these two sets of q_B and q_F are somewhat different, the corresponding $B(E2)$ values differ by less than 5%, except for one case. The validity of the supersymmetry is indicated by the rate $(\sum_i |B(E2)_{Th.i}-B(E2)_{Ex.i}|)/\sum_i B(E2)_{Ex.i}=0.35$, which again is comparable to the value 0.39 obtained for ^{190}Os and ^{191}Ir assuming U(6|4) supersymmetry [2].

One-nucleon transfer reactions connect neighbouring nuclei that are members of the same supermultiplet. Such transitions are described by operators of the type (3), in which L and s are coupled to total angular momentum J:

$$D^{(\lambda,\mu)Jt}_{M_J m_t}(lLs) = \sum_{M_L m_s} \langle L M_L s m_s | J M_J\rangle D^{(\lambda,\mu)Lst}_{M_L m_s m_t}(l) \ . \quad (8)$$

One-nucleon spectroscopic factors C^2S are available from the ground state of ^{20}Ne to several states of ^{19}F [21,23]. To calculate the transfer amplitudes, one has to evaluate the matrix elements of (8) with $m_t=1/2$ between the $n_\pi(\lambda,\mu)LJ=8(8,0)00$ ^{20}Ne ground state and various states of ^{19}F. (Transitions to ^{19}Ne states correspond to $m_t=-1/2$.) In these matrix elements parameters of the type $\alpha^{(\lambda,\mu)Jt}_{m_t}$ are associated with each operator (8). It is encouraging that the experimental compilation [23] contains data for transitions exactly to those α-cluster states of ^{19}F which can be reached from the ground state of ^{20}Ne with the generators (8). All other α-cluster states can

Table 2. E2 transitions for ^{20}Ne and ^{19}F. The $B(E2)_{sep}$ are theoretical values fitted only to the ^{20}Ne or ^{19}F transitions, while $B(E2)_{jt}$ indicates joint fits of the two systems. Transitions in ^{19}F involve states from the $K^\pi = \frac{1}{2}^+_1$, $\frac{1}{2}^-_1$ and $\frac{1}{2}^-_3$ bands, corresponding to $n_\pi(\lambda,\mu) = 7(6,0)$, $8(7,0)$ and $8(8,1)$. In the fitting procedure the weights $B(E2)_{Exp.}/\Delta B(E2)_{Exp.}$ were used.

$J_i^\pi(E_x)$ (MeV)	$J_f^\pi(E_x)$ (MeV)	$B(E2)_{Exp.}$ (e^2fm^4)	$B(E2)_{sep}$ (e^2fm^4)	$B(E2)_{jt}$ (e^2fm^4)
$2^+(1.63)$	$0^+(0)$	65.5 ± 3.2	71.50	72.1
$4^+(4.25)$	$2^+(1.63)$	71 ± 6	90.53	91.27
$6^+(8.78)$	$4^+(4.25)$	65 ± 10	76.70	77.32
$3^-(7.16)$	$1^-(5.79)$	161 ± 26	108.64	109.52
$\frac{5}{2}^+(0.20)$	$\frac{1}{2}^+(0)$	20.93 ± 0.24	29.21	29.31
$\frac{3}{2}^+(1.55)$	$\frac{1}{2}^+(0)$	20.5 ± 2.1	29.21	29.31
$\frac{9}{2}^+(2.78)$	$\frac{5}{2}^+(0.20)$	24.7 ± 2.7	34.00	34.11
$\frac{13}{2}^+(4.65)$	$\frac{9}{2}^+(2.78)$	16.0 ± 2.7	22.13	22.20
$\frac{7}{2}^+(5.46)$	$\frac{5}{2}^+(0.20)$	6.0 ± 1.5	3.40	3.41
	$\frac{3}{2}^+(1.55)$	42 ± 12	30.60	30.70
	$\frac{9}{2}^+(2.78)$	9 ± 6	3.29	3.30
$\frac{5}{2}^-(1.35)$	$\frac{1}{2}^-(0.11)$	60 ± 9	39.55	37.47
$\frac{3}{2}^-(1.46)$	$\frac{1}{2}^-(0.11)$	75 ± 33	43.30	41.02
	$\frac{3}{2}^-(1.46)$	30^{+27}_{-12}	50.85	48.18
$\frac{9}{2}^-(4.03)$	$\frac{5}{2}^-(1.35)$	84 ± 18	44.52	42.18
$\frac{11}{2}^-(8.95)$	$\frac{7}{2}^-(4.00)$	24.4 ± 3.6	47.21	44.74
	$\frac{9}{2}^-(4.03)$	$1.5^{+2.7}_{-1.2}$	2.52	2.39
$\frac{3}{2}^-(6.79)$	$\frac{1}{2}^-(0.11)$	2.1 ± 0.9	7.20	1.36

be reached only using higher-order operators. The operators (8) with $(\lambda,\mu) = (0,2)$ annihilate one relative excitation oscillator quantum and lead to the $J^\pi = \frac{1}{2}^+$, $\frac{3}{2}^+$ and $\frac{5}{2}^+$ ground-band states of ^{19}F with $(\lambda,\mu) = (6,0)$. Since each transition contains a separate parameter $\alpha_{\frac{1}{2}}^{(0,2)J\frac{1}{2}}$, no correlation between the data can be identified. Transitions in which the number of relative oscillator quanta n_π is left unchanged can lead to the $J^\pi = \frac{1}{2}^-$ and $\frac{3}{2}^-$ states of the $(\lambda,\mu) = (7,0)$ and (8,1) bands. Now correlations can be found between the transitions to states that have the same spin J but different (λ,μ). Data are available for both states only for $J^\pi = \frac{3}{2}^-$, and the relative intensity of the transition to the $E_x = 6.69$ and 1.46 MeV state is 3.2. The prediction 2.75 based on U(4|12) supersymmetry is fairly close to this value.

In summary, we conclude that, although there are few criteria by which the validity of the present supersymmetry can be tested, these all seem to support the idea of correlating different cluster systems under this scheme. Calculations of energy spectra and electric quadrupole transitions of the ^{16}O+α and ^{15}N+α systems show that essentially the same results are obtained from fits in which both nuclei are included, and those in which the fits are done separately. Correlations are also found for one-nucleon transfer data between α-cluster states of the two nuclei.

Further tests of this type of supersymmetry are possible for other cluster systems, for instance, ^{15}O+α (with $N_F = 1$), ^{14}C+α, ^{14}N+α and ^{14}O+α (with $N_F = 2$). Construction of other cluster supersymmetry schemes with similar nature can also be envisaged.

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