Compact Stellarator Coils

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Abstract. Experimental devices to study the physics of high-beta ($\beta \gtrsim 4\%$), low aspect ratio ($A \lesssim 4.5$) stellarator plasmas require coils that will produce plasmas satisfying a set of physics goals, provide experimental flexibility, and be practical to construct. In the course of designing a flexible coil set for the National Compact Stellarator Experiment, we have made several innovations that may be useful in future stellarator design efforts. These include: the use of Singular Value Decomposition methods for obtaining families of smooth current potentials on distant coil winding surfaces from which low current density solutions may be identified; the use of a Control Matrix Method for identifying which few of the many detailed elements of the stellarator boundary must be targeted if a coil set is to provide fields to control the essential physics of the plasma; the use of Genetic Algorithms for choosing an optimal set of discrete coils from a continuum of potential contours; the evaluation of alternate coil topologies for balancing the tradeoff between physics objective and engineering constraints; the development of a new coil optimization code for designing modular coils, and the identification of a "natural" basis for describing current sheet distributions.

1. Introduction

In the course of designing a flexible coil set for the National Compact Stellarator Experiment (NCSX), we have made several innovations that should be useful in future stellarator design efforts. Although NCSX is a quasi-axisymmetric (QA) device the methods described here should be applicable to any low-A stellarator design. A summary of five of these innovations is given below. A complete summary will be given in the Nuclear Fusion proceedings of this conference.

2. Current Sheet Coil Improvements using SVD[1]

The NESCOIL code[2] has been an important coil design tool for larger aspect ratio stellarators, and continues to be used in the design of NCSX. A coil winding surface (CWS) is chosen that encloses the plasma and has realistic coil-to-plasma separations. A current potential $\Phi(u,v)$ representing a surface current distribution $\vec{j}' = \hat{n}' \times \nabla\Phi(u,v)$ is sought such that the normal component of the magnetic field, $b \equiv \vec{B} \cdot \hat{n}$ vanishes in the least-squares sense at the plasma boundary ($\hat{n}$ and $\hat{n}'$ are unit normals to the plasma and coil winding surface, and u, v are poloidal and toroidal angles per field period on the CWS). Once the potential is determined, discrete coils are obtained by selecting an appropriate set of contours of Φ and interpreting each contour as a filamentary coil carrying an amount of current that is proportional to the change in potential midway between the chosen contour and its two chosen neighbours. Problems can be encountered with this standard NESCOIL procedure when it is applied to a CWS distant from the plasma: Numerical difficulties are associated with ill-conditioning of the inductance equations which relate the Fourier components of the current potential to the normal component of the magnetic field at the plasma boundary, and these can result in excessively large current densities in the current sheet solution. To overcome this problem and to obtain smooth current potential solutions we have implemented a Singular Value Decomposition (SVD) method for solving the inductance equations. By varying the number of singular values retained in the SVD

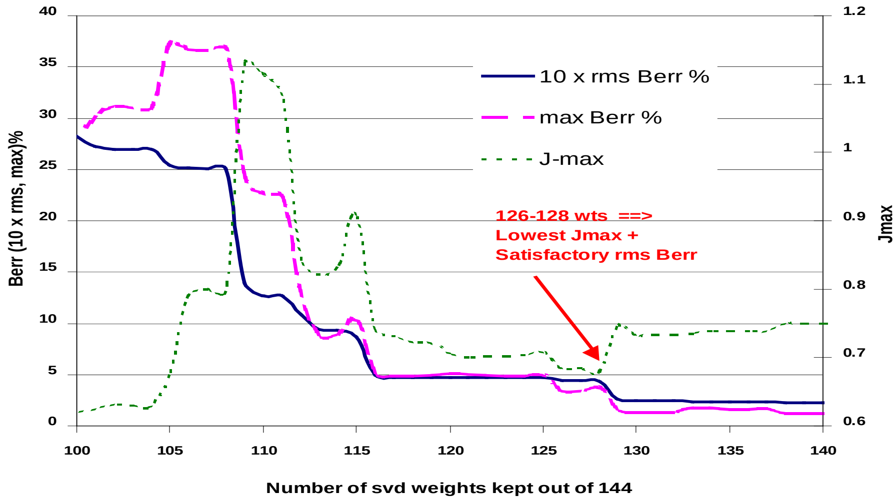

Figure 1: *SVD Scan for LI383 Saddle Current Sheet*

solution, a family of current sheet solutions is obtained which represents a tradeoff between the fitting error (related to the accuracy of reconstructing the target plasma from the coils) and engineering criteria such as coil complexity and current density.

Figure 1 shows results from the calculation of a family of current sheet solutions with saddle topology for a candidate NCSX plasma named LI383 ($< R >= 1.73m, < a >= 0.40m, \beta = 4\%, I_p = 150kA, B_T = 1.2T$). A CWS with a coil-to-plasma normal separation distance varying between 18 cm and 25 cm was used. The inductance equations, $\mathbf{LI} = \mathbf{b}$, relating the Fourier coefficients $\mathbf{I} = \{\Phi_{mn}\}$ of the current potential to the normal component of the magnetic field $\mathbf{b} = \{(\vec{B} \cdot \hat{n})(u, v)_i\}$ on the plasma boundary were solved using SVD. The $\mathbf{b}$ data were evaluated on a 64 x 64 mesh of points on the plasma boundary, uniformly spaced in u, v. Maximum poloidal and toroidal modenumbers used in the Fourier representation of Φ were $M = 8, N = 8$ which lead to row and column dimensions for $\mathbf{L}$ of $N_b = 4096$ and $N_\Phi = 144$, respectively. $N_\Phi = 144$ approximate solutions of the inductance equation were obtained using the following procedure: For $j = 1, ..., N_\Phi$, all but the largest j singular values are set to zero for the calculation of the pseudoinverse of $\mathbf{L}$. Once the pseudoinverse is calculated, the solution vector $\mathbf{I}$ for the given value of j is determined by standard back-substitution. For $j = N_\Phi$, the solution is identical to the standard NESCOIL least-squares solution. For $j < N_\Phi$ the solutions are further regularized by the SVD cutoff, allowing a trade-off between accuracy of solution and maximum current density of the current sheet. The accuracy of solution is characterized by B_{err}^{rms} and B_{err}^{max}, the r.m.s. and maximum normal components of the magnetic field error on the plasma boundary normalized by the local total magnetic field. B_{err}^{rms} in particular is a measure of how well the current sheet solution can reconstruct the shape of the target equilibrium, and in practice we find that $B_{err}^{rms} \lesssim 1\%$ is required for accurate reconstruction of QA configurations. From Fig. 1 we see an essentially monotonic dependence of the fitting errors on the number of singular values retained. However, the dependence of the calculated sheet current density, J^{max}, on j is non-monotonic. By selecting the particular current sheet obtained by retaining 126 finite singular values, and using this sheet for cutting coils (see Sec. 4), the current sheet density is reduced by approximately 10% compared with the standard least-squares NESCOIL solution. In other cases, reductions in current sheet density of up to 50% have been achieved.

3. A Genetic Algorithm for Cutting Discrete Coils[3]

Once the current potential $\Phi(u,v)$ is calculated from NESCOIL, a set of discrete coils can be obtained by selecting N_c appropriate contours of Φ and interpreting each as a filamentary coil. In the limit as $N_c \to \infty$ the discrete coil system reproduces exactly the magnetic field of the current sheet. For a practical coil system, however, we must choose a coil set with the following minimum set of properties: (1) the number of coils should be small, to allow for heating and diagnostics; (2) the reconstruction errors (measured by how well the boundary conditions $\mathbf{b} = \mathbf{0}$ are satisfied at the plasma boundary) should satisfy $B_{err}^{rms} \lesssim 1\%$, and (3) the maximum coil current should be small ($< 20kA/cm^2$) to minimize resistive dissipation which limits the flat-top time of the magnetic field. Various algorithms have been explored for choosing the optimum set of contours to consider as coils. Among these, a Genetic Algorithm(GA)[4], which is an adaptive search and optimization method that simulates natural evolution processes of biological organisms, has been found to greatly improve our ability to find coil designs which realize the coil design targets (maximum current density, coil complexity,...).

GA's work with a population of 'individuals' each of which represents a possible solution to the optimization problem. An individual is assigned a 'fitness' according to how well it satisfies the optimization targets. In the present application, an individual is defined to be a particular subset of potential contours, and the fitness measure is a linear combination of B_{err}^{rms} and I_c^{max}. The fittest individuals are allowed to reproduce by cross-breeding, thereby producing a new generation of individuals (population of new solutions) that contains a high proportion of the best characteristics of the previous generation. In this way, over successive generations, good characteristics are spread throughout the population and the most promising areas of the search space are explored. The GA incorporates "mutation" during evolution, which encourages finding the global, rather than a local, minimum state.

Full details of the GA applied to the problem of cutting stellarator coils are presented in ref. [3]. Here, we simply demonstrate the usefulness of this coil-cutting algorithm for the c82 plasma configuration, for saddle coil topology, by comparing results using the GA with those from the conventional algorithm which chooses N_c contours equally spaced in Φ, having equal currents in each of the coils. To achieve $B_{err}^{rms} \leq 1\%$ with equally spaced contours, it was found necessary to have $N_c = 13$ coils per half-period. This gives a corresponding maximum coil current density of $I_c^{max} = 14.7kA/cm^2$. Table 1 shows results from a sequence of GA runs assuming different values of N_c and targeting a linear combination of B_{err}^{rms} and I_c^{max} in the cost (fitness) function. The GA is seen to reduce the number of required coils by a factor of 3 while achieving equal, or somewhat lower, values for the targeted quantities.

Method	N_c	$B_{err}^{rms}\%$	$B_{err}^{max}\%$	$I_c^{max}[kA/cm^2]$
Equi-Φ	13	0.95	7.0	14.7
GA	7	0.52	2.8	14.2
GA	6	0.61	3.8	12.7
GA	5	0.77	5.7	13.2
GA	4	0.92	5.0	14.2

Table 1: *Comparison of mean and max fitting errors at plasma boundary, and max coil current density for various numbers of coils per half-period.*

4. COILOPT: A code for Designing Modular Coils[5]

The coil design techniques discussed in previous sections have focussed largely on the use of saddle coils with a background toroidal field. Modular coils provide both poloidal and toroidal magnetic field components and pose additional design issues. Unlike saddle coils, no acceptable modular coil designs based on conformal winding surfaces have been found for compact stellarator configurations: magnetic field errors for a reasonable number of coils are simply too large. This results from a tradeoff between current density, which requires a relatively close plasma-coil spacing, and ripple errors that favor large plasma-coil separations.

To address these issues, COILOPT, a coil optimization code was developed. The primary difference between this and similar codes such as ONSET[6] is in the representation of the coils. Coils lie on a winding surface with (typically) the toroidal location being given as a Fourier series in the poloidal angle. The winding surface is described by the usual Fourier series in the poloidal and toroidal angles (θ and ϕ respectively) for R and Z, namely,

$$R = \sum R_{mn} \cos(m\theta + n\phi), Z = \sum Z_{mn} \sin(m\theta + n\phi). \quad (1)$$

The toroidal position of a coil on this surface, the winding law, is given by

$$\phi(\theta) = \phi_0 + \sum_k [a_k \cos(k\theta) + b_k sin(k\theta)]. \quad (2)$$

This representation leads to a coil set that depends on, typically, of order a hundred independent parameters for the winding surface and the coil winding law. This is a factor 10-100 less than the number of parameters that are required to describe a coil set composed of a set of short segments. As a result of this reduction in the number of independent parameters, coil designs can be produced using a few hours of IBM RISC 6000 time.

The optimization uses the Levenberg-Marquardt algorithm. In addition to targeting the magnetic field error B_{err}^{rms}, measures of plasma coil separation and coil to coil separation are used to control current density. Similarly, measures of coil curvature and length are used to control the variation of the winding surface and to produce coils acceptable from an engineering standpoint. Allowing the winding surface shape to vary during the optimization is key to the successful application of COILOPT. A modular coil set for the LI383 configuration was developed using COILOPT and is illustrated in Fig. 2. The current density in the copper is about $12kA/cm^2$, in the range that would permit coils to operate at room temperature.

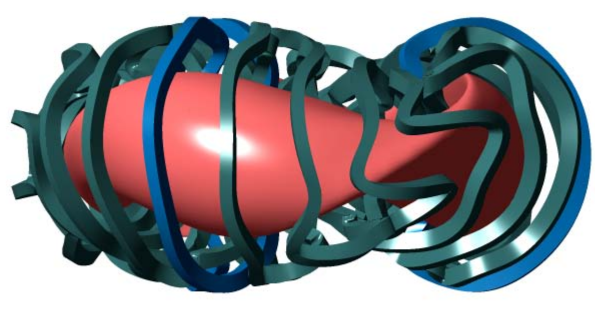

Figure 2: *Modular coils for LI383 with seven coils/period and four unique coils*

5. Integrated Coil-Plasma Design Methods

The traditional methodology for coil-plasma system design is based on a two-stage approach. In the first stage, a plasma configuration is identified by the fixed-boundary plasma optimizer. This provides a target boundary surface and normal distribution of $\vec{B}$ on the plasma surface derived from currents flowing in the plasma volume. In the second stage coils are sought which attempt to match normal magnetic fields on the specified plasma boundary. Once the coil geometry is determined engineering figures of merit are analyzed, such as current density, minimum radius of curvature, etc. If these are unsatisfactory, the configuration is modified and the process is repeated. Iteration usually leads to a solution that meets a set of engineering requirements.

In NCSX design we have had success incorporating some engineering constraints into the fixed-boundary plasma optimizer, a procedure which greatly improves the efficiency of the two-stage coil design process. For example, a call to NESCOIL at each major step of the physics optimizer provides input to an auxiliary penalty function which measures the magnitude of J^{max}, the maximum current sheet density, and $\mathcal{C}$, the current sheet "complexity"[8] (mathematically the enstrophy of the current potential, a measure of the lumpiness of the current potential).

Additional penalty function strategies are being developed which relate to issues of plasma control. The basic idea is to penalize configurations developed by the optimizer which require short wavelength magnetic fields for their reconstruction. Any distribution of current on a CWS can be expanded is a complete set of "natural functions"[7]. Each function is an eigenfunction of an in-surface Helmholtz operator and is associated with an eigenvalue which tells us how rapidly the magnetic field strength due to that distribution of current decreases with distance from the surface. The lowest order natural functions have the smallest eigenvalues and decay most slowly with distance from the surface. We are presently exploring various methods for constraining configuration shapes such that their associated current sheet solutions can be fit exactly with low-order natural functions (say the lowest $N_f \sim 50$ or so). This can be done within the context of a free- or fixed-boundary optimization code. For example, once a CWS is defined the natural function current distributions can be calculated and a free-boundary optimizer can vary the N_f coefficients of the natural functions to minimize physics penalty functions (measuring quasi-axisymmetry, stability and other physics or engineering measures such as current sheet complexity). Alternatively, in a fixed-boundary optimizer, for each step that the plasma configuration changes shape a CWS can be generated, and the associated natural functions calculated. A penalty function can then be evaluated representing the failure to fit the calculated normal $\vec{B}$ at the plasma boundary with the lowest N_f of these functions.

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