Nonlinear Features of the Electron Temperature Gradient Mode and Electron Thermal Transport in Tokamaks

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Abstract. Analytical investigations of several linear and nonlinear features of ETG turbulence are reported. The linear theory includes effects such as finite beta induced electromagnetic shielding, coupling to electron magnetohydrodynamic modes like whistlers etc. It is argued that nonlinearly, turbulence and transport are dominated by radially extended modes called 'streamers'. A nonlinear mechanism generating streamers based on a modulational instability theory of the ETG turbulence is also presented. The saturation levels of the streamers using a Kelvin Helmholtz secondary instability mechanism are calculated and levels of the electron thermal transport due to streamers are estimated.

1. Introduction

The physics of electron and ion heat transport in tokamaks is a problem of considerable current interest. Experiments and simulations have convincingly demonstrated that ion heat transport is dominated by the ion temperature gradient (ITG) instability. Excellent ion confinement (comparable to neoclassical predictions) has been observed in negative central shear plasmas where the ITG mode is stabilized by a combination of strong velocity shear and reverse magnetic shear and an internal transport barrier (ITB) is formed [1]. A comparable improvement of electron heat transport is typically not observed in ITB plasmas. It is only in cases of strong negative magnetic shear that some improvement of electron heat transport is seen [1]. It thus seems likely that electron heat transport is determined by short scale fluctuations ($k_\perp \rho_i \gg 1$, ρ_i being the ion Larmor radius) which do not influence ion heat transport, are unaffected by magnitudes of velocity shear which stabilize the ITG mode (i.e., growth rate $\gamma > \gamma_{ITG} \sim \omega_{E\times B}$), and are stabilized by strong negative magnetic shear. A mode which has all these features is the electron temperature gradient mode (ETG) driven by field line curvature effects. The linear and nonlinear theories of this mode are thus of considerable interest and form the subject of the current paper.

The possible relevance of electrostatic ETG modes to electron heat transport in tokamaks was investigated by several authors [2]. It was shown that when $\eta_e = d\ln T_e/d\ln n$ exceeds exceeds a critical value, short-scale ($\rho_e \leq \lambda_\perp \ll \rho_i$) fast growing ($\gamma \sim |\omega| \sim c_e/(L_T R)^{1/2}$, c_e is the electron thermal speed and L_T, R are the temperature and curvature scale lengths) electrostatic modes are excited. These modes typically leave the ion transport unaffected and give a quasilinear mixing length estimate of electron thermal conductivity $\chi_{ETG} \sim \rho_e^2 c_e/L_n$; this coefficient is however, too small to explain the observations. Ohkawa [3] pointed out that inclusion of electromagnetic effects may enhance the transport to $\chi_e \sim (c^2/\omega_{pe}^2)(c_e/qR)$ which is closer to empirical observations. Detailed calculations for ETG modes with electromagnetic effects were carried out by Guzdar [4], Horton [2] and made attempts to justify Ohkawa's phenomenological estimates again using quasi linear mixing length arguments. These attempts were however, not very convincing. More recently [5, 6] it has been argued on the basis of particle and fluid simulations that temperature gradient driven modes are nonlinearly dominated by radially extended nonlinear perturbations called 'streamers'. These streamers saturate by a secondary Kelvin Helmholtz (K-H) like instability mechanism [7] and can lead to transport values much larger than the mixing length estimates.

In this paper we have analytically re-examined aspects of linear and nonlinear features of the ETG instabilities. Starting with the Braginskii fluid equations, we first look at the linear theory of these instabilities taking account of coupling to magnetic flutter perturbations ($B_\perp$ effects) as well as to compressible magnetic perturbations ($B_\parallel$ effects). We find that coupling to flutter perturbations is stabilizing for the ETG mode. We also consider the unexplored case of $\gamma/kv_i > 1$, when the ETG mode directly couples to whistler like perturbations (because ion response is negligible). In this limit, conditions for electron magnetohydrodynamic (EMHD) ballooning instabilties which arise when curvature effects overcome the restoring forces due to EMHD effects are derived. We next argue from general considerations that nonlinear ETG structures should be dominated by radially extended streamers. The generation of such streamer like states from a homogeneous isotropic turbulent state through a modulational instability mechanism is examined by a kinetic wave equation treatment. To estimate the saturated level of streamer like perturbation we carry out a secondary K-H instability analysis of such structures. Since the primary streamer structures are periodic in space, their secondary instability theory can be carried out by using a Floquet type analysis. Using such techniques, we estimate the growth rates of secondary instabilities. It is argued that the primary streamer instability will saturate when the growth rate of the secondary instability matches that of the primary instability. This allows us to estimate the saturation level of streamers as well as the magnitude of transport due to them.

2. Basic Equations

The ETG modes satisfy the following frequency and wavelength restrictions : $\Omega_i \leq \omega \sim \omega_* \ll \Omega_e$, $kc_i > \omega > k_\parallel c_e$, $\rho_i \gg \lambda_\perp \geq \rho_e$. Here Ω_j are the respective cyclotron frequencies, $\rho_j = c_j/\Omega_j$ the Larmor radii and $c_j = \sqrt{T_j/m_j}$ the thermal velocities. $\omega_* \sim k_\theta \rho_e c_e/L_n$ is the diamagnetic drift frequency and other relevant parameters are $\omega_{*T} = \eta_e \omega_*$, $\omega_{*p} = (1+\eta_e)\omega_*$, $\eta_e = L_n/L_{T_e}$, $\omega_{de} = \epsilon_n \omega_*$, $\epsilon_n = 2L_n/R$, L_n, L_T, R being respectively, the density, temperature and curvature scale-lengths typically satisfying the condition $R > L_n > L_T$.

Our chief objective is to present a set of equations which takes account of the coupling of the ETG modes to the perpendicular and parallel magnetic field perturbations $(\delta B_\perp, \delta B_\parallel)$. We shall use the reduced Braginskii 2-fluid equations to derive a set of model equations for the ETG mode in the hydrodynamic approximation. Under the conditions outlined above $(\omega \geq \Omega_i, k_\perp \rho_i \gg 1, \omega < kv_i)$, ions are an unmagnetized species and satisfy the Boltzmann condition $\tilde{n}_i = -\tau\tilde{\phi} \simeq \tilde{n}_e$, where $\tilde{n_j} = \delta n_j/n_{0j}$, $\tau = T_e/T_i$, $\tilde{\phi} = e\delta\phi/T_e$ and the last approximate equality follows from requirement of quasi-neutrality. For very short scales such that $k_\perp \lambda_{De} \sim 1$, we may consider deviations from quasi-neutrality, in which case, we will write $\tilde{n}_i = -\tau\tilde{\phi} = \tilde{n}_e - \lambda_D \nabla^2 \tilde{\phi}$. The fluid model for electrons is analogous to the one used for ions in the ITG mode [9] and consists of the electron continuity equation, parallel equation of motion and the temperature equation coupled with expressions for perpendicular drifts of the electron fluid. These drifts may be written as

$$v_{e\perp} = -\frac{c}{B^2}\bar{B}\times\left(\bar{E}+\frac{\bar{\nabla}p_e}{en}\right)\left(1-\frac{\delta B_{\parallel}}{B_0}\right) - \frac{c}{B\Omega_e}\left(\frac{d}{dt}+v_{*p}\cdot\nabla\right)\bar{E}_{\perp} + v_{\parallel}\frac{\delta\bar{B}_{\perp}}{B_0} \tag{1}$$

where in addition to the well known $\bar{E}\times\bar{B}$, diamagnetic and polarization drifts (modified by $\delta B_{\parallel}$ effects) we have included the last term which is a perpendicular drift due to magnetic flutter perturbations. Note that $d/dt = \partial/\partial t + \bar{v}_E\cdot\bar{\nabla}$ and in writing the polarization drift term, we have used the well known cancellation between the convective diamagnetic contributions and drifts due to stress tensor. Using Eq. (1) to eliminate $\tilde{n}_e$ and the definition $\tilde{v}_{e\parallel} \simeq -\tilde{J}_{\parallel}/en_0 = (c_e^2 c/\omega_p^2)\nabla_{\perp}^2\tilde{A}_{\parallel}$ (where, ion parallel response currents are negligible because of Boltzmann equilibration and $\tilde{A}_{\parallel} = eA_{\parallel}/T$ is the normalized parallel component of vector potential) to eliminate $\tilde{v}_{e\parallel}$, we get the normalized equations

$$\begin{aligned}\left(\tau-\nabla_{\perp}^2\right)\partial_t\tilde{\phi} - \left[1-\tilde{\epsilon}_n\left(1+\tau\right)+\left(1+\eta_e\right)\nabla_{\perp}^2\right]\nabla_y\tilde{\phi} - \tilde{\epsilon}_n\partial_y\tilde{T}_e - \nabla_{\parallel}\nabla_{\perp}^2\tilde{A}_{\parallel} + \partial_t\delta\tilde{B}_{\parallel} \\ = \left[\tilde{\phi},\nabla_{\perp}^2\tilde{\phi}\right] - (\beta_e/2)\left[\tilde{A}_{\parallel},\nabla_{\perp}^2\tilde{A}_{\parallel}\right] - \left[\tilde{\phi},\delta\tilde{B}_{\parallel}\right]\end{aligned} \tag{2}$$

$$\begin{aligned}\left[\left(\beta_e/2-\nabla_{\perp}^2\right)\partial_t + \left(1+\eta_e\right)(\beta_e/2)\nabla_y\right]\tilde{A}_{\parallel} + \nabla_{\parallel}\left(\tilde{\phi}-\tilde{p}_e\right) \\ = -\left(1+\tau\right)(\beta_e/2)\left[\tilde{\phi},\tilde{A}_{\parallel}\right] + (\beta_e/2)\left[\tilde{T}_e,\tilde{A}_{\parallel}\right] + \left[\tilde{\phi},\nabla_{\perp}^2\tilde{A}_{\parallel}\right]\end{aligned} \tag{3}$$

$$\partial_t\tilde{T} + (5/3)\tilde{\epsilon}_n\nabla_y\tilde{T} + \left(\eta_e-(2/3)\right)\nabla_y\tilde{\phi} + (2\tau/3)\partial_t\tilde{\phi} = -\left[\tilde{\phi},\tilde{T}\right] \tag{4}$$

where $[a,b] = (\partial_x a\partial_y b - \partial_x b\partial_y a)$ is the Poisson bracket typical of convection, and other fluid plasma nonlinearities, x and y are normalized to ρ_e, z to L_n (x, y, z being respectively, the radial, poloidal and toroidal coordinates), we have defined normalized variables $\left(\tilde{\phi},\tilde{n},\tilde{T},\delta\tilde{B}_{\parallel}\right) = L_n/\rho_e\left(e\delta\phi/T_{e0},\delta n/n_0,\delta T_e/T_{e0},\delta B_{\parallel}/B\right)$, $\tilde{A}_{\parallel} = (2c_eL_n/\beta_e c\rho_e)eA_{\parallel}/T_{e0}$, $\tilde{p}_e = \tilde{n}+\tilde{T}_e$, $\beta_e = 8\pi nT_e/B_0^2$. Note that as in the standard ballooning formalism, we may interpret (in the linear approximation), $\nabla_{\perp}^2 f = -k_{\perp}^2 f = -k_{\theta}^2 f\left(1+(s\hat{\theta}-\alpha\sin\hat{\theta})^2\right)$, $\tilde{\epsilon}_n = \epsilon_n\left[\cos\theta+(s\theta-\alpha\sin\theta)\sin\theta\right]$, $\nabla_{\parallel}f = ik_{\parallel}f \simeq (1/qR)\left(\partial f/\partial\theta\right)$ where θ is the extended coordinate in the ballooning formalism, $q \equiv rB_{\phi}/RB_{\theta}$ is the safety factor, r and R are the minor and major radius of the machine, B_{θ} and B_{ϕ} are the poloidal and toroidal magnetic fields, $s = r\,d\ln q/dr$ is the magnetic shear parameter, $\alpha = \left(2\beta_e q^2/\epsilon_n\right)\left(1+\eta_e+\left(1+\eta_i\right)/\tau\right)$ is the Shafranov shift. The above equations are to be supplemented with an equation for $\delta B_{\parallel} = (\nabla\times A)\cdot\hat{e}_{\parallel}$ related to $\bar{A}_{\perp}$, the perpendicular component of vector potential satisfying the equation $\nabla_{\perp}^2\bar{A}_{\perp} = -\left(4\pi/c\right)\bar{J}_{\perp} = \left(4\pi en_0/c\right)\left(\underline{v}_E+\underline{v}_{*p}\right)$. For $k_{\parallel} < k_{\perp}$, this equation may be written as $\delta\tilde{B}_{\parallel} = \beta_e\left(\tilde{p}_e-\tilde{\phi}\right)/2$. Note that in this approximation $\delta\tilde{B}_{\parallel}$ does not have an evolution equation and that our equation is a fluid analogue of Eq. (6) of Jenko [6].

3. Linear Theory

We use a semi-local theory for obtaining the eigenvalue solutions of above coupled Eqs. (2)-(4) with $\delta\tilde{B}_{\parallel} = 0$. For that we take a strongly ballooning function of the form $\tilde{f}(\theta) = (1+\cos\theta)\tilde{f}/\sqrt{3\pi}$; $\theta < \pi$, and substituting $\nabla_{\parallel} = -i\langle k_{\parallel}\rangle$, $\nabla_{\perp}^2 = -\langle k_{\perp}^2\rangle$, and $\tilde{\epsilon}_n = \langle\epsilon_n\rangle$ in Eqs. (2)-(4), where $\langle\cdots\rangle = \int_0^{2\pi}(f^*\cdots f)d\theta/\int_0^{2\pi}|f|^2 d\theta$ denotes an average over the eigenfunction [8], the linear semi-local dispersion relation can be obtained as

$$\omega^2\left(\tau+\langle k_\perp^2\rangle\right)+\omega k_\theta\left[1-\langle\tilde\epsilon_n\rangle\left(1+(10\tau/3)\right)-\langle k_\perp^2\rangle\left(1+\eta_e+(5\langle\tilde\epsilon_n\rangle/3)\right)\right]+\langle\tilde\epsilon_n\rangle k_\theta^2\times$$
$$\left[\eta_e-(7/3)+(5/3)\langle\tilde\epsilon_n\rangle\left(1+\tau\right)+(5/3)\langle k_\perp^2\rangle\left(1+\eta_e\right)\right]=\frac{\omega-\frac{5}{3}\langle\tilde\epsilon_n\rangle k_\theta}{\omega}\frac{\langle k_\perp^2\rangle\langle k_\parallel^2\rangle\bar\omega}{\left(\langle k_\perp^2\rangle\omega+\frac{\beta_e}{2}\bar\omega\right)} \quad (5)$$

where, $\bar\omega=\omega-(1+\eta_e)\,k_y$, $\langle k_\parallel^2\rangle=\epsilon_n^2/12q^2$, $\langle k_\perp^2\rangle=k_\theta^2\left[1+\frac{s^2}{3}\left(\pi^2-7.5\right)-\frac{10}{9}s\alpha+\frac{5}{12}\alpha^2\right]$, $\langle\tilde\epsilon_n\rangle=\epsilon_n\left[\frac{2}{3}+\frac{5}{9}\hat s-\frac{5}{12}\alpha\right]$, and $\tilde p_e\approx(\bar\omega/\omega)\,\tilde\phi$ is assumed in the parallel compression term of Eq. (3). The Eq. (5) contains both the slab and toroidal version of electromagnetic ETG mode. The slab branch of ETG mode has been studied extensively earlier by various authors. Here, we discuss the toroidal ETG mode in semi-local limits. In the ballooning limit $k_\theta^2>\epsilon_T/2q$, the parallel compression term on the r.h.s of Eq. (5) can be treated perturbatively. Eq. (5) to the leading order gives the growth rate $\gamma_0\simeq k_\theta\left(\epsilon_n/\tau\right)^{1/2}\left(\eta_e-\eta_{th}\right)^{1/2}$ where, $\eta_{th}=2/3-1/2\tau+1/4\epsilon_n\tau+\epsilon_n\left(1/4\tau+10\tau/9\right)$ and the terms proportional to $k_\theta^2\rho_e^2$ are neglected. Including the r.h.s perturbatively we find that the parallel electron motion and electromagnetic shielding to $E_\parallel$ results in a shift in real frequency and stabilizing contribution to the growth of the mode.

We now discuss the numerical solutions of the toroidal branch of ETG mode in local limit ($\theta=0$), neglecting parallel electron motion (with $k_\parallel=0$). We benchmark the fluid model by comparisions with the gyrokinetic results of Horton et al. Figure (a) gives the plot of γ versus η_e for three different values of $k_\theta=0.1,0.3,0.5$, and in Fig. (b) the plot of γ versus L_n/R is displayed with $\eta_e=1.5$, 2, and 2.5, (the other plasma parameters are $\epsilon_n=0.3$, $\tau=1$). Note that the gyrokinetic results of Horton et al (see their Figs. 4 and 5) are reproduced with the fluid theory. The dependence on s and $\alpha(\beta_e)$ is shown in Figs. (c) and (d). The growth rate versus k_θ from the full semi-local dispersion relations are illustrated. For $s=1$, $q=1.4$, low $\beta_e=0.01$, $L_n/R=0.1$, and $L_n/R=0.3$, the growth is found to reach a maximum around $k_\theta=0.6$, and complete stabilization of the mode occurs for $k_\theta<0.9$. However, the model includes only first order FLR effects and is not accurate in this regime. Note that for $L_n/R=0.1$ and high value of β_e such as 0.05, and 0.08, the growth of the mode is reduced to a low value. For $L_n/R=0.3$, similar reduction in the growth rate of ETG mode is observed at very large value of $\beta_e\geq 0.08$. Thus magnetic flutter effects which are important at high β_e produce a stabilizing influence on the ETG mode.

We have also investigated the dispersion relation in the limit $|\omega|>kv_i$, taking account of coupling to $\delta\tilde B_\parallel$, $\delta\tilde B_\perp$ perturbations. This is the limit in which the density response is negligible ($\tilde n_i\to 0$, $\tilde n_e\sim k^2\lambda_D^2\tilde\phi\ll\tilde\phi$) and the ETG instability gets coupled to EMHD physics and the whistler mode. For $\eta_e\gg 1$, the local dispersion relation takes the form

$$(1+k_\perp^2\lambda_s^2)\omega^2-\omega\omega_{*T}\left(2+k_\perp^2\lambda_s^2\right)+2\omega_d\omega_{*T}/\beta_e-\left(k_\parallel k_\perp\lambda_s^2\right)^2\Omega_e^2\left(1-\omega_{*T}/\omega\right)/$$
$$\left(1+k_\perp^2\lambda_s^2-\omega_{*T}/\omega\right)=0 \quad (6)$$

For $\omega_{*T}\to 0$, this gives the whistler wave dispersion relation $\omega=k_\parallel k\lambda_s\Omega_e/(1+k_\perp^2\lambda_s^2)$, which in the electrostatic limit $k_\perp\lambda_s\gg 1$ reduces to the mode $\omega=\Omega_e k_\parallel/k$. For $k_\perp^2\lambda_s^2\leq 1$, and $\omega_{*T}\neq 0$, we get the EMHD ballooning instability

$$\omega=\omega_{*T}\pm\left(k_\parallel^2k_\perp^2\lambda_s^4\Omega_e^2-2\omega_d\omega_{*T}/\beta_e\right)^{1/2} \quad (7)$$

giving a threshold condition $L_T/q^2R \sim \epsilon_T/q^2 < c_e^2/c^2$ and a typical growth rate $\sim (2\omega_d\omega_{*T}/\beta)^{1/2} \sim c_e c k_y/\omega_p\sqrt{RL_T} \leq c_e/\sqrt{RL_T}$. The condition for neglect of ion response requires $\gamma > kv_i$ or $(c/\omega_{pi})(1/\sqrt{RL_T}) > 1$ which requires strong temperature gradients. In the electrostatic limit, the threshold condition is $\epsilon_T/q^2 < k^2\rho_e^2$ and the $\gamma > kv_i$ condition takes the form $(c/\omega_{pi})(1/\sqrt{RL_T}) > kc/\omega_{pe} \gg 1$.

4. Streamer Physics

In this section we elucidate several aspects of streamer physics relevant to the ETG mode. Eqs. (2)-(4) show that if $\partial/\partial x = 0$, all nonlinear terms vanish. Thus, such modes will grow indefinitely, as predicted by the linear theory. This indicates that modes with large radial extent may grow to large values. A similar result may be inferred from a scaling transformation analysis of Eqs. (2)-(4). For simplicity, we restrict our attention to electrostatic perturbations in which effects due to parallel and perpendicular compression effects are neglected. We thus look at Eqs. (2) and (4) only, take $A_{\|} \to 0$ and also ignore the polarization drift nonlinearities. The only nonlinearity retained is then the convection nonlinearity in the T equation. The resulting equations can be shown to be invariant under the following scale transformations : $x \to ax, \quad y \to by, \quad , z \to cz, \quad t \to bt, \quad \phi \to a\phi, \quad T \to aT$. The amplitude at saturation thus linearly scales with the x scale. This shows that modes with larger radial extent will have larger saturation amplitudes. We may thus form large amplitude anisotropic eddies in the xy plane with a $q_y \gg q_x$ where $\bar{q}$ is the wave-vector for streamers. Such eddies will be called 'streamers'.

We now carry out a modulational instability analysis [10] which shows that homogeneous isotropic turbulence of ETG modes is unstable to the formation of streamers. A wave kinetic description is used to describe the background short scale ETG turbulence; the basic variable describing these waves is the action density $N_k = E_k/|\omega_r| \simeq \varepsilon_0|\phi_k|^2/|\omega_r|$, where $\omega_r \simeq (k_y/2)(13\epsilon_n/3 - 1)$ and $\varepsilon_0 = \tau + k_\perp^2 + \eta_e k_y^2/|\omega_r| + 2k_{\|}k_\perp^2/\beta_e|\omega_r|$ and we have used the linearized expressions from ETG theory to express the energy density in potential, temperature and electromagnetic field fluctuations in terms of $|\phi_k|^2$. The wave kinetic equation may be written as

$$\partial_t N_k + \bar{v}_g \cdot \bar{\nabla} N_k - \nabla_r \left(\omega + \bar{k} \cdot \bar{v}_E\right) \cdot \nabla_k N_k = \gamma_k N_k - \Delta\omega_k N_k^2 \tag{8}$$

where γ_k is the linear growth rate and $\Delta\omega_k N_k$ is a model nonlinear damping rate accounting for local couplings, which in standard theory leads to the mixing length saturation amplitude $N_{k0} \simeq 2\gamma_k/\Delta\omega_k$. We now perturb the background turbulence with long wave modulations (Ω, q) described by Eqs. (2) to (4). The perturbation in the action density $\delta N_k(q, \Omega)$ will be driven by the slow variation of $\omega + \bar{k} \cdot \bar{v}_E$ viz,

$$\delta\omega + \bar{k} \cdot \delta\bar{v}_E = [1/2 + 13\epsilon_n/6]\, k_y \partial_x n_q + (5\epsilon_n/3)(\rho/L_n) k_y T_q - k_x \partial_y \phi_q \tag{9}$$

and for $q_y \gg q_x$ is approximately given by $\delta N_k(q, \Omega) = q_y^2 k_x \phi_q \mathrm{Re}(q, \Omega)\left(\partial N_{k0}/\partial k_y\right)$, where $\mathrm{Re}(q, \Omega) = \gamma_k / \left[(\Omega - \bar{q} \cdot \bar{v}_g)^2 + \gamma_k^2\right]$. The return coupling of the fast wave action density on the slow wave modulation equations (2)-(4) will enter through the nonlinear coupling terms which may be identified as follows : Reynolds stress term $[\phi, \nabla^2\phi] \simeq -q_y^2 \sum_k k_x k_y \frac{|\omega_r|}{\varepsilon_0} \delta N_k(q, \Omega)$, Magnetic stress term $\left[A_{\|}, \nabla^2 A_{\|}\right] \simeq -q_y^2 \sum_k k_x k_y \Lambda_0 \frac{|\omega_r|}{\varepsilon_0} \delta N_k(q, \Omega)$, Convection nonlinearity effect $[\phi, T] \simeq$

$-iq_y\eta_e \sum_k \frac{k_y}{\gamma_k}\frac{|\omega_r|}{\varepsilon_0}\delta N_k\left(q,\Omega\right)$ where

$$\Lambda_0 = |A_{\|}|^2/|\phi|^2 \equiv \left(k_{\|}^2/\gamma_k^2\right)\left(\gamma_k^2 + \left(1+\eta\right)^2 k_y^2\right) / \left(\gamma_k^2\left(k_\perp^2 + \beta_e/2\right)^2 + \beta_e^2\left(1+\eta\right)^2 k_y^2/4\right),$$

denotes the relative contribution due to magnetic flutter nonlinearity (magnetic stresses), and it has been assumed that $\omega_r < \gamma_k$.

Following standard stability analysis, we may finally write the dispersion relation for streamers, (viz. modulations with $q_y \gg q_x$) as

$$\Omega^2\left(\tau + q_\perp^2\right) + \Omega q_y\left[1 - \epsilon_n\left(1 + 10\tau/3\right) - \left(1 + \eta_e + 5\epsilon_n/3\right)q_\perp^2 - i\sigma_1/q_y\right] + \epsilon_n q_y^2\left[\eta_e - 7/3 + \left(5\epsilon_n/3\right)\left(1+\tau\right) + (5/3)q_\perp^2\left(1+\eta_e\right) + \sigma_2/q_y^2\right] = 0 \qquad (10)$$

where $\sigma_1 = -q_y^4\sum(1-\beta_e\Lambda_0/2)(k_x^2|\omega_r|/\varepsilon_0)(k_y\partial N_{k0}/\partial k_y)\mathrm{Re}(q,\Omega)$, $\sigma_2 = -\eta_e q_y^3(\sum k_x/\gamma_k)\times(|\omega_r|/\varepsilon_0)(k_y\partial N_{k0}/\partial k_y)\mathrm{Re}(q,\Omega)$. For $\partial N_k/\partial k_y < 0$, we see two simple regimes of modulational instability. One is the regime where σ_1, the drive due to Reynolds stresses dominates and we get from the first two terms of the dispersion relation $\gamma_q \cong \sigma_1/\left(\tau + q_\perp^2\right)$. Note that the inclusion of magnetic flutter effects through $\beta_e\Lambda_0$ terms provides a stabilizing influence on the modulational instability complete stabilization may result. A second regime is one where the convection nonlinearity in the pressure equation plays the dominant role and we have $\gamma_q \cong \left(\epsilon_n\sigma_2\right)^{1/2}$.

These considerations show that modulations with $q_y \gg q_x$ which are like radially extended streamers will grow on the background turbulence with significant growth rate. If we use mixing length argument $e\phi/T \sim 1/(q_x L_T)$ to estimate the saturation amplitude of these modes that tends to be a large value. It is therefore of interest to examine the conditions under which the strong velocity shear in the streamers may lead to their breakup due to the excitation of a secondary Kelvin Helmholtz instability. This may fix the saturation amplitude of the streamers at a more reasonable lower value and thence provide us with a method of estimating the anomalous transport caused by them. For simplicity, we consider a 1-d streamer $\phi_0 \cos qy$ which has no x dependence and consider perturbation governed by Eqn. (5) (in which we restore n to its original form) and write

$$\partial_t\delta n + [\phi_0,\delta n] + [\delta\phi, n_0] + \partial_t\nabla^2\delta\phi + \left[\phi_0,\nabla^2\delta\phi\right] + \left[\delta\phi,\nabla^2\phi_0\right] = 0 \qquad (11)$$

We now use the Poisson equation $\delta\nabla^2\phi = n + \phi/\tau$ where $\delta = \lambda_D^2/\rho_e^2 = \Omega_e^2/\omega_{pe}^2$ and the ion density n_i is replaced by $-\tau\phi$. The final equation takes the form (for $\tau = 1$)

$$-\partial_t\delta\phi + (1+\delta)\left\{\partial_t\nabla^2\delta\phi + \left[\phi_0,\nabla^2\delta\phi\right] + \left[\delta\phi,\nabla^2\phi_0\right]\right\} = 0 \qquad (12)$$

Since ϕ_0 is a periodic function of y, we may use the Floquet theorem to write $\delta\phi = \sum_n \delta\phi_n \sin\left[\left(k_y + nq\right)y + k_x x\right]\mathrm{e}^{\gamma t}$. Substituting in equation (12) we can get an infinite determinant for finding the eigenvalue γ. When ϕ_0 is small, the determinant may be truncated to a 3×3 and we get the secondary K-H instability dispersion relation :

$$\gamma_{KH}^2 = \left(1+\delta\right)^4\frac{k_x^2q^2\left(q^2-k^2\right)}{1+\left(1+\delta\right)k^2}\frac{\phi_0^2}{4}\left[\frac{2k_yq + k^2}{1+\left(1+\delta\right)\left(k^2+q^2+2k_yq\right)} + \frac{k^2 - 2k_yq}{1+\left(1+\delta\right)\left(k^2+q^2-2k_yq\right)}\right] \qquad (13)$$

Eq. (13) leads to the following conclusions. The secondary instabilities are restricted to scale $k^2 \le q^2$ and have a typical growth rate (since $q^2 \le 1$) $\gamma_{KH} \sim (1+\delta)^2 k_x k q^2\phi_0/\sqrt{2}$.

Equating this growth rate to that of the larger of modulational instability/background ETG instability we get an estimate of the saturation amplitude of streamers : $e\phi_{st}/T \sim (\rho_e/L_T)(1/q\rho_e))^3$.

5. Transport Estimates and Conclusions

The heat flux due to electrostatic ETG modes may be estimated as $\langle \tilde{v}_r \tilde{T} \rangle$ giving an electron thermal conductivity coefficient $\chi \sim L_T\langle(\omega_{*T}/\omega)(e\phi/T)(ik_y\phi c/B)\rangle \simeq L_T|(e\phi/T)|^2 c_e k_y \rho_e$ Taking $k_y c\phi/B\omega \sim 1/k_x$ gives a saturation amplitude $e\phi/T \sim 1/k_x L_T$ which when plugged into the above expression, gives $\chi \sim \rho_e^2\,(c_e/L_T)$ (since $k_y\rho_e \sim 1$). We now wish to make an estimate of the transport due to streamers. These fluctuations have large amplitudes and saturate only due to K-H instabilities. The saturation amplitude is given by $e\phi_{st}/T \sim (\rho_e/L_T)(1/q\rho_e)^3$ and leads to a thermal conductivity coefficient $\chi \sim (c_e\rho_e^2/L_T)(1/q\rho_e)^5$. For $q\rho_e \sim 0.5$ (note $q \leq k_{max}$ where k_{max} is the wave-vector of the maximally growing linear mode), this gives an enhancement by a factor of 64 over the mixing length estimate. The scaling of χ with parameters is difficult to determine, since the value of $q\rho_e$ where the turbulence has dominant streamers is determined by properties of growth of background ETG turbulence and the nonlinear effects due to modulational instabilities.

In conclusion we have examined certain interesting features of the ETG mode believed to be responsible for electron thermal transport in tokamaks. We have shown that finite beta coupling to magnetic flutter perturbation stabilizes the linear mode. We have given general arguments which indicate that the turbulent transport will be dominated by radially extended large amplitude nonlinear structures called streamers. This is followed up with a modulational instability calculation which estimates the growth rate of streamers. The saturation level of streamers and transport due to them is finally estimated from a Kelvin Helmholtz like secondary instability mechanism. It is shown that streamer transport can readily exceed the mixing length ETG transport by one to two orders of magnitude - thus coming close to explaining experimental observations and recent particle simulations. In the modulational instability calculations it is shown that magnetic flutter nonlinearities at finite β stabilize the modulational instability. This suggest that at higher β, streamer transport may be less virulent, as also seen in particle simulations.

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Fig.a

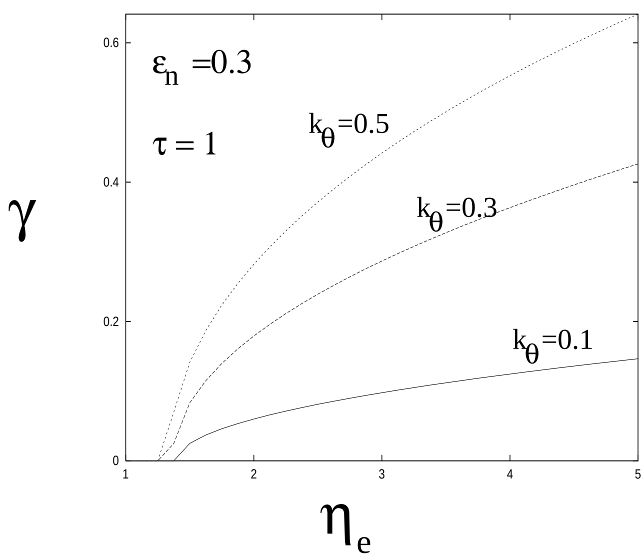


Fig.b

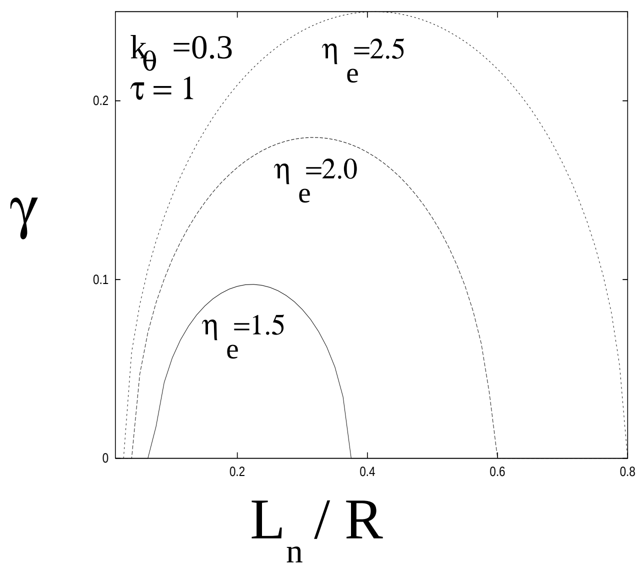


Fig.c

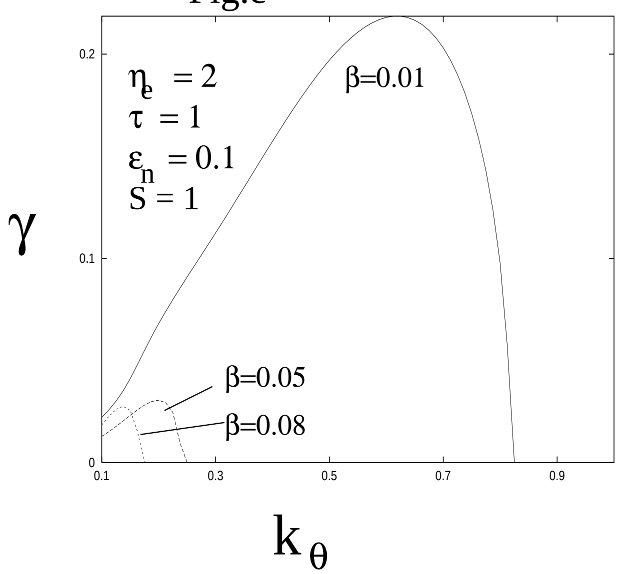
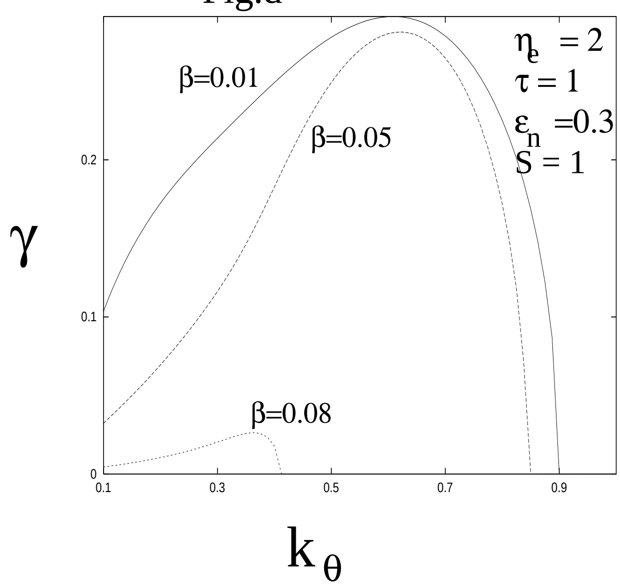


Fig.d



Figs: (a) and (b) Plot of growth rate for local (at $\theta=0$) dispersion relation.
(c) and (d) Plot of growth rate for full dispersion relation assuming wave function $\phi \sim (1+ \cos\theta)/\sqrt{3\pi}$ (strong balooning limit).