Electrostatic Turbulence with Finite Parallel Correlation Length and Radial Electric Field Generation

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Abstract. Particle diffusion in a given electrostatic turbulence with a finite correlation length along the confining magnetic field is studied in the test particle approach. An anomalous diffusion regime of amplified diffusion coefficient is found in the conditions when particle trapping in the structure of the stochastic potential is effective. The auto-generated radial electric field is calculated.

1. Introduction

The amplitudes and spectra of plasma turbulence are measured in a large number of experiments on tokamak devices. They determine, by means of theoretical models, the transport coefficients. However, the effective transport can also be strongly influenced by other physical factors which are often neglected in such estimates. We determine here the effect of a finite correlation length of the stochastic potential along the confining magnetic field. This problem has already been studied in the literature but only in the quasilinear limit corresponding to high frequency or low amplitude turbulence. These results are extended here to the nonlinear regimes characterizing the low frequency turbulence. The physical process which is specific to the low frequency turbulence is the dynamical trapping of the trajectories around the extrema of the stochastic potential [1], [2]. We show here that the trapping process combined with the parallel motion determines an anomalous transport regime of increased diffusion. We also determine the radial electric field E_r generated for restoring the ambipolarity of the radial fluxes and show that trajectory trapping produces a change in the direction of E_r. We use our recently developed statistical approach which describes the complicated process of trajectory trapping [3] and determines an exact expression for the 2-point Lagrangian velocity correlation (LVC) in Gaussian fields.

2. Effect of parallel motion in electrostatic turbulence

We consider a constant confining magnetic field directed along z axis $\mathbf{B}_0 = B_0\mathbf{e}_z$ (slab geometry) and an electrostatic turbulence represented by an electrostatic potential $\phi(\mathbf{x}, z, t)$, where $\mathbf{x} \equiv (x, y)$ are the Cartesian coordinates in the plane perpendicular to $\mathbf{B}_0$ and z along $\mathbf{B}_0$. The test particle approach of the turbulent transport relies on the following Langevin equations :

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}[\mathbf{x}(t), z(t), t], \qquad \mathbf{x}(0) = \mathbf{0} \tag{1}$$

$$\frac{dz}{dt} = V_{th}, \qquad z(0) = 0 \tag{2}$$

where $\mathbf{x}(t)$, $z(t)$ represent the trajectory of the particle guiding center. The velocity $\mathbf{v}(\mathbf{x}, z, t)$ is determined by the electrostatic potential as:

$$\mathbf{v}(\mathbf{x}, z, t) = -\nabla\Phi(\mathbf{x}, z, t) \times \mathbf{e}_z \tag{3}$$

where the reduced potential Φ is defined by $\Phi(\mathbf{x},z,t) \equiv \phi(\mathbf{x},z,t)/B_0$ and ∇ is the gradient in the (x,y) plane. The motion along the confining magnetic field $B_o\mathbf{e}_z$ can be approximated by Eq.(2) where $V_{th} \equiv \sqrt{2T/m}$ is the thermal velocity of the particles. The variation of the velocity determined by the electrostatic force $-\partial_z\phi$ is negligible compared to the large thermal velocity both for electrons and ions. The potential $\Phi(\mathbf{x},z,t)$ is a stochastic field considered to be Gaussian, stationary and homogeneous. The two-point Eulerian correlation (EC) of the potential is modelled by:

$$E(\mathbf{x},t) \equiv \langle \Phi(\mathbf{0},0,0)\, \Phi(\mathbf{x},z,t) \rangle = \beta^2 \mathcal{E}(\mathbf{x}) \exp\left(-\frac{|z|}{\lambda_\parallel}\right) \exp\left(-\frac{|t|}{\tau_c}\right) \tag{4}$$

Angular brackets denote statistical average over the realizations of the stochastic potential field, β is the root mean square of Φ, τ_c is its correlation time and $\lambda_\parallel$ its correlation length along the confining magnetic field. $\mathcal{E}(\mathbf{x})$ is a dimensionless function which decays from $\mathcal{E}(\mathbf{0}) = 1$ to zero when $|\mathbf{x}| \to \infty$. Its Fourier transform is the spectrum in the perpendicular plane (x,y) and the characteristic wavelength of the spectrum defines the perpendicular correlation length λ. A dimensionless parameter, *the Kubo number* can be defined as: $K = V\,\tau_c/\lambda$, where $V = \beta/\lambda$ measures the amplitude of the fluctuating velocity. The Kubo number characterizes particle motion in the plane (x,y) perpendicular to the confining magnetic field. The parallel motion (along the z axis) depends on a similar parameter, *the parallel Kubo number*, defined by: $K_\parallel \equiv V_{th}\tau_c/\lambda_\parallel$. The mean square displacement (MSD) of the particles and the running diffusion coefficient are determined from the LVC.

The equation of motion along z axis (2) is a deterministic equation. Its solution $z(t) = V_{th}t$ can be introduced in the equations of the motion in the (x,y) plane and thus the problem is reduced to two space variables but with an additional time dependence in the potential which is introduced by the parallel motion.

We determine the running diffusion coefficient for any value of K and $K_\parallel$ using *the decorrelation trajectory method* developed in Refs.[3] for 2-dimensional turbulence. It applies to Gaussian stochastic fields which are homogeneous, stationary and have an EC of the type (4). Very recently we have proved that, under the assumptions listed above, the decorrelation trajectory method yields the *exact* analytical expression for the LVC and for the running diffusion coefficient, valid for arbitrary values of Kubo number. Following calculation steps similar to those presented in [3], we obtain the following result for the LVC and for the running diffusion coefficient :

$$L_{ij}(t) = \delta_{ij} V^2 F'\left(K\tau(t,K_\parallel)\right) \exp\left[-(K_\parallel+1)\,|t|\right] \tag{5}$$

$$D(t;K,K_\parallel) = \left(\frac{\lambda^2}{\tau_c}\right) KF\left[K\tau(t,K_\parallel)\right] \tag{6}$$

where the function $F(\theta)$ is given by :

$$F(\theta) \equiv \frac{1}{\sqrt{2\pi}} \int\int_0^\infty dp\,du\,u^3 \exp\left(-\frac{u^2(1+p^2)}{2}\right) X(u\theta;p) \tag{7}$$

and $F'(\theta)$ is the derivative of $F(\theta)$. The asymptotic diffusion coefficient is:

$$D(K) = \left(\frac{\lambda^2}{\tau_c}\right) KF\left(\frac{K}{K_\parallel+1}\right) \tag{8}$$

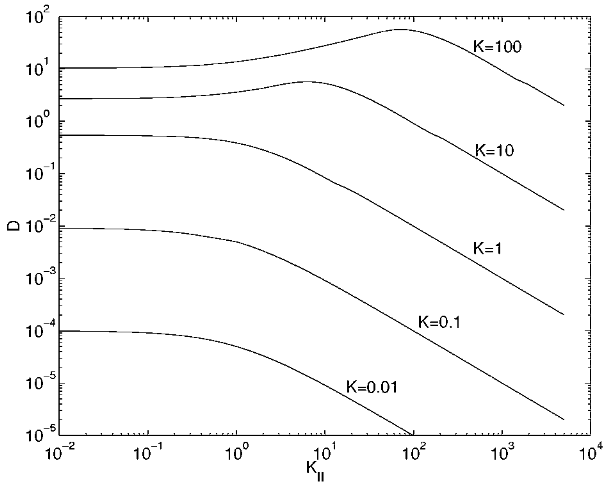

Figure 1: The diffusion coefficient as a function of $K_{\|}$ for several values of K.

The function $F(\theta)$ is obtained in [3] for an electrostatic turbulence with infinite parallel correlation length. At $\theta \ll 1$, $F(\theta) \cong \theta$ and at $\theta \gg 1$, $F(\theta)$ decays algebraically as $F(\theta) \approx \theta^{-\nu}$. For an EC of the potential $\mathcal{E}(\mathbf{x}) = 1/(1+\mathbf{x}^2/2)$, we have obtained $\nu = 0.38$. The variation of the shape of $\mathcal{E}(\mathbf{x})$ determines modification of ν around this value [3]. The function $F(\theta)$ describes the trajectory trapping : the long (algebraic) decay of this function results from the elimination of the contribution of the trapped trajectories (which vanishes in the integrals in Eq.(7) by an incoherent mixing). The diffusion coefficient at large K is thus determined by a small part of the trajectories which are not trapped. Equation (8) shows that the parallel motion determines a modification of the argument of the function F which becomes $K_{eff} = K/\left(K_{\|}+1\right)$.

At small values of the parallel Kubo number, $K_{\|} \ll 1$, this effective value is $K_{eff} \simeq K$ and Eq.(8) is identical with the result obtained in [3] for 2-dimensional turbulence. Thus the diffusion coefficient is not influenced by the parallel motion. It is determined only by K : $D = D_{QL} = (\lambda^2/\tau_c)K^2$ if $K \ll 1$, or $D = (\lambda^2/\tau_c)K^{1-\nu}$ if $K \gg 1$.

At large values of $K_{\|}$, $K_{\|} \gg Max[K,1]$, the effective Kubo number is small ($K_{eff} \simeq K/K_{\|} \ll 1$) and the diffusion coefficient can be approximated according to (8) by:

$$D \simeq \left(\frac{\lambda^2}{\tau_c}\right)\frac{K^2}{K_{\|}} = \frac{V^2\lambda_{\|}}{V_{th}}, \qquad K_{\|} \gg K. \tag{9}$$

The parallel motion is the dominant decorrelation mechanism in these conditions and the diffusion coefficient is independent of the correlation time τ_c. This is the quasilinear result which is thus reproduced by the general equation (8). It corresponds to the absence of trajectory trapping.

When $1 \ll K_{\|} \ll K$, K_{eff} is large ($K_{eff} \simeq K/K_{\|} \gg 1$) and Eq.(8) can be approximated by:

$$D \simeq \left(\frac{\lambda^2}{\tau_c}\right)K\left(\frac{K_{\|}}{K}\right)^{\nu}, \qquad 1 \ll K_{\|} \ll K \tag{10}$$

In these case, both the parallel motion and the dynamic trapping influence the diffusion coefficient. It is interesting to note that the process of trajectory trapping actually inverses the dependence of the diffusion coefficient on $K_{\|}$: the decrease of D as $1/K_{\|}$ appearing in Eq.(9) is transformed into an "anomalous" increase due to trapping in Eq.(10).

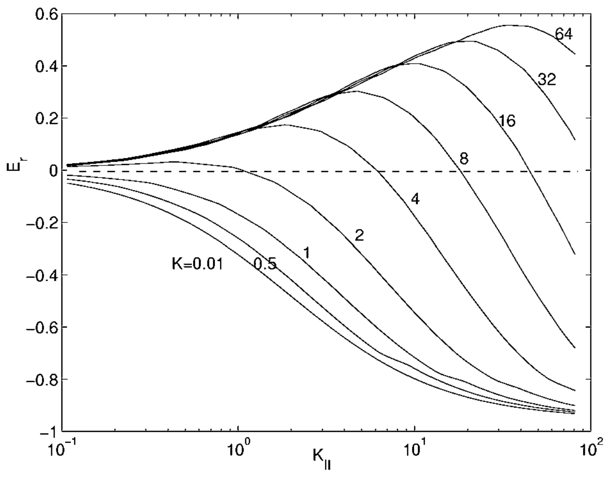

Figure 2: E_r as function of $K_{\parallel}$ for several values of K.

The diffusion coefficient obtained in Eq.(8) is presented in Fig.1 as a function of $K_{\parallel}$ for various values of K. The three regimes mentioned above are clearly identified in this figure. Typical tokamak plasmas are characterized by Kubo numbers in the range $[0.1, 100]$ and by values of the parallel Kubo number of the order $\left[10^{-2}, 10^{-1}\right]$ for the ions and $[1, 100]$ for electrons. Thus the parallel motion is important for electrons and can be neglected for ions.

3. Radial electric field

Important differences between the transport coefficients of the electrons and ions appear due to the thermal velocity contained in $K_{\parallel}$. Consequently, a quasistationary radial electric field E_r is generated to ensure the ambipolarity of the particle diffusive fluxes [4]. Considering a hydrogen plasma with $T_e = T_i = T$ and introducing the ratio of the electron and ion diffusion coefficients from (8):

$$\alpha \equiv \frac{D_e}{D_i} = \frac{F\left(\frac{K}{K_{\parallel e}+1}\right)}{F\left(\frac{K}{\mu K_{\parallel e}+1}\right)}, \tag{11}$$

the radial electric field and the effective diffusion coefficient are obtained as:

$$eE_r = \frac{T}{L_n}\frac{\alpha - 1}{\alpha + 1}, \quad D^{eff} = D_e \frac{2}{\alpha + 1} \tag{12}$$

where L_n is the characteristic length of the density and $\mu \equiv \sqrt{\frac{m_e}{m_i}}$. The value of D^{eff} is situated between D_e and D_i close to the smaller of the two diffusion coefficients.

At small values of the Kubo number K, the diffusion coefficient of the electrons is smaller than that of ions and a negative electric field is generated. The nonlinear anomalous regime found in Sec.2 determines an important effect: for $K > 1$, the sign of the generated electric field is reversed since the diffusion coefficient of the electrons becomes larger than that of ions. Fig.2 presents the radial electric field (12) normalized by T/L_n as a function of $K_{\parallel e}$ for several values of the Kubo number. A positive electric field is generated when $K/K_{\parallel e} \gtrsim 1$ and $K > 1$.

The effect of a radial electric field on transport processes in tokamak plasmas appears at two levels: first, due to the fact that it is sheared (K and $K_{\parallel e}$ are radius dependent) it destroys the large size structures of the stochastic potential and reduces the level of the turbulence. In the mean time, E_r can have a direct effect on the transport coefficients (even if the level of the turbulence is unchanged) since the deterministic poloidal motion with the velocity E_r/B_0 represents a mechanism of decorrelation. This effect is important for small scale turbulent structures which are not affected by the shear of the electric field. The following qualitative analysis shows that the radial electric field genereted in quasilinear conditions has negligible influence on the effective diffusion coefficients but the reversed field appearing when particle trapping is effective can influence the transport and determine improved confinement regimes. There are three decorrelation mechanisms which compete in determining the diffusion coefficient. The time variation of the stochastic field (with the characteristic time τ_c), the parallel motion (on the scale $\tau_{\|} = \lambda_{\|}/V_{th} = \tau_c/K_{\|}$) and the poloidal drift V_E induced by E_r (with the characteristic time $\tau_E = \lambda_{\perp}/V_E = \tau_c V/V_E K$, where $V_E \approx T/L_n B_0 e$). When K is small ($K \ll 1$), $D_e \ll D_i$ and the effective diffusion coefficient (12) is $D^{eff} \cong 2D_e$. Due to the large value of $K_{\|e}$ the decorrelation mechanism is the parallel motion. Thus, the effective decorrelation is by the parallel motion and the radial electric field (which is negative) does not play a role. When the trajectory trapping is important ($K \gg 1$), the diffusion coefficient of the electrons is larger than that of ions and the effective diffusion is dominated by that of ions $D^{eff} \cong 2D_i$. The latter corresponds to small $K_{\|}$ and is independent of the parallel motion which is too slow to compete with the time variation of the stochastic field ($\tau_{\|i} \gg \tau_c$). The poloidal drift V_E can be the dominant decorrelation mechanism in this case. The condition is that $V_E \tau_c/\lambda > 1$ which is true if $V_E \simeq V$. Thus, when the generated electric field is large enough the diffusion coefficient will be influenced by E_r. This happens in the presence of trapping when the electric field is positive. A reduction of the diffusion can be expected in these conditions. This qualitative estimate is in agreement with recent measurements on Alcator C-Mod, Tore-Supra and JET [5] which have shown that the transition from low to high confinement regime is correlated with the change from negative to positive radial electric field.

4. Conclusions

In conclusion, we have shown in this paper that particle diffusion in electrostatic turbulenece can strongly be influenced by the motion along the confining magnetic field if the parallel correlation length is finite. An anomalous diffusion regime in which the diffusion coefficient increases with the parallel Kubo number was found. It is determined by the dynamic trapping of the trajectories in the structure of the stochastic potential. A radial electric field is generated for restoring the ambipolarity of particle flux. It is negative in the quasilinear regime and positive in the nonlinear, trapping dominated regime. A qualitative analysis shows that in the latter case the radial electric field could influence the diffusion coefficient. The dependence of the diffusion coefficient and of the generated electric field on the two Kubo numbers is determined.

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