EVALUATION OF THE PROBABILITY OF CRACK INITIATION AND CRACK INSTABILITY FOR A PIPE WITH A SEMI-ELLIPTICAL CRACK

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ABSTRACT

For cracked components, the fracture mechanics theory provides deterministic relationship between the maximum permissible external loading and some parameters of the component : its dimensions, the material properties, the crack size and its location. However, due to uncertainties of some of those parameters (for instance the material properties, the applied loading ...), a purely deterministic approach provides an incomplete picture of the reality. Therefore, a probabilistic approach may be necessary on this field. Such an approach allows us to estimate the failure probability of a structure subjected to a certain external loading.

When dealing with ductile fracture, both initiation of the crack and its instability have to be examined. Moreover, studied defects for industrial cases are often semi-elliptical and loading conditions can sometimes be very complex. Therefore, it is very important for the development of probabilistic fracture mechanics and reliability methods to developed tools being able to calculate such complex configurations.

This paper presents some work conducted at EDF R&D Division to evaluate the probability that a semi-elliptical crack in a pipe not only initiates but also propagates when submitted to mechanical loading such as bending and pressure combined or not with a thermal shock. The first part is related to the description of the mechanical model : the simplified methods included in the French RSE-M Code used to evaluate the J-integral as well as the principle of the determination of the crack propagation. Then, the way this deterministic approach is combined to a reliability code is described. Finally, an example is shown : the initiation and the instability of a semi-elliptical crack in a pipe submitted to combined pressure and bending moment.

INTRODUCTION

For cracked components, the fracture mechanics theory provides deterministic relationship between the maximum permissible external loading and some parameters of the component : its dimensions, the material properties, the crack size and its location. However, due to uncertainties of some of those parameters (for instance the material properties, the applied loading ...), a purely deterministic approach provides an incomplete picture of the reality. Therefore, a probabilistic approach may be necessary on this field. Such an approach allows us to estimate the failure probability of a structure subjected to a certain external loading.

When dealing with ductile fracture of cracked pipes, both initiation of the crack and its instability have to be examined. Moreover, studied defects for industrial cases are often semi-elliptical and loading conditions can sometimes be very complex. Therefore, it is very important for the development of probabilistic fracture mechanics and reliability methods to developed tools being able to calculate such complex configurations.

A first alternative is to combine directly the reliability code with a finite element code as described in different papers (Rahman 1997, Hornet 1998, Pendola 2000). But for industrial problems, this can conduct to very large computing time since the finite element evaluation has to be made number of times. Another possibility is to use some analytical models. Such work has already been published in the past (Rahman 1995, Heinfling 1999, Warke 1999).

This paper presents some work conducted at EDF R&D Division to evaluate the probability that a semi-elliptical crack in a pipe not only initiates but also propagates when submitted to mechanical loading such as bending and pressure combined or not with a thermal shock. The first part is related to the description of the mechanical model : the simplified methods included in the French RSE-M Code used to evaluate the J-integral as well as the principle of the determination of the crack propagation. Then, the way this deterministic approach is combined to a reliability code is described. Finally, as an example, the initiation and the instability of a semi-elliptical crack in a pipe submitted to combined pressure and bending moment are studied.

MECHANICAL MODELS

The RSE-M Code (RSE-M, 2000) provides simplified methods to calculate the stress intensity factor K_I and the J integral in a pipe containing a circumferentially oriented surface crack submitted to mechanical loads (in-plane bending and torsion moments, pressure, tension), thermal loads as well as to the combination of these loads. These methods have been jointly developed by CEA, EDF and Framatome for several years (Chapuliot, 1998 ; Chapuliot, 1999 ; Le Delliou, 2000 ; Papin, 2001), and new options are in progress (longitudinal surface cracks in straight pipes, surface cracks in elbows...).

Table 1 gives the nomenclature used throughout the paper :

Parameter	Definition	Parameter	Definition
r_i	Inner radius of pipe	E	Young's modulus
r_e	Outer radius	ν	Poisson's ratio
r_m	Mean radius	S_y	Yield stress (at 0.2 %)
t	Pipe wall thickness	α	mean coefficient of thermal expansion
a	Crack depth	M_2	In-plane bending moment

c	Half-crack length	P	Pressure
β	Half-crack angle		

Figure 1 shows a semi-elliptical crack on the outside surface of a pipe. The crack angle β is defined by : $\beta = \frac{c}{r_e}$ ($\beta = \frac{c}{r_i}$ if the crack is on the inside surface).

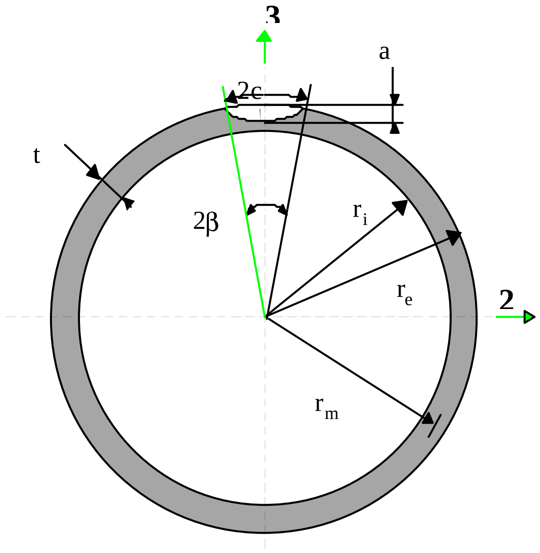


Figure 1 - A semi-elliptical circumferential crack in a pipe

Estimation of the J-integral under mechanical loading

For mechanical loads two methods - named CLC and CEP - are available in the Code. The CLC method, based on a corrected limit load, will be shortly presented here (for the second one, see Papin, 2001). The equation used to calculate L_r for a pressure combined with an in-plane bending moment is :

$$L_r = \sqrt{\left[\frac{m_2}{q_m \mu_{em} \mu_t}\right]^2 + \left[(1-\mu_{ti})\frac{p}{\mu_{ep}}\right]^2} + \mu_{ti}\frac{p}{\mu_{ep}}$$

where m_2 and p are adimensional loading :

$$m_2 = \frac{M_2}{4 r_m^2 t S_y} \qquad p = \frac{\sqrt{3}}{2}\frac{P r_m}{t S_y}$$

and q_m, μ_{em}, μ_t, μ_{ep} and μ_{ti} are coefficients depending only on geometrical parameters. These coefficients are given in Table 2.

Table 1 - Coefficients for the CLC method

Coefficient	$\beta < 2\pi\frac{a}{t}$	$\beta \geq 2\pi\frac{a}{t}$
q_m	$\cos\left(\frac{\beta}{2}\frac{a}{t}\right) - \frac{1}{2}\frac{a}{t}\sin(\beta)$	$1 - \frac{a}{t}$

μ_{em}	$1-0.16\sqrt{\frac{a}{t}}$	1
μ_{ep}	1 (inner surface crack) 0.9 (outer surface crack)	
μ_t	$1+\frac{1}{2}\frac{a}{t}\frac{\beta}{\pi}$	$1+\frac{1}{2}\frac{a}{t}$
μ_{ti}	$\mu_{ti}=\frac{\sqrt{\mu_t^2-1}}{\mu_t}$	

Then K_r is calculated from L_r by :

$$K_r=\left[\frac{E\,\varepsilon_{ref}}{L_r\,S_y}+\frac{1}{2}\frac{L_r^2}{L_r^2+1}\right]^{-\frac{1}{2}}$$

where ε_{ref} is the strain corresponding to the stress $\sigma_{ref} = L_rS_y$ on the true stress - true strain curve of the material (same approach as Option 2 of the R6 rule). Finally, an estimation of the elastic-plastic J-integral is obtained by :

$$J=\frac{J_e}{K_r^2}$$ where J_e is the elastic value of J

Estimation of the J-integral under thermal loading

The RSE-M Code provides a method to calculate J for an inside circumferential surface crack subjected to a cold thermal transient. It is assumed that the temperature distribution through the pipe wall and the elastic value of J - denoted J_e^{th} - are known. J_e^{th} can be obtained by an elastic analysis of the uncracked body and the use of influence coefficients. First, a thermal strain ε_{th} is calculated :

$$\varepsilon_{th}=0.85\,\frac{\alpha\left(\frac{\Delta T_1}{2}+\Delta T_2\right)}{\frac{4}{3}(1-\nu)}\,f\!\left(\frac{a}{t}\right)$$

where ΔT_1 and ΔT_2 are the values of ΔT_1(t) and ΔT_2(t) at the time of the transient when J_e^{th} is maximum, and f a parameter given in Table 3. Figure 2 shows the definition of ΔT_1 and ΔT_2. They are respectively the linear thermal gradient and the non-linear thermal gradient defined in ASME Section III NB-3600 (1998). Their absolute value should be used in the above equation.

Table 3 - Values of the parameter f

$\frac{a}{t}$	$\leq\frac{1}{30}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2.5}$	$\frac{1}{2}$
f - deepest point	1	1.06	1.10	0.86	0.63	0.54	0.54
f - surface point	1.6	1.6	1.6	1.6	1.6	1.6	1.6

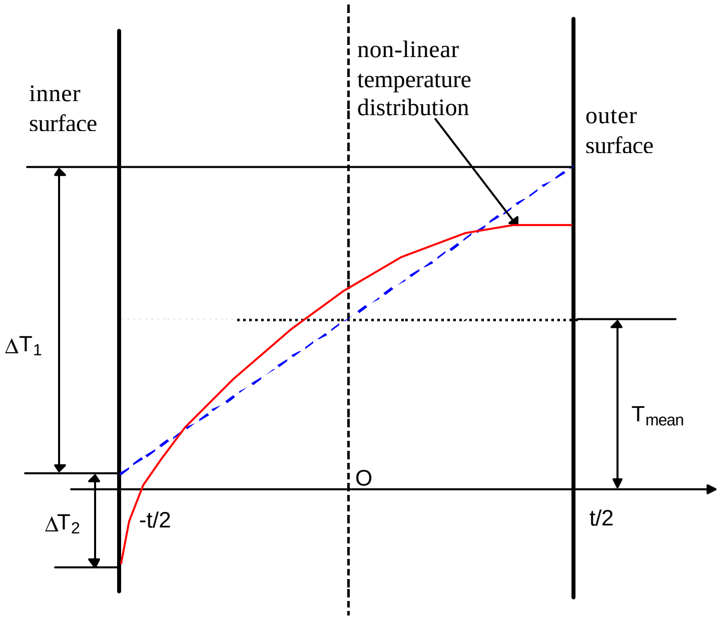


Figure 2 - Through wall temperature distribution in a pipe

Then an elastic-plastic corrective coefficient k_{th} is determined by :

$$k_{th} = \mathrm{Max}\left[\begin{array}{l} \sqrt{128\,\dfrac{\sigma_{th}}{E\,\varepsilon_{th}} - 0.28\left(\dfrac{\sigma_{th}}{E\,\varepsilon_{th}}\right)^2} \\ 0.5 \end{array}\right]$$

where σ_{th} is the stress corresponding to the strain ε_{th} on the true stress - true strain curve of the material. Finally, an estimation of the elastic-plastic J-integral is obtained by :

$$J_{th} = k_{th}^2\, J_e^{th}$$

Estimation of the J-integral under combined loading

To take into account the interaction between the stress systems, a parameter k^*_{th} is introduced to calculate J :

$$J = \left[\sqrt{J^m} + k^*_{th}\sqrt{J^{th}_{el}}\right]^2$$

This parameter k^*_{th} is defined as follows :

1) $k^*_{th} = k_{th}$ (no interaction) if at least one of the following conditions is verified :

   a) $L_r \leq 0.5$
   b) $L_r(P=0) \leq 2p$

where $L_r(P=0)$ is calculated by the formula given for the mechanical loads with P = 0 and p is the adimensional pressure previously defined :

$$p = \frac{\sqrt{3}}{2}\,\frac{P r_m}{t S_y}$$

2) $k^*_{th} = 1 - \dfrac{2p}{L_r(P=0)}(1 - k_{th})$ if neither of the above conditions a) and b) is verified.

Description of the OSTAND software

These estimation schemes have been implemented in a software named OSTAND. Its present features are as follows :

- calculation of stress intensity factors for circumferential or longitudinal surface cracks in straight pipes (based on the influence coefficients given in the RSE-M Code),
- calculation of the temperature and the stresses in a pipe submitted to any kind of thermal (axisymmetrical) transient on its inner surface. The transient is described by the evolution of the fluid temperature and the heat exchange coefficient versus time. The thermal calculation can be non-linear,
- calculation of the J-integral for a circumferential crack submitted to either mechanical loading, thermal loading or combined loading (as described previously).

COMBINATION OF THE DETERMINISTIC MODEL WITH THE RELIABILITY CODE

The combination of the reliability code PROBAN (Olesen 1992) with the deterministic code OSTAND has been made in a direct way as it has already been done in case of a finite element code (Hornet 1998, Pendola 2000). In other word, each time the evaluation of J is necessary the reliability code creates the corresponding input file with the right values of the different probabilistic variables for the deterministic code. Then the deterministic code is executed. The output parameter (J in our case) is finally read from the output file of the deterministic code.

In case of determination of the probability that a crack initiates, this combination is very simple since the limit state function is given by the comparison between the applied J-Integral (evaluated by OSTAND software) and the fracture toughness J0.2. If the applied J is greater than the fracture toughness, the crack initiates.

When studying the crack instability an iterative method is necessary to evaluate the crack propagation and to determine whether this crack is stable or not. Moreover, for semi-elliptical crack, the crack propagation has to be considered not only at the deepest point but also all along the crack front. Important hypotheses have been used for dealing with this problem. Firstly, the J resistance curve is taken similar when considering the crack propagation at the deepest point or at the surface. Secondly, the crack remains semi-elliptical that is to say that the crack propagation can be defined by the crack propagation at the deepest point and the crack propagation at the surface. Thirdly, a stable crack growth greater than 3 mm is considered to be unstable since 3 mm is considered to be the validity limit of the J resistance curve determined using CT specimens.

Figure 3 illustrates the case of a stable crack. The crack propagation in depth and in length directions (respectively Δa and Δc) have to be determined by the intersection of the applied J curve at the deepest point and at the surface point as a function of crack propagation and the J resistance curve. It has to be noted that the iterative process is not so simple since the applied J curve at the deepest point (or at the surface) is a function of the crack propagation at the deepest point and at the surface.

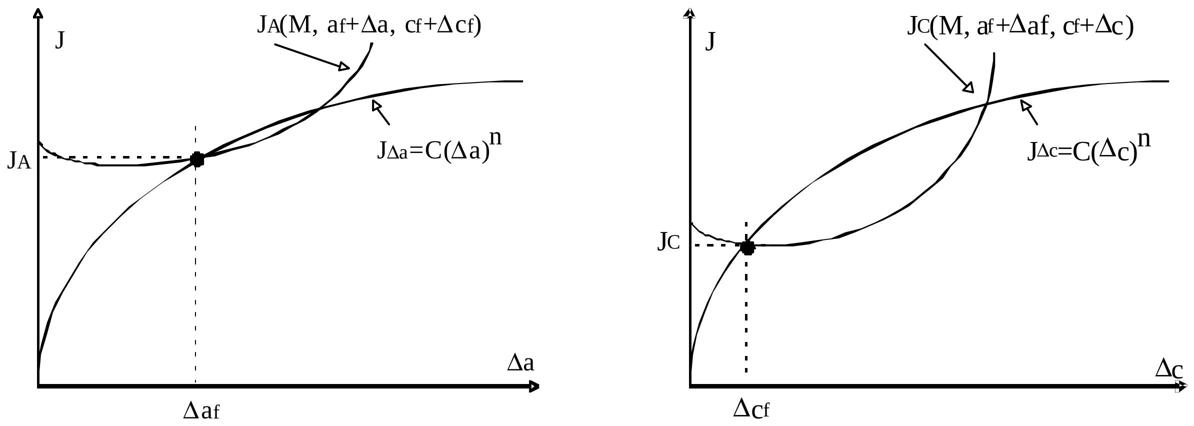


Figure 3 - illustration of a stable crack growth

Figure 4 presents an illustration of the way the reliability code and the deterministic code have been combined to be able to deal with the crack initiation and the crack instability. The subroutine *JOSTAND.f* is able to write the input data of OSTAND, to execute it and to read in the output file of OSTAND the J value at the deepest point (Jfond) and at the surface point (Jbord). The subroutine *Amorcage.f* makes the link with PROBAN and takes the maximum of the J values at the deepest point and at the surface and compares it with the fracture toughness (J0.2) The subroutine *PROPAG.f* corresponds to the iterative process for evaluating the crack propagation at the deepest point (Δafond) and at the surface (Δabord). This subroutine uses the *JOSTAND.f* subroutine. The subroutine *Propagation.f* makes the link with PROBAN and takes the maximum of the crack propagation at the deepest point and at the surface point.

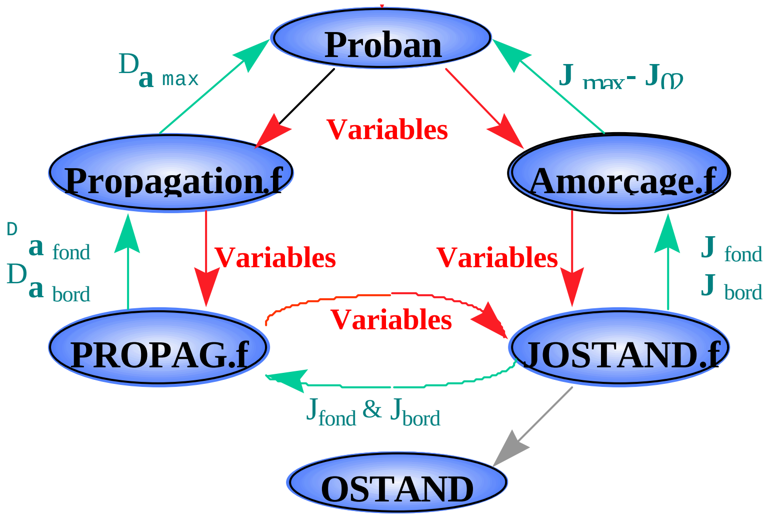


Figure 4 - illustration of the combination between the reliability code PROBAN and the deterministic code OSTAND

CASE STUDY

Data of the studied case

The objective is to evaluate the probability of initiation and instability of a semi-elliptical crack in a pipe subjected to internal pressure and in-plane bending moment. Figure 5 presents the geometry of the cracked pipe. The pipe is made of a Carbon Manganese steel and the crack is considered to be in the weld joint. Therefore, the stress-strain curve used is the one of the base metal and the resistance curve as well as the fracture toughness is from the weld metal. The materials properties are taken from Appendix 5.6 of RSE-M Code. Table 4 contains the deterministic and probabilistic data based on expert judgment or experimental measurement.

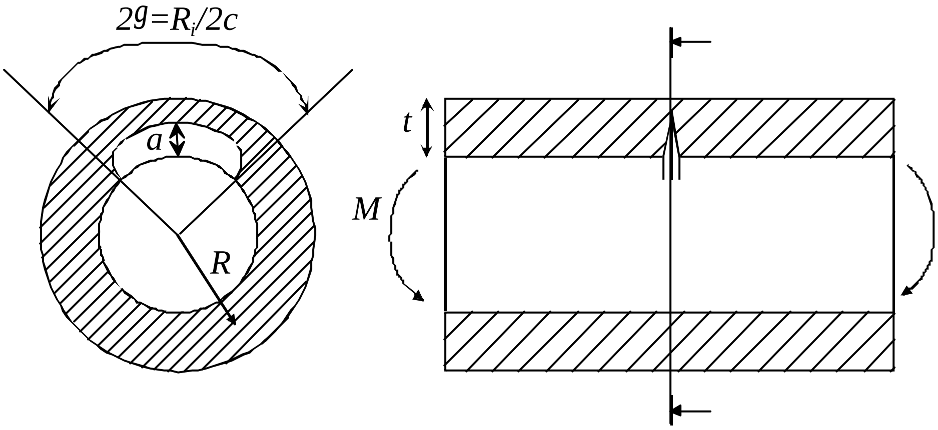


Figure 5 - Geometry of the pipe considered

Table 4 - Data of the case presented

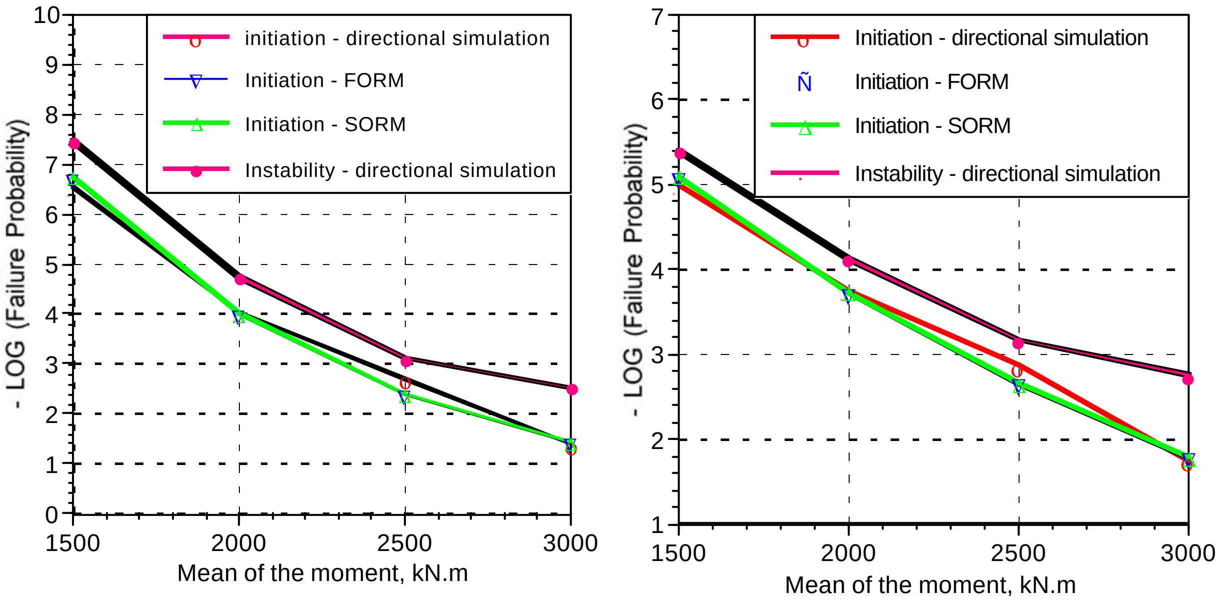
Parameter	distribution law	mean value	standard deviation or coefficient of variation
Young’s modulus E, MPa	Lognormale	185 000	5 %
Yield strength Sy, MPa	Normale	213.57	5 %
Fracture toughness J0.2, MPa.mm	Lognormale	74.1	20 MPa.mm
Exponent of the fracture resistance curve, nj	deterministic	0.32	--
crack depth a, mm	Normale	8	1 mm
half crack length c, mm	Normale	24	1 mm
Poisson’s ratio ν	deterministic	0.3	--
Pipe’s internal radius Ri, mm	deterministic	374.4	--
Thickness t, mm	deterministic	32	--
Internal pressure P, MPa	Lognormale	15.5	5 %
Applied moment M, kN.m	Lognormale	1500-3000	20 %

Results

The probability of initiation and the probability of instability of the crack have both been computed using different reliability methods such as First Order Reliability Method (FORM), Second Order Reliability Method (SORM) and directional simulations. Such calculations have been conducted for four different levels of applied moment : 1500 kN.m ; 2000 kN.m ; 2500 kN.m and 3000 kN.m.

For the initiation case, 20 directional simulations have been used whereas only 5 directional simulations have been considered for instability problems because of the large computation time requested. Indeed, the evaluation of the crack instability needs about 30 hours on an Ultra60 workstation when using 5 directional simulations. This long duration for the calculation could be reduced by improving the convergence algorithm used for instability calculation and compiling the analytical models used in OSTAND with Proban instead of writing several data files and reading the results. The FORM and SORM methods did not converge correctly for the instability case due to the non regularity of the limit state function.

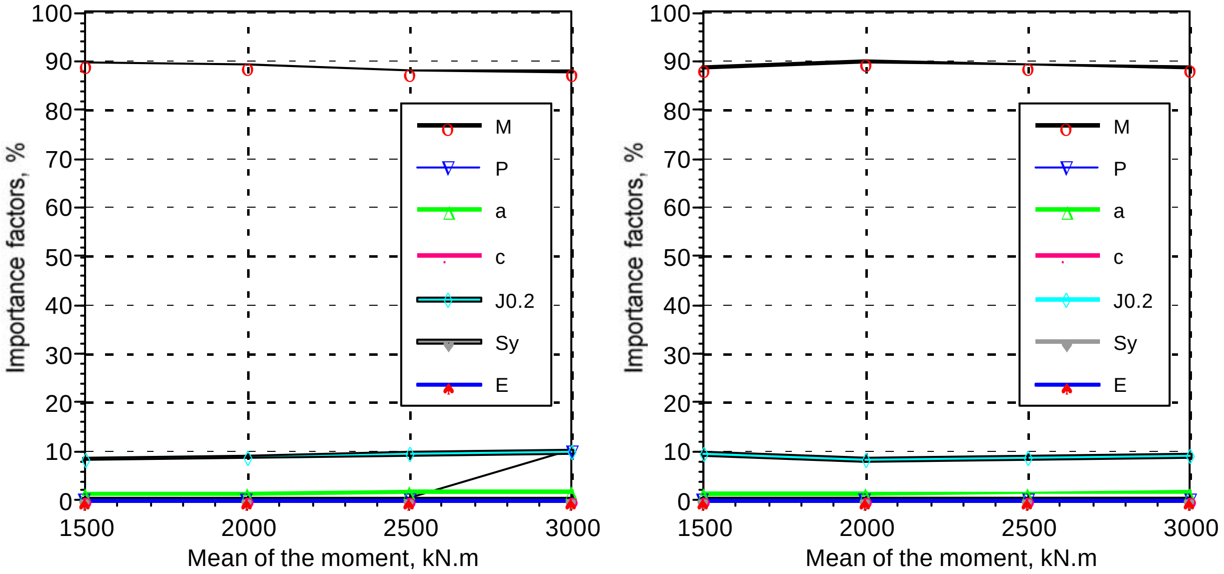
Figure 6 compares the results obtained in term of failure (initiation or instability) probability and reliability index for different values of the applied moment. For initiation, all the different methods (FORM, SORM, and directional simulations) used give similar results. Moreover, there is about one decade between the probability that a crack initiates and the probability that it becomes unstable (crack propagation greater than 3 mm). The difference between the probability than the crack initiates and the probability that the crack is unstable is not very important because the J resistance curve is relatively flat (depending on the value of the exponent of the fracture resistance curve, nj).



a) failure probability b) reliability index

Figure 6 - Evolution of the probability of initiation and instability as a function of the mean value of the moment

The importance factors obtained are given in Figure 7 for the initiation and in Figure 8 for the instability event. When comparing the importance factors obtained by directional simulation (Figure 7a) and FORM-SORM method (Figure 7b) for the initiation, we can note that the results are similar. Moreover, for both, initiation and instability, the most important parameter is the moment applied to the structure with a value of about 80 % (Figure 7 and 8). The second most important parameter with a value of about 10 % is the fracture toughness. In other words, the probability for the crack to initiate or to be unstable is mostly governed by the variability of the applied moment.



a) initiation - directional simulation b) initiation - FORM/SORM

Figure 7 - Importance factors calculated for the initiation

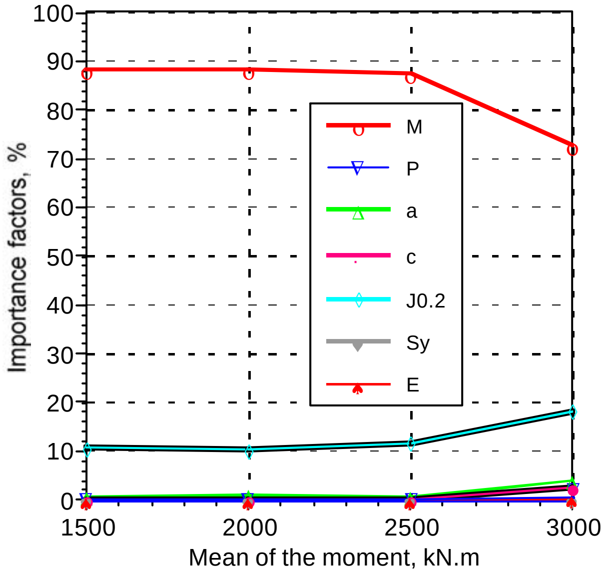


Figure 8 - Importance factors calculated for the instability

CONCLUSIONS

A general software based on simplified methods in fracture mechanics has been developed to evaluate the applied J. This software has been combined with the general probabilistic code Proban. This way the probability for a crack in a pipe to initiate or to be unstable can be evaluated for different loading combination including mechanical and thermal loading. As an example, the case of a semi-elliptical crack in a pipe subjected to internal pressure and in-plane moment is studied. This study shows the importance of the loading variability.

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