D2-2001-292

Kh. M. Beshtoev

NEUTRINO OSCILLATIONS.
THEORY AND EXPERIMENT

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2001

1. Introduction

The suggestion that, by analogy with $K^o, \bar{K}^o$ oscillations, there could be neutrino oscillations (i.e., that there could be neutrino-antineutrino oscillations $\nu \to \bar{\nu}$) was considered by Pontecorvo [1] in 1957. It was subsequently considered by Maki et al. [2] and Pontecorvo [3] that there could be mixings (and oscillation) of neutrinos of different aromas (i.e., $\nu_e \to \nu_\mu$ transitions).

The problem of solar neutrinos arose after the first experiment performed to measure the flux of neutrinos from the Sun by the $^{37}Cl-^{37}Ar$ [4] method. The flux was found to be several times smaller than expected from calculations made in accordance with the standard solar model (SSM) [5]. It was suggested in [6] that the solar neutrino deficit could be explained by neutrino oscillations. Subsequently, when the result of the experiment at Kamiokande [7] confirmed the existence of the deficit relative to the SSM calculations, one of the attractive approaches to the explanation of the solar neutrino deficit became resonant enhancement of neutrino oscillations in matter [8]. Resonant enhancement of neutrino oscillations in matter was obtained from Wolfenstein's equation for neutrinos in matter [9]. It was noted in Ref. [10] that Wolfenstein's equation for neutrinos in matter is an equation for neutrinos in matter in which they interact with matter not through the weak but through a hypothetical weak interaction that is left-right symmetric. Since in the standard weak interactions participate only left components of neutrinos the results obtained from Wolfenstein's equation have no direct relation to real neutrinos.

Later experimentalists obtained the first results on the Gran Sasso $^{71}Ga -^{71}Ge$ experiment [11], that within a 3σ limit did not disagree with the SSM calculations. The new data from the SAGE experiment [12] are fairly close to the Gran Sasso results.

In Ref. [13], the author of this article proposed a new mechanism of enhancement of neutrino oscillations in matter that is realized through the weak interaction of oscillation neutrinos with matter if the thickness of this matter is sufficiently great. Later in works [14]

it was shown that since the standard weak interactions cannot generate masses, the resonance enhancement of neutrino oscillations in matter cannot be realized without violation of the energy-momentum conservation law.

Besides the experimental devices marked above, at present there are working Super-Kamiokande [15-17] and SNO [18] detectors. The experimental results obtained with SNO detector present a great interest since they can be used for modelness analysis of neutrino oscillations.

After the discovery of neutrino oscillations on Super-Kamiokande [19] (by non-direct method) and on SNO [20] (by direct method) it is necessary to analyze the situation which arises in the problem of neutrino oscillations.

In this work theoretical schemes of neutrino oscillations and their analyses are considered. Also the experimental data obtained on Super-Kamiokande (Japan) and SNO (Canada) are given. Comparison of these data with consequences in the theoretical schemes has been carried out.

2. Theory

2.1. Distinguishing Features of Weak Interactions

The strong and electromagnetic interaction theories are left-right systemic theories (i.e. all components of the spinors participate in these interactions symmetrically). In contrast to this only the left components of fermions participate in the weak interaction. We will consider some consequences deduced from this specific feature of the weak interaction.

The local conserving current $j^{\mu i}$ of the weak interaction has the following form:

$$j^{\mu i} = \bar{\Psi}_L \tau^i \gamma^\mu \Psi_L, \tag{1}$$

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where $\bar{\Psi}_L, \Psi_L$ are lepton or quark doublets

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix}_{iL} \tag{2}$$

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_{iL}, \qquad i = 1 - 3,$$

where i is aromatic number of quarks or leptons.

The currents S_i^μ obtained from the global abelian transformation by using Neuter theorem [21] are

$$S_i^\mu = i(\bar{\Psi}_i \partial_\mu \Psi_i), \tag{3}$$

(where i characterizes the type of the gauge transformation) and the corresponding conserving current (the forth component of S_i^μ) is

$$I_i = \int S_i^0 d^3x = \int \epsilon \bar{\Psi}_i \Psi_i d^3x, \tag{4}$$

where ϵ is the energy of fermion Ψ_i.

Since we cannot switch off the weak interactions, then while the particle is moving in vacuum all the effects connected with these interactions will be realized.

If now we take into account that the right components of fermions $\bar{\Psi}_{iR}, \Psi_{iR}$ do not participate in the weak interaction, then from (4) for abelian currents we get

$$I_i = \int \epsilon \bar{\Psi}_{iL} \Psi_{iL} d^3x \equiv 0, \tag{5}$$

i.e. (in contrast to the strong and electromagnetic interactions) no conserving additive numbers appear in the weak interaction. However, we can see from experiments that the hierarchical violation of these additive numbers takes place here (see [22] and references there).

2.2. About Neutrino mass

a) Hypothesis: A massless free particle cannot have a charge. An example of this case is photon (carrier of the electromagnetic interactions), which has no charge. To gluons, which are in the confining state, this hypothesis cannot be applied. In application to neutrino having a weak charge, this hypothesis drives to a conclusion: the neutrino participating in weak interactions cannot be massless. In work [23] this hypothesis at sufficiently common suppositions was proved.

b) The discovery of neutrino oscillations is an additional confirmation of the conclusion that it is a massive particle.

2.3. Theory of Neutrino Oscillations

In the old theory of neutrino oscillations [24, 6], constructed in the framework of Quantum theory in analogy with the theory of $K^o, \bar{K}^o$ oscillation, it is supposed that mass eigenstates are ν_1, ν_2, ν_3 neutrino states but not physical neutrino states ν_e, ν_μ, ν_τ, and that the neutrinos ν_e, ν_μ, ν_τ are created as superpositions of ν_1, ν_2, ν_3 states. This means that the ν_e, ν_μ, ν_τ neutrinos have no definite mass, i.e. their masses may vary in dependence on the ν_1, ν_2, ν_3 admixture in the ν_e, ν_μ, ν_τ states. Naturally, in this case the law of conservation of the energy and the momentum of the neutrinos is not fulfilled. Besides, every particle must be created on its mass shell and it will be left on its mass shell while passing through vacuum. It is clear that this picture is incorrect.

In the modern theory on neutrino oscillations [25]-[26], constructed in the framework of the particle physics theory it is supposed that:

1) The physical states of the ν_e, ν_μ, ν_τ neutrinos are eigenstates of the weak interaction and, naturally, the mass matrix of ν_e, ν_μ, ν_τ neutrinos is diagonal. All the available experimental results indicate that the lepton numbers l_e, l_μ, l_τ are well conserved, i.e. the standard weak interactions do not violate the lepton numbers.

2) Then, to violate the lepton numbers, it is necessary to introduce

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an interaction violating these numbers. It is equivalent to introducing nondiagonal mass terms in the mass matrix of ν_e, ν_μ, ν_τ. Diagonalizing this matrix we go to the ν_1, ν_2, ν_3 neutrino states. Exactly like the case of K^o mesons created in strong interactions, when mainly $K^o, \bar{K}^o$ mesons are produced, in the considered case ν_e, ν_μ, ν_τ, but not ν_1, ν_2, ν_3, neutrino states are mainly created in the weak interactions (this is so, because the contribution of the lepton numbers violating interactions in this process is too small). And in this case no oscillations take place.

3) Then, when the ν_e, ν_μ, ν_τ neutrinos are pass through vacuum, they will be converted into superpositions of the ν_1, ν_2, ν_3 owing to the presence of the interactions violating the lepton number of neutrinos and will be left on their mass shells. And, then, oscillations of the ν_e, ν_μ, ν_τ neutrinos will take place according to the standard scheme [24-26]. Whether these oscillations are real or virtual, it will be determined by the masses of the physical neutrinos ν_e, ν_μ, ν_τ.

i) If the masses of the ν_e, ν_μ, ν_τ neutrinos are equal, then the real oscillation of the neutrinos will take place.

ii) If the masses of the ν_e, ν_μ, ν_τ are not equal, then the virtual oscillation of the neutrinos will take place. To make these oscillations real, these neutrinos must participate in the quasielastic interactions, in order to undergo transition to the mass shell of the other appropriate neutrinos by analogy with $\gamma - \rho^o$ transition in the vector meson dominance model. In case ii) enhancement of neutrino oscillations will take place if the mixing angle is small at neutrinos passing through a bulk of matter [13, 27].

So, the mixings (oscillations) appear since at neutrinos creating eigenstates of the weak interaction are realized (i.e. ν_e, ν_μ, ν_τ neutrinos) but not the eigenstates of the weak interaction violating lepton numbers (i.e. ν_1, ν_2, ν_3 neutrinos) and then, when passing through vacuum, they are converted into superpositions of ν_1, ν_2, ν_3 neutrinos. If ν_1, ν_2, ν_3 neutrinos were originally created, then the mixings (oscillations) would not have taken place since the weak interaction conserves the lepton numbers.

Now we come to a more detailed consideration of the oscillations. For simplification we consider the oscillation of two types of neutrinos ν_e, ν_μ having l_{ν_e}, l_{ν_μ} numbers which can transit each into the other. We can use the mass matrix of ν_e, ν_μ neutrinos to consider transitions between these particles in the framework of the quantum theory (or particle physics) since the mass matrix is an eigenstate of the type of interaction which creates these particles (see below).

The mass matrix of ν_e and ν_μ neutrinos has the form

$$\begin{pmatrix} m_{\nu_e} & 0 \\ 0 & m_{\nu_\mu} \end{pmatrix}. \tag{6}$$

Due to the presence of the interaction violating the lepton numbers, a nondiagonal term appears in this matrix and then this mass matrix is transformed into the following nondiagonal matrix (CP is conserved):

$$\begin{pmatrix} m_{\nu_e} & m_{\nu_e \nu_\mu} \\ m_{\nu_\mu \nu_e} & m_{\nu_\mu} \end{pmatrix}, \tag{7}$$

then the lagrangian of mass of the neutrinos takes the following form ($\nu \equiv \nu_L$):

$$\begin{aligned} \mathcal{L}_M &= -\tfrac{1}{2} \left[m_{\nu_e} \bar{\nu}_e \nu_e + m_{\nu_\mu} \bar{\nu}_\mu \nu_\mu + m_{\nu_e \nu_\mu} (\bar{\nu}_e \nu_\mu + \bar{\nu}_\mu \nu_e) \right] \equiv \\ &\equiv -\tfrac{1}{2} (\bar{\nu}_e, \bar{\nu}_\mu) \begin{pmatrix} m_{\nu_e} & m_{\nu_e \nu_\mu} \\ m_{\nu_\mu \nu_e} & m_{\nu_\mu} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \end{aligned}, \tag{8}$$

which is diagonalized by turning through the angle θ and (see ref. in [24]) and then this lagrangian (8) transforms into the following one:

$$\mathcal{L}_M = -\frac{1}{2} \left[m_1 \bar{\nu}_1 \nu_1 + m_2 \bar{\nu}_2 \nu_2 \right], \tag{9}$$

where

$$m_{1,2} = \frac{1}{2} \left[(m_{\nu_e} + m_{\nu_\mu}) \pm \left((m_{\nu_e} - m_{\nu_\mu})^2 + 4 m_{\nu_\mu \nu_e}^2 \right)^{1/2} \right],$$

and angle θ is determined by the following expression:

$$tg 2\theta = \frac{2 m_{\nu_e \nu_\mu}}{(m_{\nu_\mu} - m_{\nu_e})}, \tag{10}$$

$$\nu_e = cos\theta\nu_1 + sin\theta\nu_2,$$
$$\nu_\mu = -sin\theta\nu_1 + cos\theta\nu_2. \tag{11}$$

From eq.(10) one can see that if $m_{\nu_e} = m_{\nu_\mu}$, then the mixing angle is equal to $\pi/4$ independently of the value of $m_{\nu_e\nu_\mu}$:

$$sin^2 2\theta = \frac{(2m_{\nu_e\nu_\mu})^2}{(m_{\nu_e} - m_{\nu_\mu})^2 + (2m_{\nu_e\nu_\mu})^2}, \tag{12}$$

$$\begin{pmatrix} m_{\nu_1} & 0 \\ 0 & m_{\nu_2} \end{pmatrix}.$$

It is interesting to remark that expression (12) can be obtained from the Breit-Wigner distribution [28]

$$P \sim \frac{(\Gamma/2)^2}{(E - E_0)^2 + (\Gamma/2)^2}, \tag{13}$$

by using the following substitutions:

$$E = m_{\nu_e}, \ E_0 = m_{\nu_\mu}, \ \Gamma/2 = 2m_{\nu_e,\nu_\mu},$$

where $\Gamma/2 \equiv W(...)$ is a width of $\nu_e \to \nu_\mu$ transition, then we can use a standard method [26, 29] for computing this value.

The expression for time evolution of ν_1, ν_2 neutrinos (see (9), (11)) with masses m_1 and m_2 is

$$\nu_1(t) = e^{-iE_1 t}\nu_1(0), \qquad \nu_2(t) = e^{-iE_2 t}\nu_2(0), \tag{14}$$

where

$$E_k^2 = (p^2 + m_k^2), k = 1, 2.$$

If neutrinos are propagating without interactions, then

$$\nu_e(t) = cos\theta e^{-iE_1 t}\nu_1(0) + sin\theta e^{-iE_2 t}\nu_2(0),$$
$$\nu_\mu(t) = -sin\theta e^{-iE_1 t}\nu_1(0) + cos\theta e^{-iE_2 t}\nu_2(0). \tag{15}$$

Using the expression for ν_1 and ν_2 from (11), and putting it into (15), one can get the following expression:

$$\nu_e(t) = \left[e^{-iE_1 t}cos^2\theta + e^{-iE_2 t}sin^2\theta \right]\nu_e(0)+$$

$$+ \left[e^{-iE_1t} - e^{-iE_2t} \right] sin\theta \cos\theta \nu_\mu(0), \qquad (16)$$
$$\nu_\mu(t) = \left[e^{-iE_1t} sin^2\theta + e^{-iE_2t} cos^2\theta \right] \nu_\mu(0)+$$
$$+ \left[e^{-iE_1t} - e^{-iE_2t} \right] sin\theta cos\theta \nu_e(0).$$

The probability that neutrino ν_e created at the time $t = 0$ will be transformed into ν_μ at the time t is an absolute value of amplitude $\nu_\mu(0)$ in (16) squared, i. e.

$$P(\nu_e \rightarrow \nu_\mu) = |(\nu_\mu(0) \cdot \nu_e(t))|^2 =$$
$$= \tfrac{1}{2} \sin^2 2\theta \left[1 - \cos((m_2^2 - m_1^2)/2p)t \right], \qquad (17)$$

where it is supposed that $p \gg m_1, m_2; E_k \simeq p + m_k^2/2p$.

The expression (17) presents the probability of neutrino aroma oscillations. The angle θ (mixing angle) characterizes value of mixing. The probability $P(\nu_e \rightarrow \nu_\mu)$ is a periodical function of distances where the period is determined by the following expression:

$$L_o = 2\pi \frac{2p}{|m_2^2 - m_1^2|}. \qquad (18)$$

And probability $P(\nu_e \rightarrow \nu_e)$ that the neutrino ν_e created at time $t = 0$ is preserved as ν_e neutrino at time t is given by the absolute value of the amplitude of $\nu_e(0)$ in (16) squared. Since the states in (16) are normalized states, then

$$P(\nu_e \rightarrow \nu_e) + P(\nu_e \rightarrow \nu_\mu) = 1. \qquad (19)$$

So, we see that aromatic oscillations caused by nondiagonality of the neutrinos mass matrix violate the law of the $-\ell_e$ and ℓ_μ lepton number conservations. However in this case, as one can see from exp. (19), the full lepton numbers $\ell = \ell_e + \ell_\mu$ are conserved.

We can also see that there are two cases of ν_e, ν_μ transitions (oscillations) [26], [29].

1. If we consider the transition of ν_e into ν_μ particle, then

$$sin^2 2\beta \cong \frac{4m_{\nu_e,\nu_\mu}^2}{(m_{\nu_e} - m_{\nu_\mu})^2 + 4m_{\nu_e,\nu_\mu}^2}, \qquad (20)$$

if the probability of the transition of ν_e particles into ν_μ particles through the interaction (i.e. m_{ν_e,ν_μ}) is very small, then

$$sin^2 2\beta \cong \frac{4m^2_{\nu_e,\nu_\mu}}{(m_{\nu_e} - m_{\nu_\mu})^2} \cong 0. \tag{21}$$

How can we understand this $\nu_e \to \nu_\mu$ transition?

If $2m_{\nu_e,\nu_\mu} = \frac{\Gamma}{2}$ is not zero, then it means that the mean mass of ν_e particle is m_{ν_e} and this mass is distributed by $sin^2 2\beta$ (or by the Breit-Wigner formula) and the probability of the $\nu_e \to \nu_\mu$ transition differs from zero and it is defined by masses of ν_e and ν_μ particles and m_{ν_e,ν_μ}, which is computed in the framework of the standard method, as pointed out above.

So, this is a solution of the problem of the origin of mixing angle in the theory of vacuum oscillations.

In this case the probability of $\nu_e \to \nu_\mu$ transition (oscillation) is described by the following expression:

$$P(\nu_e \to \nu_\mu, t) = sin^2 2\beta sin^2 \left[\pi t \frac{\mid m^2_{\nu_1} - m^2_{\nu_2} \mid}{2p_{\nu_e}} \right], \tag{22}$$

where p_{ν_e} is a momentum of ν_e particle.

Originally it was supposed [6, 24] that these oscillations are real oscillations. However we see that these oscillations are virtual because when ν_e really transits into ν_μ, then it can decay into electron neutrino plus something, i.e. we gain the energy from vacuum, which is equal to the mass difference $\Delta m = m_{\nu_\mu} - m_{\nu_e}$ (momenta of ν_e and ν_μ are equal at oscillations). Then it is clear that at real $\nu_e \to \nu_\mu$ transition the law of energy conservation is violated. This law can be fulfilled only at virtual $\nu_e \to \nu_\mu$ transitions.

2. If we consider the virtual transition of ν_e into ν_μ neutrino at $m_{\nu_e} = m_{\nu_\mu}$ (i.e. without changing the mass shell), then

$$tg2\beta = \infty,$$

$\beta = \pi/4$, and

$$sin^2 2\beta = 1. \tag{23}$$

In this case the probability of the $\nu_e \rightarrow \nu_\mu$ transition (oscillation) is described by the following expression:

$$P(\nu_e \rightarrow \nu_\mu, t) = \left[\pi t \frac{4m^2_{\nu_e,\nu_\mu}}{2p_a} \right]. \tag{24}$$

To make these virtual oscillations real, their participation in quasielastic interactions is necessary for the transitions to their own mass shells [29].

It is clear that the $\nu_e \rightarrow \nu_\mu$ transition is a dynamical process.

Now let us consider the common case. In this case the mass lagrangian has the following form:

$$\mathcal{L}_M = -\bar{\nu}_R M \nu_L + H.c. \equiv$$
$$\equiv \Sigma_{l,l'=e,\mu,\tau} \, \nu_{l'R} M_{l'l} \nu_{lL} + H.c., \tag{25}$$

M is a complex 3×3 matrix. It is necessary to remark that the ν_R is absent in the weak interactions lagrangian. By using the expression

$$M = VmU^+, \tag{26}$$

(where V, U - unitary matrices) we transform $\mathcal{L}_M$ to a diagonal form

$$\mathcal{L}_M = -\bar{\nu}_R m \nu_L + H.c. \equiv$$
$$\equiv \Sigma_{k=1}^3 \, m_k \bar{\nu}_k \nu_k + H.c., \tag{27}$$

where

$$m_{ik} = m_k \delta_{ik},$$

and

$$\nu'_L = U^+ \nu_L, \quad \nu'_R = V^+ \nu_R, \quad \nu' = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \tag{28}$$

We can see that the lagrangian (25) is invariant at the global gauge transformation

$$\nu_k(x) \rightarrow e^\Lambda \nu_k(x) \tag{29}$$

or $l(x) \rightarrow e^\Lambda l(x), \quad l = e, \mu, \tau$, i.e. lepton numbers are not conserved separately (i.e. neutrino is mixed) but there appears a lepton

number l related with the common gauge transformation which is conserved.

2.4. Schemes of Neutrino Oscillations

Let us consider different schemes of neutrino oscillations.

2.4.a. Neutrino-Antineutrino Oscillations

The suggestion that, by analogy with $K^o, \bar{K}^o$ oscillations, there could be $\nu, \bar{\nu}$ oscillations, was considered by B. Pontecorvo in work [1].

In this case the mass lagrangian of neutrinos has the following form:

$$\mathcal{L}'_M = -\tfrac{1}{2}(\bar{\nu}_e, \nu_e) \begin{pmatrix} m_{\nu_e \nu_e} & m_{\bar{\nu}_e \nu_e} \\ m_{\nu_e \bar{\nu}_e} & m_{\bar{\nu}_e \bar{\nu}_e} \end{pmatrix} \begin{pmatrix} \nu_e \\ \bar{\nu}_e \end{pmatrix} . \tag{30}$$

Diagonalizing this mass matrix by standard methods one obtains the following expression:

$$\mathcal{L}'_M = -\tfrac{1}{2}(\bar{\nu}_1, \bar{\nu}_2) \begin{pmatrix} m_{\nu_1} & 0 \\ 0 & m_{\bar{\nu}_2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} , \tag{31}$$

where

$$\nu_1 = cos\theta \nu_e - sin\theta \bar{\nu}_e,$$
$$\nu_2 = sin\theta \nu_e + cos\theta \bar{\nu}_e.$$

These neutrino oscillations are described by expressions (14)-(19) with the following substitution of $\nu_\mu \to \bar{\nu}_e$.

It is necessary to remark that if these neutrinos are Dirac ones, then the probability to observe $\bar{\nu}_e$ is much smaller than the probability to observe ν_e (such neutrinos can be named the "sterile" neutrinos (see ref. [3]). It is clear that in this case the lepton numbers are not conserved, i.e. gauge invariance is violated since the particle transforms into antiparticle in contrast to the $\nu_e \to \nu_\mu$ transitions where only aromatic numbers are violated.

2.4.b. Oscillations of aromatic neutrinos

In the work [2] Maki et al. supposed that there could exist transitions between aromatic neutrinos ν_e, ν_μ. Afterwards ν_τ was found and then ν_e, ν_μ, ν_τ transitions could be possible. The author of this work has developed this direction (see [30]). It is necessary to remark that only this scheme of oscillations is realistic for neutrino oscillations (see also this work). The expressions which described neutrino oscillations in this case are given above in expressions (14)-(19).

2.4.c. Majorana neutrino oscillations

Before discussion of neutrino oscillations in this scheme we give definitions of Majorana neutrinos (more common consideration, in a formal form, of this question is discussed in [6, 24]). Majorana fermion in Dirac representation has the following form [24, 31]:

$$\chi^M = \frac{1}{2}[\Psi(x) + \eta_C \Psi^C(x)], \tag{32}$$
$$\Psi^C(x) \to \eta_C C \bar{\Psi}^T(x),$$

where η_C is a phase, C is a charge conjunction, T is a transposition.

From Exp. (32) we see that Majorana fermion χ^M has two spin projections $\pm\frac{1}{2}$ and then the Majorana spinor can be rewritten in the following form:

$$\chi^M(x) = \begin{pmatrix} \chi_{+\frac{1}{2}}(x) \\ \chi_{-\frac{1}{2}}(x) \end{pmatrix}. \tag{33}$$

The mass Lagrangian of Majorana neutrinos in the case of two neutrinos χ_e, χ_μ ($-\frac{1}{2}$ components of Majorana neutrinos, and $\bar{\chi}_{...}$ is the same Majorana fermion with the opposite spin projection) in the common case has the following form:

$$\mathcal{L}'_M = -\frac{1}{2}(\bar{\chi}_e, \bar{\chi}_\mu) \begin{pmatrix} m_{\chi_e} & m_{\chi_e\chi_\mu} \\ m_{\chi_\mu\chi_e} & m_{\chi_\mu} \end{pmatrix} \begin{pmatrix} \chi_e \\ \chi_\mu \end{pmatrix}. \tag{34}$$

Diagonalizing this mass matrix by standard methods one obtains the following expression:

$$\mathcal{L}'_M = -\tfrac{1}{2}(\bar{\nu}_1, \bar{\nu}_2) \begin{pmatrix} m_{\nu_1} & 0 \\ 0 & m_{\nu_2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}, \tag{35}$$

where

$$\nu_1 = cos\theta\chi_e - sin\theta\chi_\mu,$$

$$\nu_2 = sin\theta\chi_e + cos\theta\chi_\mu.$$

These neutrino oscillations are described by expressions (14)-(19) with the following substitution of $\nu_{e\mu} \rightarrow \chi^M_{e\mu}$.

The standard theory of weak interactions is constructed on the base of local gauge invariance of Dirac fermions. In this case Dirac fermions have the following lepton numbers l_l, which are conserved (however, see Sect. 2.1),

$$l_l, l = e, \mu, \tau, \tag{36}$$

and Dirac antiparticles have lepton numbers with the opposite sign

$$\bar{l} = -l_l. \tag{37}$$

Gauge transformation of Majorana fermions can be written in the form:

$$\chi'_{+\frac{1}{2}}(x) = exp(-i\beta)\chi_{+\frac{1}{2}}(x),$$

$$\chi'_{-\frac{1}{2}}(x) = exp(+i\beta)\chi_{-\frac{1}{2}}(x). \tag{38}$$

Then lepton numbers of Majorana fermions are

$$l^M = \sum_i l^M_i(+1/2) = -\sum_i l^M_i(-1/2),$$

i. e., antiparticle of Majorana fermion is the same fermion with the opposite spin projection.

Now we come to discussion of the problem of the place of Majorana fermion in the standard theory of weak interactions [32].

To construct the standard theory of weak interactions [33] Dirac fermions are used. The absence of contradiction of this theory with the experimental data confirms that all fermions are Dirac particles. And in this theory there are numbers which can be connected with conserving currents. As it is stressed above these numbers are violated (Sect. 2.1).

Now, if we want to include the Majorana fermions into the standard theory we must take into account that, in the common case, the gauge charges of the Dirac and Majorana fermions are different (especially it is well seen in the example of Dirac fermion having an electrical charge since it cannot have a Majorana charge (it is worth to remind that in the weak currents the fermions are included in the couples form)). In this case we cannot just include Majorana fermions in the standard theory of weak interactions by gauge invariance manner. Then in the standard theory the Majorana fermions cannot appear.

2.4.d. Neutrino Oscillations in the case of Dirac-Majorana mixing type

We do not discuss this mechanism due to the reason mentioned above. Consideration of this mechanism can be found in [24].

2.4.e. Neutrino Oscillation Enhancement in Matter

At present there exist two mechanisms of neutrino oscillation enhancement in matter. A short consideration of these mechanisms is given below.

2.4.e.1. Resonant Mechanism of Neutrino Oscillation Enhancement in Matter

In strong and electromagnetic interactions the left-handed and right-handed components of spinors participate in a symmetric manner. In contrast to these interactions only the left-handed components of spinors participate in the weak interactions as it is mentioned above. This is a distinctive feature of the weak interactions.

In the ultrarelativistic limit, the evolution equation for the neutrino wave function ν_Φ in matter has the form [8], [9]

$$i\frac{d\nu_{Ph}}{dt} = (p\hat{I} + \frac{\hat{M}^2}{2p} + \hat{W})\nu_{Ph}, \tag{39}$$

where $p, \hat{M}^2, \hat{W}$ are, respectively, the momentum, the (nondiagonal) square mass matrix in vacuum, and the matrix, taking into account neutrino interactions in matter,

$$\nu_{Ph} = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \qquad \hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\hat{M}^2 = \begin{pmatrix} m^2_{\nu_e \nu_e} & m^2_{\nu_e \nu_\mu} \\ m^2_{\nu_\mu \nu_e} & m^2_{\nu_\mu \nu_\mu} \end{pmatrix}.$$

If we suppose that neutrinos in matter behave analogously to the photon in matter and the neutrino refraction indices are defined by the expression

$$n_i = 1 + \frac{2\pi N}{p^2} f_i(0) = 1 + 2\frac{\pi W_i}{p}, \tag{40}$$

(where i is a type of neutrinos (e, μ, τ), N is density of matter, $f_i(0)$ is a real part of the forward scattering amplitude), then W characterizes polarization of matter by neutrinos (i.e. it is the energy of matter polarization).

The electron neutrino (ν_e) in matter interacts via $W^\pm, Z^0$ bosons and ν_μ, ν_τ interact only via Z^0 boson. These differences in interactions

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lead to the following differences in the refraction coefficients of ν_e and ν_μ, ν_τ

$$\Delta n = \frac{2\pi N}{p^2} \Delta f(0), \tag{41}$$

$$\Delta f(0) = -\sqrt{2} \frac{G_F}{2\pi},$$

where G_F is the Fermi constant.

Therefore the velocities (or effective masses) of ν_e and ν_μ, ν_τ in matter are different. And at the suitable density of matter this difference can lead to a resonance enhancement of neutrino oscillations in "matter" [8, 34].

As we can see from the form of Eq. (39), this equation holds the left-right symmetric neutrinos wave function $\Psi(x) = \Psi_L(x) + \Psi_R(x)$. This equation contains the term W, which arises from the weak interaction (contribution of W boson) and which contains only a left-side interaction of the neutrinos, and is substituted in the left-right symmetric equation (33) without indication of its left-side origin. Then we see that equation (39) is an equation that includes term W which arises not from the weak interaction but from a hypothetical left-right symmetric interaction (see also works [10, 30, 35]). Therefore this equation is not the one for neutrinos passing through real matter. The problem of neutrinos passing through real matter has been considered in [10, 30, 35, 36].

In three different approaches: by using mass Lagrangian [35, 30], by using the Dirac equation [35, 30], and using the operator formalism [36], the author of this work has discussed the problem of the mass generation in the standard weak interactions and has come to a conclusion that the standard weak interaction cannot generate masses of fermions since the right-handed components of fermions do not participate in these interactions. Also it is shown [37] that the equation for Green function of the weak-interacting fermions (neutrinos) in the matter coincides with the equation for Green function of fermions in vacuum and the law of conservation of the energy and the momentum of neutrino in matter will be fulfilled [36] only if the energy W of

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polarization of matter by the neutrino or the corresponding term in Wolfenstein equation, is zero (it means that neutrinos cannot generate permanent polarization of matter). These results lead to the conclusion: resonance enhancement of neutrino oscillations in matter does not exist.

The simplest method to prove the absence of the resonance enhancement of neutrino oscillations in matter is:

If we put an electrical (or strong) charged particle in matter, there arises polarization of matter. Since the field around the particle is spherically symmetrical, the polarization must also be spherically symmetrical. Then the particle will be left at rest and the law of energy and momentum conservation is fulfilled.

If we put a weakly interacting particle (a neutrino) in matter then, since the field around the particle has a left-right asymmetry (weak interactions are left interactions with respect to the spin direction), polarization of matter must be nonsymmetrical, i.e. on the left side there arises maximal polarization and on the right there is zero polarization. Since polarization of the matter is asymmetrical, there arises asymmetrical interaction of the particle (the neutrino) with matter and the particle cannot be at rest and will be accelerated. Then the law of energy momentum conservation will be violated. The only way to fulfil the law of energy and momentum conservation is to demand that polarization of matter be absent in the weak interactions. The same situation will take place in vacuum.

It is interesting to remark that in the gravitational interaction the polarization does not exist either [38].

2.4.e.2. Enhanced Oscillation of Neutrinos of Different Masses in Matter

The oscillation probability is estimated for neutrinos of different masses in their passing through matter of different thickness, including the Sun [13, 27].

1) So, if neutrinos of different types have equal masses, real oscillations are possible for different types of neutrinos by analogy with $K^o, \bar{K}^o$ oscillation;

2) if neutrino masses are different for different neutrino types, only virtual neutrino oscillations are possible while real oscillations require participation of neutrinos in interactions for their transition to the respective mass shells by analogy with transition of a γ-quantum to the ρ -meson in the vector dominance model.

We shall estimate the probability for neutrinos to change from one type ν_l to another $\nu_{l'}(m_{\nu_l} \neq m_{\nu_{l'}})$ in passing through matter. Neutrino transition to the mass shell will occur via the weak neutrino-matter interaction (by analogy with the $\gamma - \rho^o$ transition or $K_1^o, \bar{K}_2^o$ oscillation). We shall assume that difference in mass of $\nu_l, \nu_{l'}$ neutrinos is small enough to consider $\nu_{l'}$ the probability of transition to the mass shell proportional to the total elastic cross section $\sigma^{el}(p)$ for the weak interaction (for simplicity we shall deal with the oscillation of two types of neutrinos). Then the length of the elastic interaction of the neutrino in the matter of density charge Z, atomic number A and momentum p will be defined as

$$\Lambda_0 \sim \frac{1}{\sigma^{el}(p)\rho(z/A)}.$$

If the neutrino mass difference is fairly large, it can be taken into account by the methods of the vector dominance model [39]. As pointed out above, we shall assume that this difference is very small and employ above formula.

The real part of forward scattering amplitude $Re f_i(p, 0)$ is responsible for elastic neutrino scattering in matter (it is supposed that at low energies the coherent process takes place). It is related to the exponential phase term $exp(-p\Delta_i r)$ (as factor to momentum) in the wave function of particle $\Psi(r, ...)$ and has the following form:

$$p\Delta_i \simeq \frac{2\pi N_e f_i(p, 0)}{p}, \qquad i = \nu_e, \nu_\mu, \nu_\tau. \qquad (42)$$

Keeping in mind that [33]

$$f_i(p, 0) \simeq \sqrt{2} G_F p \left(\frac{M_W^2}{M_i^2} \right), \qquad (43)$$

if $i = \nu_e$, $\qquad M_i^2 = M_W^2$,

if $i = \nu_\mu, \nu_\tau$, $\qquad M_i^2 = M_{Z^0}^2$,

we obtain

$$p(\Delta_i) \simeq \sqrt{2} G_F N_e \left(\frac{M_W^2}{M_i^2} \right).$$

The phase of the elastic scattering amplitude changes by 2π over the length

$$\Lambda_{i0} \simeq \frac{2\pi}{\sqrt{2} G_F \rho(z/A) \left(\frac{M_W^2}{M_i^2} \right)} = 2\pi L_{i0} \sim \Lambda_0. \qquad (44)$$

For simplification further we will suppose that $M_e^2 \simeq M_\mu^2 \simeq M_\tau^2$ and then $\Lambda_i = \Lambda$. (Absorption or the imaginary part of the forward scattering amplitude can be ignored for low-energy neutrinos.)

Knowing that the length of elastic neutrino-matter interaction is Λ_0, we must estimate the oscillation probability for the neutrino passing through the matter of thickness L. The probability of the elastic ν_l interaction in matter of thickness L is

$$P(L) = 1 - exp(-2\pi L/\Lambda_0). \qquad (45)$$

Then, using formulae (44), (45), we can find the neutrino oscillation probability $\rho_{\nu_l \nu_{l'}}(L)$ at different thickness L. Averaging the expression for neutrino oscillation probability [13] over R

$$P_{\nu_l \nu_{l'}}(R) = \frac{1}{2} sin^2 2\theta_{\nu_l \nu_{l'}} (1 - cos 2\pi \frac{R}{L_0}), \qquad (46)$$

where $L_0 = \frac{4\pi p}{\Delta m^2}$,

then we obtain

$$\bar{P}_{\nu_l \nu_{l'}}(R) = \frac{1}{2}\sin^2 2\theta_{\nu_l \nu_{l'}}.$$

Then the oscillation probability $\rho_{\nu_l \nu_{l'}}(L)$ or the mixing angle β at $\Lambda_0 \geq L_0$ will be defined by the expressions (for simplicity it is supposed that $\Lambda_0 = \Lambda_e = \Lambda_\mu = \Lambda_\tau$):

a) for L comparable with Λ_0,

$$\rho_{\nu_l \nu_{l'}}(L) = \frac{1}{2}\sin^2 2\beta \simeq \bar{P}_{\nu_l \nu_{l'}} = \frac{1}{2}\sin^2 2\theta_{\nu_l \nu_{l'}}, \qquad (47)$$

where $\beta \simeq \theta_{\nu_l \nu_{l'}}$;

b) for very large L, $\frac{L}{\Lambda_0} > \frac{1}{\sin^2 2\theta_{\nu_l \nu_{l'}}} \gg 1$,

$$\rho_{\nu_l \nu_{l'}}(L) = \frac{1}{2}\sin^2 2\beta \simeq \frac{1}{2}, \qquad (48)$$

and $\beta \simeq \frac{\pi}{4}$;

c) for intermediate L,

$$\rho_{\nu_l \nu_{l'}}(L) = \frac{1}{2}\sin^2 2\theta_{\nu_l \nu_{l'}} \leq \rho_{\nu_l \nu_{l'}}(L) \leq \frac{1}{2}, \qquad (49)$$

and $\qquad \theta_{\nu_l \nu_{l'}} \leq \beta \leq \frac{\pi}{4}$.

If $L_0 \geq \Lambda_0$ the expressions like (47)-(49) are also true, but Λ_0 should be replaced by L_0 and the thickness of matter will be determined in units of L_0. Also, since the oscillation length L_0 increases with the neutrino momentum (see (46)), the number of oscillation lengths $n = L/L_0$ fitting in the given thickness L decreases with increasing neutrino momentum as, accordingly, the neutrino oscillation probability $\rho_{\nu_l \nu_{l'}}(L)$ does.

Let us consider the neutrino oscillation probability for intermediate interaction numbers n. The distribution probability of n-fold elastic neutrino interaction for thickness L with the mean value $\bar{n} = L/\Lambda_0$ at not very large $\bar{n}$ is determined by the Poisson distribution

$$f(n, \bar{n}) = \frac{(\bar{n})^n}{n!}e^{-\bar{n}}. \qquad (50)$$

At large $\bar{n}$ it changes to the Gaussian distribution:

$$f(n, \bar{n}, \bar{n}) = \frac{1}{\sqrt{2\pi\bar{n}}} e^{\frac{-(n-\bar{n})^2}{2\bar{n}}}. \tag{51}$$

The probability of neutrino conversion from ν_l to ν_l and $\nu_{l'}$ n-fold elastic interaction is determined by recursion relations (where $\theta \equiv \theta_{\nu_l \nu_{l'}}$) given in works [13, 27].

Here we give the expression for probability for neutrino conversion at $\sin^2 2\theta \ll 1$ for two types of neutrinos (ν_e, ν_μ)

$$\rho(\nu_e \to \nu_e) = 1 - \bar{n}\frac{1}{2} \sin^2 2\theta, \tag{52}$$

$$\rho(\nu_e \to \nu_\mu) = \bar{n}\frac{1}{2} \sin^2 2\theta.$$

Then enhancement of neutrino oscillation in matter will take place, i.e. ν_e neutrinos will transit in ν_μ, ν_τ neutrinos, but it is necessary to take into account that mean numbers of interaction lengths L_μ^o, L_τ^o of ν_μ, ν_τ will be δ times less and then, correspondingly, $\bar{n}$ in (52) will be changed for $\bar{n}_\mu, \bar{n}_\tau$:

$$\delta = \bar{n}_e/\bar{n}_\mu = \bar{n}_e/\bar{n}_\tau \simeq 2.49. \tag{53}$$

The mean number of elastic interactions of electron neutrinos produced in the Sun is

$$\Lambda_{Sun} \simeq 1.7 \cdot 10^7 m, \qquad \bar{n}_e^{Sun} \simeq 40, \qquad \bar{n}_\mu^{Sun} \simeq 16, \bar{n}_\tau^{Sun} \simeq 16.$$

It is necessary to mention that the considered mechanisms of enhancement of neutrino oscillation in matter lead only to changing the mixing angles and for their realizations the vacuum mixing angle of neutrino oscillations must differ from zero.

2.4.f. Neutrino Oscillations in Supersymmetrical Models

Neutrino oscillation in supersymmetrical models is considered in works (and see references there) [40-42]. Here we do not fulfil detailed considerations of these schemes but want to remark that in

these schemes side by side the neutrino oscillations the superpartner of the fermions and bosons must be observed.

3. Experimental Data

3.a. Neutrino Experimental Data from SNO (Canada)

The SNO detector [18], containing 1000 tons of heavy water (D_2O), is placed in a shaft in Sudbury at depth 6010 m water equivalent (Sudbury Neutrino Observatory).

The neutrinos are detected in the following reactions:

1. $\nu_x + e^- \rightarrow \nu_x + e^-$, $\quad E_{thre} \simeq 6$ MeV,
2. $\nu_e + d \rightarrow p + p + e^-$, $\quad E_{thre} \simeq 1.45$ MeV,
3. $\nu_x + d \rightarrow p + n + \nu_x$, $\quad E_{thre} \simeq 2.23$ MeV,

$x = e, \mu, \tau$.

Reaction 1 goes through charged and neutral currents, if $x = e$, and neutral if $x = \mu, \tau$; reaction 2 goes through charged current, and reaction 3 through neutral current. Using any couple of the reactions we can find the primary flux of the Sun neutrinos. Sudbury reported first results in [20, 43]. These results are obtained for the Sun neutrinos with threshold $E_{eff} \geq 6.75 MeV$.

Figure 1 shows the distribution of $cos\theta$ (a), and kinetic energy spectrum with a statistical error (b) with the ^8B spectrum [44] scaled to the data. The ratio of the data to the prediction [45] is shown in (c). The bands represent the 1σ uncertainties derived from the most significant energy-dependent systematic errors. There is no evidence for a deviation of the spectral shape from the predicted shape on the non-oscillation hypothesis.

Normalized to the integrated rates above the energy $E_{eff} = 6.75$ MeV, the flux of neutrinos is (from reactions 2 and 1):

$$\phi_{SNO}^{CC}(\nu_e) = 1.75 \pm 0.07(stat.) + 0.12(-0.11)(syst.) \qquad (54)$$

$$\pm 0.05(theor.) \times 10^6 cm^{-2} s^{-1} ,$$

$$\phi_{SNO}^{ES}(\nu_x) = 2.39 \pm 0.34(stat.) + 0.16(-0.14)(sys.) \times 10^6 cm^{-2} s^{-1}, \quad (55)$$

where the theoretical uncertainty is the CC cross section uncertainty. The neutrinos flux (55) measured on SNO is consistent with the same flux measured on Super-Kamiokande (58).
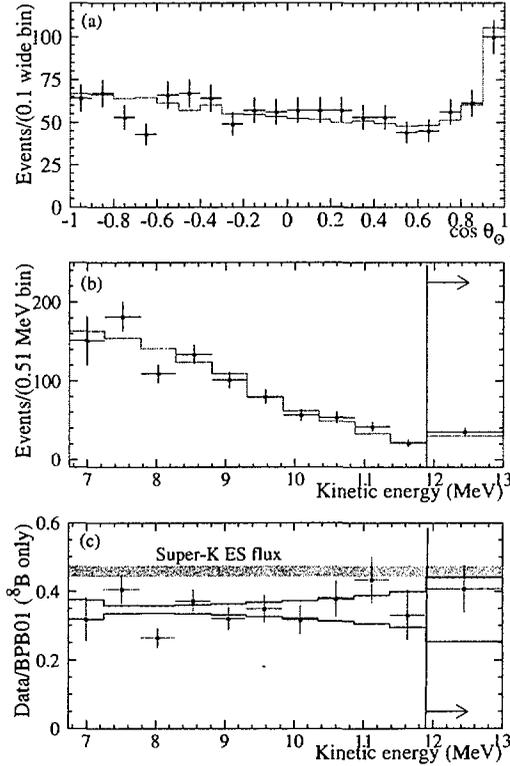

Figure 1: Distributions of (a) $cos\theta_{sun}$, and (b) extracted kinetic energy spectrum for CC events with $R \leq 5.50$ m and $T_{eff} \geq 6.75$ MeV. The Monte Carlo simulations for an undistorted ^8B spectrum are shown as histograms. The ratio of the data to the expected kinetic energy distribution with correlated systematic errors is shown in (c). The uncertainties in the ^8B spectrum have not been included

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The difference between ν flux deduced from the ES rate and that deduced from the CC rate is

$$\phi_{SNO} = 0.64 \pm 0.40 \times 10^6 cm^{-2} s^{-1}.$$

It is the ν_μ, ν_τ flux measured through NC.

The best fit to the $\phi_{SNO}(\nu_{\mu\tau})$ flux is:

$$\phi_{SNO}(\nu_{\mu\tau}) = 3.69 \pm 1.13 \times 10^6 cm^{-2} s^{-1}. \tag{56}$$

The ratio of the SNO CC flux to the solar model [44] is

$$\frac{\phi_{SNO}^{CC}}{\phi_{BPB00}} = 0.347 \pm 0.029.$$

The total flux of active ^8B neutrinos is determined to be:

$$\phi_{SNO}(\nu_x) = 5.44 \pm 0.99 \times 10^6 cm^{-2} s^{-1}. \tag{57}$$

This result is in a good agreement with prediction of the standard solar models [45, 46].

The SNO results are the first direct indication of the non-electron flavor components in the solar neutrino fluxes, and it is, practically [45, 46], the total flux of ^8B neutrinos generated by the Sun.

3.b. Neutrino Experimental Data from Super-Kamiokande (Japan)

The Super-Kamiokande detector [15, 16] is a cylindrically-sharped water Cherenkov detector with 50000 ton of ultra-pure water. It is located about 1000m (2700 m.w.e.) underground in the Kamioka mine. Super-Kamiokande is a multipurpose experiment, and solar and atmospheric neutrino physics is one of its main topics.

i) The Sun neutrino fluxes measured in Super-Kamiokande detector [47] through the electron scattering reaction 1. $\nu_x + e^- \rightarrow \nu_x + e^-$ $_{thre} \simeq 5$ MeV are as follows:

$$\phi_{SK}^{ES}(\nu_e) = 2.32 \pm 0.03(stat.) + 0.08(-0.07)(syst.) \times 10^6 cm^{-2} s^{-1}. \tag{58}$$

These fluxes are in a good consistence with the Sun neutrino fluxes measured in SNO.
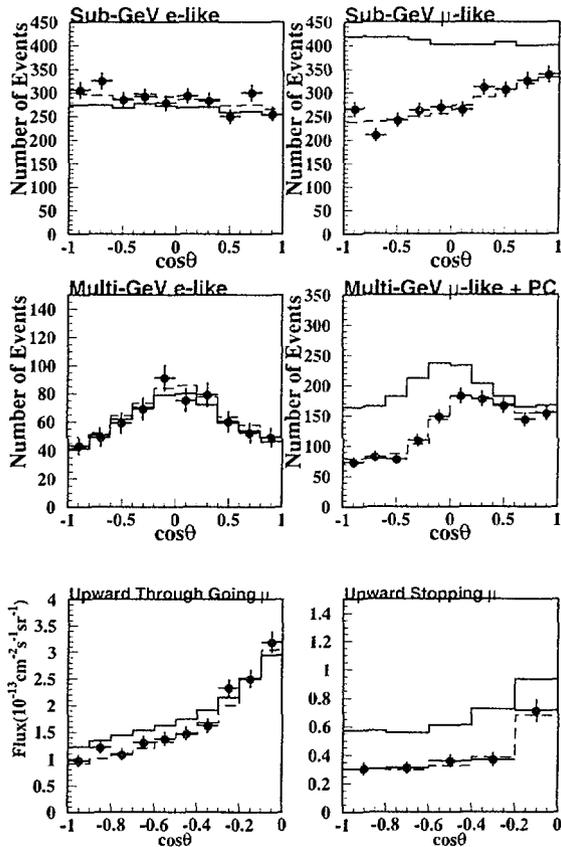

Figure 2: Zenith angle distribution of Super-Kamiokande 1289 days FC, PC and UPMU samples. Dots, solid and dashed lines correspond to data, MC with no oscillation and MC with best fit oscillation parameters, respectively

The day-night asymmetry A is

$$A = \frac{(\Phi_n - \Phi_d)}{((\Phi_n + \Phi_d)/2)} = 0.033 \pm 0.022(stat.) + 0.013(-0.012)(syst.).$$

This is 1.3σ from the zero asymmetry.

ii) Atmospheric neutrinos are produced by the interactions of the primary cosmic rays on nuclei of the Earth atmosphere. The atmospheric neutrinos at a few GeV have ratio 2 $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$. The events observed in Super-Kamiokande are categorized into four types: (1) Fully Contained (FC) events, which have their vertex in the detector and all visible particles contained in the detector. (2) Partially Contained (PC) events, which have their vertex in the detector and at least one visible particle exits from the detector. (3) Upward through-going muons which are produced by the ν_μ charged current interaction in the rock surrounding the detector and go through the detector. (4) Upward stopping muons which are produced by the ν_μ charged current interaction in the rock surrounding the detector but stop in the detector. The primary neutrino (ν_e-like and ν_μ-like) energy are divided in two regions: (1) $E_\nu \leq 1.33$ GeV sub-GeV, (2) $E_\nu > 1.33$GeV multi GeV.

Figure 2 gives the zenith angle distribution of Super-Kamiokande 1289 days samples [48]. Dots, solid and dashed lines correspond to the data, MC with no oscillation and MC best oscillation parameters [49], respectively ($\Delta m = 2.5 \times (10)^{-3}eV^2$, $sin^2 2\theta = 1.00$) . These data are well explained by $\nu_\mu \rightarrow \nu_\tau$ 2-flavor oscillations and are consistent with ν_τ appearance roughly at the two-sigma level.

4. Conclusions from Comparison of the Experimental Data with Theoretical Scheme Predictions on Neutrino Oscillations

1. In the Super-Kamiokande experiment on atmospheric neutrinos the deficit of muonic neutrinos is detected. The analysis shows that they can transit only in ν_τ neutrinos. The $\nu_\mu \rightarrow \nu_e$ transition in this experiment is not observed. From this fact we can conclude (taking into account SNO results) that the length of $\nu_\mu \rightarrow \nu_\tau$ transitions is of the order of the Earth diameter, and the angle θ of $\nu_\mu \rightarrow \nu_\tau$ transitions

is near to the maximal mixing angle $\theta \cong \pi/4$. Then the length of $\nu_\mu \to \nu_e$ transitions is much more than the Earth diameter. The SNO experimental data also confirm, through neutral current registration, $\nu_\mu \to \nu_\tau$ transitions with the same mixing angle.

2. From the SNO experimental data on straight registration neutrinos (by neutral and charge currents in case ν_e and neutral current in the case ν_μ, ν_τ) we can come to the following conclusion: the primary ν_e neutrinos transit in equally proportions in μ_e, ν_μ, ν_τ neutrinos, i. e., mixing angles $\theta_{(...)}$ of ν_e, ν_μ, ν_τ are equal to the maximal angles of mixing. The length of $\nu_e \to \nu_\mu, \mu_\tau$ oscillations is less than the distance to the Sun.

Come to comparison of these results with the predictions in the above-considered theoretical schemes on neutrino oscillations.

4.a. Neutrino-Antineutrino Oscillations

In the existing experimental results the neutrinos disappearance has not been detected (see above), i. e., this mechanism is not confirmed.

4.b. Aromatic Neutrino Oscillations

This scheme was confirmed by experiments.

Pontecorvo-Gribov type oscillations for aromatic neutrinos maximal mixing angle can be realized only at neutrino masses equality $m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau}$. It is hardly probable that the neutrino masses are equal. The length of $\mu_\mu \to \nu_\tau$ oscillations nearly is equal to the Earth diameter, and the length of $\nu_e \to \nu_\tau$ oscillations is more much than the Earth diameter. Then more probable is the type of oscillations suggested by the author [36], $\theta_{(...)} = \pi/4$ (see Sect. 2 and below) and the transition between oscillating neutrinos is virtual. Here neutrino oscillations can take place in the charge mixings scheme [50]. It is supposed that the neutrinos are mixed via weak interactions and therefore if we consider charge mixings of two neutrinos-a, b, then the mixing

angles must be

$$sin\theta \cong \frac{g_w(a)}{\sqrt{g_w^2(a) + g_w^2(b)}} \cong \frac{1}{\sqrt{2}},$$

since $g_w(a) \cong g_w(b)$, where $g_w(a), g_w(b)$ are weak couple constants of a, b neutrinos.

4.c. Majorana Neutrino Oscillations

From the above-considered discussion (see Sect. 2.4.c) we can come to a conclusion that the Dirac and Majorana gauge charges are different and therefore we cannot put Majorana fermions in the Dirac theory. Then it is obvious that this scheme of neutrino oscillations cannot be realized.

4.d. Neutrino Oscillations in the Scheme of Majorana-Dirac Mixing Type

We do not discuss this scheme for the reason given above (see Sect. 2.4.c). It is clear that this scheme cannot be realized in the experiment either.

4.e. Mechanisms of Neutrino Oscillations Enhancement in Matter

4.e.1. Mechanism of Resonance Enhancement of Neutrino Oscillations in Matter

The experimental data on energy spectrum and day-night effect obtained in Super-Kamiokande (energy spectrum of neutrinos is not distorted, day-night effect is within the experimental mistakes) and the results obtained in SNO have not confirmed this effect. Besides, this effect can be realized only at the violation of the law of the energy-momentum conservation (see Section 2.4.e.1 in this work and Ref. [26]).

4.e.2. Mechanism of Accumulation of the Neutrino Different Masses in Matter

This mechanism effectively works only at small mixing angles. Since the mixing angles discovered in SNO and Super-Kamiokande are maximal, then we can neglect the contribution of this mechanism to the neutrino oscillations.

4.f. Neutrino Oscillation in Supersymmetric Models

This type of oscillations can be confirmed only in case of discovery of the superpartners of fermions and bosons besides the neutrino oscillations.

5. Conclusion

The theoretical schemes on neutrino oscillations are considered. The experimental data on neutrino oscillations from Super-Kamiokande (Japan) and SNO (Canada) are given. The comparison of these data with theoretical schemes has been done. We have come to a conclusion: The experimental data confirm only the scheme with transitions (oscillations) between aromatic ν_e, ν_μ, ν_τ neutrinos with maximal mixing angles. This scheme was suggested by Z. Makki et al., in 1962 [2] and repeated by B. Pontecorvo in 1967 [3] and subsequently is developed by Kh. Beshtoev (see references in this work). Besides, this mechanism of a neutrino oscillations is the only one which is theoretically substantiated.

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Бештоев Х. М.   Д2-2001-292
Осцилляции нейтрино. Теория и эксперимент

Рассматриваются теоретические схемы, и приводятся экспериментальные данные по осцилляциям нейтрино, полученные в экспериментах супер-Камиоканде (Япония) и SNO (Канада). Проводится сравнение этих данных с предсказаниями, полученными в теоретических схемах. Сделан вывод о том, что эксперимент подтверждает только схему с переходами (осцилляциями) между электронными, мюонными и тау-нейтрино с максимальными углами смешивания.

Работа выполнена в Лаборатории физики частиц ОИЯИ.

Beshtoev Kh. M.   D2-2001-292
Neutrino Oscillations. Theory and Experiment

Theoretical schemes on neutrino oscillations are considered. The experimental data on neutrino oscillations obtained in the Super-Kamiokande (Japan) and SNO (Canada) experiments are given. Comparison of these data with the predictions obtained in the theoretical schemes is done. Conclusion is made that the experimental data confirm only the scheme with transitions (oscillations) between aromatic ν_e, ν_μ, ν_τ neutrinos with maximal mixing angles.

The investigation has been performed at the Laboratory of Particle Physics, JINR.

Редактор Е. И. Кравченко. Макет Т. Е. Попеко