Probabilistic Analysis of Strength and Thermal-Physic WWER Fuel Rod Characteristics Using START-3 Code

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1. Introduction

During the last years probability methods of studying were widely used to determine the influence exerted by the geometry, technology and performance parameters of a fuel rod on the characteristics of its condition. Geometry, technology and performance fuel rod parameter and parameter of calculation models presentation as random numbers is the base of probabilistic analysis for investigation fuel rod conditions. Fuel rod conditions are random numbers too.

Despite the diversity of probability methods the simplest schema of the Monte-Carlo method (MC) forms their basis. This schema assumes a great number of the realizations of a random value and the statistical assessment of its characteristics. To generate random values, use is usually made of a pseudo-random number generator. The application of the quasi-random sequence elements in place of the latter substantially reduces the machine time since it promotes a quicker convergence of the method.

Probability methods used to study the characteristics of a fuel rod condition can be considered to be an auxiliary means of deterministic calculations that allows the assessment of the conservatism degree of design calculations.

2. Description of Probability Study Procedure

The stage of computations in the probability analysis of the fuel rod condition characteristics by the Monte-Carlo method is multiple calculations using the deterministic code START-3 [1,2,3] where as input parameters use is made of random values having characteristics similar to the probability characteristics of the geometry, technology and performance parameters of a fuel element.

The START-3 code is designed for the strength and thermophysics calculations with the aim of studying, validating and licensing fuel rods for nuclear power reactors on thermal and fast neutrons under normal and off-normal operating conditions. The code is commercially applied by the enterprise-the Lead designer-technologist of power reactor fuel rods – as an instrument for investigating, designing and licensing fuel rods.

2.1. General Schematics of Quasi-Monte-Carlo Method (QMCM)

The simplest schematics of QMCM assumes the acquisition of a large number of random value realizations and statistic assessment of its characteristics.

The standard procedure used to acquire random values consists in using a pseudo-random value generator. As it is known the rate of the convergence of the assessment acquired via the standard MC schematics is in proportion to $1/\sqrt{N}$ where N is the number of computations. There are other procedures promoting a higher rate of the MC method convergence as well as a reduced dispersion of an assessment, which results in low machine time spent on calculations at the constant accuracy. Among those methods the most efficient procedure is the one that assumes the use of uniform sequences of points of integration formula with equal weights in the MC methods algorithm in place of random (pseudo-random) values.

Recently this method has been adequately applied to investigate the probability characteristics of fuel rods [4-5]. Mostly studied and qualitative are quasi-random sequences of Halton and Sobol; both the sequences in a 1-dimensional case coinciding.

In many instances the error order in QMCM is proportional $N^{(1-\varepsilon)}$ (ε>0) for the above sequences which is much more than in MC method using random (pseudo-random) values.

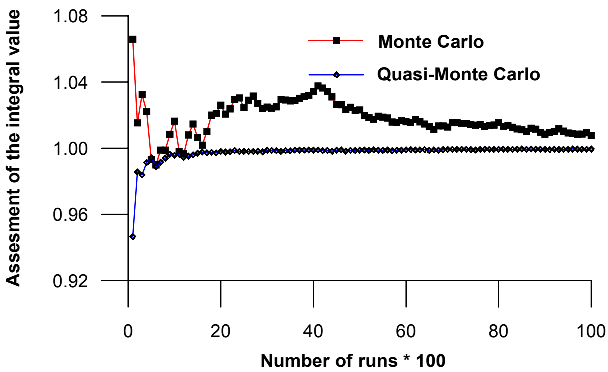

Figure 1. The assessment of the integral value using the MC and QMC methods vs. the number of points used in the calculation for the first 10 000 computations

To illustrate the advantage of using quasi-random sequences in place of pseudo-random ones two trial calculations using both the procedures were implemented.

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Figure 1 shows the assessed mean value for the integral of (1):

$$\varphi = 5{,}818605(x_1 - 0{,}1)\sqrt{|x_2 - 1/9|\sqrt{|x_3 - 1/8|}} \quad (1)$$

by (2)

$$\iiint_{K^n} \varphi(P)dP \approx \frac{1}{N}\sum_{i=1}^{N}\varphi(P_i) \quad (2)$$

(its exact value is unity) vs. the number of points used in the calculation for the first 10 000 points. The figure illustrates the qualitatively more quicker convergence of the results acquired by QMCM compared to those acquired when standard random values are used in MC method.

2.2. Construction of Quasi-Random Sequences in Order to Calculate Fuel Rod Statistic Parameters

The problem investigating the probability characteristics of the fuel rod condition in a core has the following specific features:

- A nuclear reactor core contains some tens of thousands of fuel rods having different performance parameters;
- To assess the serviceability and, hence, the number of failures, all the probability characteristics of the fuel rod condition parameters have to be valid in the low probability range.

The main requirement placed on the sequences of the numbers used to investigate the effect of uncertainties in the input parameters influencing the fuel rod condition characteristics is the requirement for the high degree of the sequence uniformity which ensures the quick convergence of the method and minimization of spent machine time. Sobol's quasi-random values conform to those requirements.

To construct the quasi-random values $Q_i^*=(q_{i,1}, \ldots, q_{i,n})$, i=1, 2, …, that form a sequence which Sobol's sequence, a special algorithm was realized [6].

The quasi-random value sequence acquired via this algorithm was checked for the distribution uniformity of its elements in a multidimensional cube using the k-uniformity test [7].

Also, tests were carried out in which the numbers are categorized according to some attribute and the empiric frequencies are compared to their mathematical expectation using χ^2 test [8].

2.2.1. The Frequency Test

The frequency test implements the check-up of the frequency of appearance of numbers in sequence intervals (the first decimal numbers are checked up).

The null hypothesis

Frequency coincidence of different digitals in investigated sequences with mathematical expectation these frequencies for random sequence.

2.2.2. The Serial Test

The frequency of various two-digit numbers among the independent pairs $\varepsilon_1\varepsilon_2$, $\varepsilon_3\varepsilon_4$, …, $\varepsilon_{N-1}\varepsilon_N$ successively formed from the sequence of the random numbers ε_1, ε_2, …, ε_N is checked up.

The null hypothesis

Frequency coincidence of different two-digit numbers among the independent pairs in investigated sequence with mathematical expectation these frequencies for random sequence.

2.2.3. The Run Test

The number of various series of a length l (ε_{k+1}= ε_{k+2}= … =ε_{k+l}) is checked up in the sequence of random values ε_1, ε_2, …, ε_N. The total number of the series is $n=n_1+\ldots+n_m+n'_{m+1}$.

The null hypothesis

The null hypothesis is a coincidence of series number in investigated sequence with mathematical expectation these series number for random sequence.

Table 1 summarizes the results of checking-up quasi-random values acquired via the above described algorithm.

Table 1. The results of checking-up quasi-random values acquired via the given algorithm using the system of tests

	Design value of the control χ^2 test	Critical value of the control χ^2 test	Result of checking-up null hypothesis
Frequency test (9 degrees of freedom)	0.0003	16.9	Not rejected
Uniformity test (99 degrees of freedom)	1609.5	10233.0	Not rejected
Serial test (99 degrees of freedom)	0.16	123.02	Not rejected
Run test (4 degrees of freedom)	1.3	9.5	Not rejected

2.3. Procedure Used to Construct Probability Characteristics of the Fuel Rod Condition

In the probability analysis of the characteristics of fuel rod conditions in a core consideration is given to a 60-degree sector of the symmetry. The WWER-1000 core sector contains 28 fuel assemblies with 312 fuel rods per each that are in operation for 3 or 4 years.

The procedure of the probability calculations involves the following: the calculations consider all fuel assemblies in the symmetry sector, however, not all fuel rods within a fuel assembly are subjected to calculations but only a number of randomly taken ones. Every power history is calculated with different values of input parameters (geometry, technology and performance parameters).

To check up the results and refine the parameters of the design schematics at the preliminary stage, the probability calculation of all fuel rods within a fuel assembly is employed.

The linear rate uncertainty is introduced into calculation by coefficient of liner rate in every calculated time point. This coefficient is random digit with normal distribution, expected value: 1 and standard deviation 0.05-0.1.

Table 2. Mean value and standard deviation in gas release based on results of calculations and post-irradiation measurements

	Mean value, [%]	Standard deviation, [%]
Calculation	0.733	0.395
Measurement	0.780	0.623

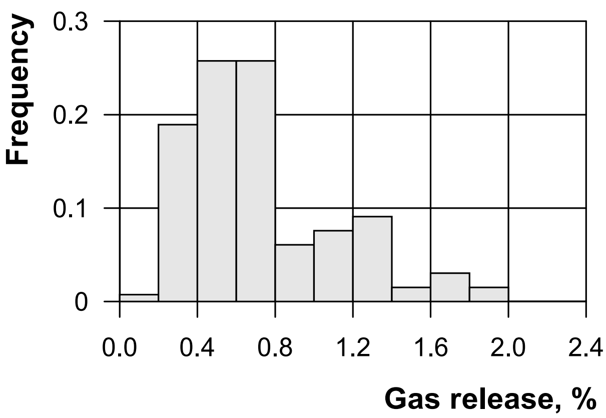

Figure 2. Gas release distribution in 44 fuel rods of fuel assembly E0325 according to calculation

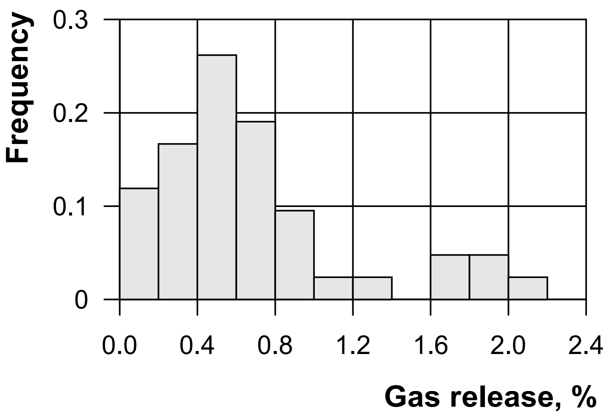

Figure 3. Gas release distribution in 44 fuel rods of fuel assembly E0325 according to PIE

3. Comparison between Results of Probability Calculations of Fuel Rods in Fuel Assembly E0325 and Results of PIE

3.1. Description of Calculations

WWER-1000 fuel assembly E0325 has operated for four fuel cycles in Zaporozhie NPP, Unit 1, to achieve the average burn-up of 48.9 MWd/kgU at the maximal linear heating up to 290 W/cm.

The following parameters were considered to be random:

- Linear heat rating of fuel rods. Its distribution was assumed normal with the dispersion of 0.07;
- Fuel rod geometry and technology parameters: fuel density, fuel densification, outer and inner radii of fuel pellet, outer and inner radii of fuel rod cladding, open porosity of fuel and gas plenum length;
- The distribution of all the parameters considered to be normal which is corroborated by statistics studies at the production plant.

In all, 44 fuel rods were subjected to calculations for which PIEs of the gas composition and pressure under claddings were implemented. Every fuel rod was calculated 3 times.

3.2. Comparison between Calculated Results and PIE Data

3.2.1. Gas Release in Fuel Rods at the End of Irradiation Cycle

The values of the estimated mean value of gas releases at the end of the irradiation cycle in the calculated and PIE results are tabulated in Table 2.

Figures 2 and 3 illustrate the frequency dependencies on gas releases at the end of irradiation cycle for the calculated results and the results of fuel rod PIE.

For gas release samples the Wilcoxon test at the confidence level of 5% does not reject the hypotheses of the calculated and experimental samples belonging to the same general population.

3.2.2. Pressure in Cold Fuel Rod at the End of its Irradiation Cycle

The values of the mean and standard deviation of the pressure (MPa) in a cold fuel rod at the end of its irradiation cycle are listed below.

Figures 4-5 are histograms of pressure distribution based on the results of the calculations and the experimentally acquired data.

It is evident from the figures that there are some discrepancies in the values of the pressure distribution between the calculated results and the data acquired experimentally.

Based on the analysis of the PIE one can assume the availability of a procedure error in the results on the gas pressure measured under the fuel rod cladding at the end of the irradiation cycle. As is known, the value of the pressure is basically determined by the percentage of a gas release. The mutual accordance between the calculated pressure and gas release values is traceable; similar laws govern the distribution. As far as the results of PIE, no interrelation of this kind is observed and the modes of the distribution of both the parameters are quite different.

It is evident from the results given above that the probability methods may be useful not only at the stage of the design calculations of a fuel rod but also for investigations and analyses of the PIE results.

4. Results of Demonstrative Probability Calculation of WWER Fuel Rods under Transient Conditions

4.1. Calculation Condition Statistics Characteristics of Technology and Geometry Parameters of Fuel Rod

Consideration is given to a 60-degree sector of the core symmetry of the series WWER-1000 of a four-year fuel cycle loaded with upgraded fuel assemblies and U-Gd fuel. The sector accommodates 28 fuel assemblies at a time. In actual fact, 3 four-year cycle and 9 three-year cycle fuel assemblies are to be calculated. Each fuel assembly contains 306 fuel rods and 6 U-Gd fuel rod.

The following parameters are considered to be random input ones:

- Inner radius of cladding;
- Outer radius of cladding;
- Outer radius of pellet;
- Inner radius of pellet;
- Fuel density;
- Volume irradiation induced sintering of fuel.

The choice of the parameters is governed by the fact that they substantially affect the level of temperatures of a fuel rod, the gas pressure under the cladding as well as the stresses and strains of a fuel rod cladding.

Two series of the calculations were implemented with 100-50-100% realization of the transient conditions at the beginning (20 eff. days) and at the end (280 eff. days) of the operation cycle. To demonstrate the features of high level stress conditions in fuel claddings the special operational algorithm was involved into consideration. The main parameter to chose the algorithm was high level of power ramps in fuel rods and correspondingly high level of PCMI.

For the statistics analysis fuel assemblies in the 3-rd and 4-th years of operation were taken since their burn-up levels are adequate for the PCI effect to show up in transients.

Table 3. Mean value and standard deviation of pressure in cold fuel rod based on calculations and PIE measurements

	Mean value, [MPa]	Standard deviation, [MPa]
Calculation	2.651	0.041
Measurement	2.590	0.083

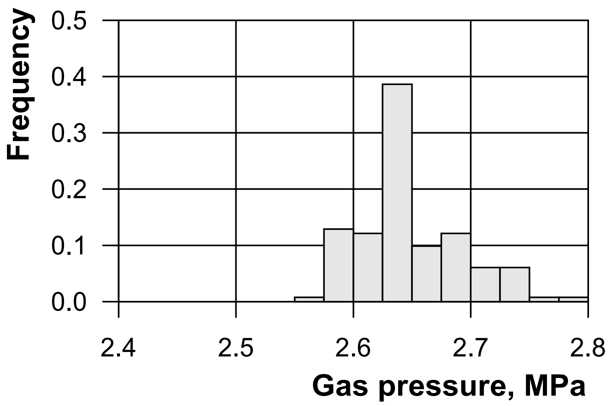

Figure 4. Gas pressure distribution in 44 fuel rods of fuel assembly E0325 according to calculation

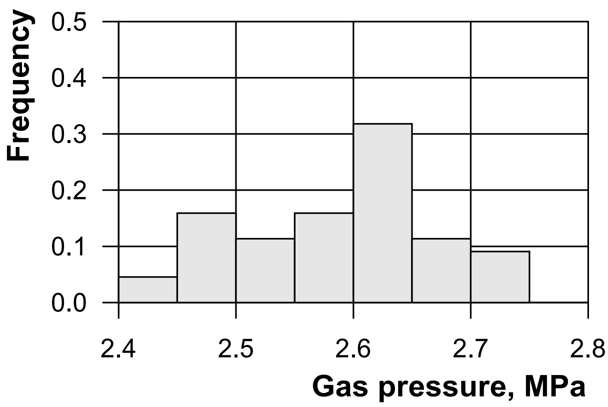

Figure 5. Gas pressure distribution in 44 fuel rods of fuel assembly E0325 according to PIE

The maximum hoop stress of a cladding in the irradiation cycle was assumed to be the fuel rod strength characteristic.

4.2. Results of Probability Calculations

4.2.1. Determination of Design Schema Parameters

At the first stage fuel rods of a 3-year cycle fuel assembly were subjected to calculations; 13 times per each with different sets of random parameters. Based on the results of the calculation, the procedure was mastered to choose the minimally adequate quantity of calculations at which the distribution mode and the values of the distribution parameter estimations remain at an adequate level of accuracy.

It is found that depending on the needed accuracy of calculations, the design schema parameters can be chosen from the following range: calculation of 50-150 fuel rods; each fuel rod being calculated from 1 to 6 times.

4.2.2. Results of Probability Calculations of Power Ramps by Suggested Procedure

75 fuel rods were subjected to calculations with each history being calculated 3 times. Altogether 12 fuel assemblies were calculated in 2 power ramp versions, namely, 20 and 280 days.

The mode of the stress distribution for the 20-day version is shown in Figure 6, for the 280-day version – in Figure 7.

From the analysis of the stress value samples follows:

- For a stress sample acquired from the calculation there is a significant symmetry in distribution (a ramp on the 280-th day) or several distinct modes (a ramp on the 20-th day);
- Mechanisms governing the behaviour of stress distributions in the range of low and mean stresses have a low influence on the stress behaviour in the high stress range;
- Probability of limit values stress exceeding at power ramp in the beginning campaign is a little and less on a several orders then in conditions

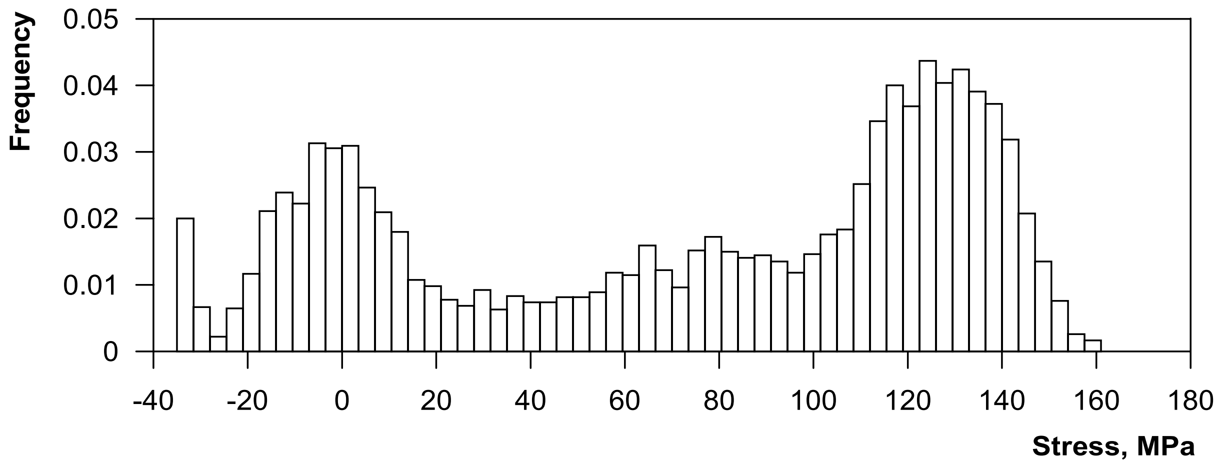

Figure 6. Maximum stress distributions in fuel rod cladding for a power ramp on the 20-th effective day

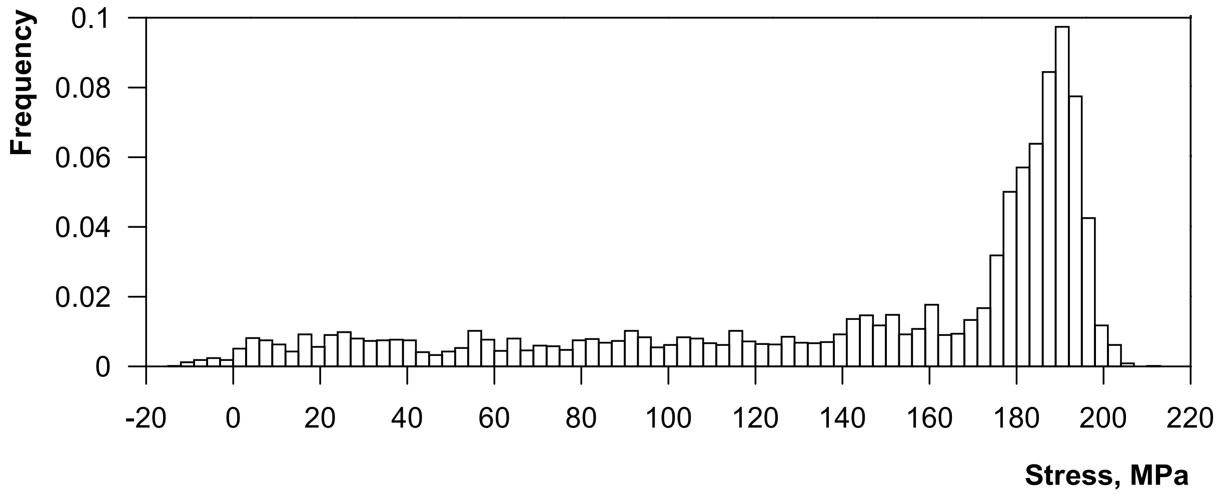

Figure7. Maximum stress distributions in fuel rod cladding for a power ramp on the 280-th effective day

of power ramp in the end of campaign.

Proceeding from the above to determine the behaviour of stresses in the high probability range (on the right hand "tail" of distribution), it is sufficient to consider random data under the condition of a ramp on the 280-th day at high stress values.

Figure 8 illustrates the randomly chosen values of stresses in excess of 200 MPa in fuel rods under power ramp conditions on the 280-th day for 3-year fuel assemblies on the probability paper of Gnedenko-Gumbel distribution (dual exponential distribution). This particular distribution is one of the distributions of extreme values [10,11].

The approximation of the empirical function of distribution with the theoretical dependence allows the computation of the probability of rare events (involving in this case the exceeding of the standard value of the design parameters which is equivalent to a fuel rod failure on the strength acceptance criterion).

Using defined theoretical distributions of extreme stress values both the probability of ultimate value of 250 MPa (SCC criteria for Zr-1%Nb alloy) and the design limit of 210 MPa exceed were estimated.

The probability of ultimate value (250 MPa) exceed is up to $1.4\cdot10^{-14}$.

The probability of design limit (208 MPa) exceed is up to $5.8\cdot10^{-4}$.

These probabilities were calculated for 60-degree symmetry sector and they are right for all core.

The results of calculations shows: probability of considered stress limits exceeding in 3-years assemblies is more on several orders then probability of similar occurrences in assemblies of other years. Thus, in calculate of occurrence probability for symmetry sector it used ratio of 3-years assemblies number to all number assemblies in symmetry sector of core.

The results of statistics calculations show that on the one hand probability of physical SCC limit exceed is negligible quantity. On the second hand probability of design limit exceed of about 1% in comparison with SCC limit could be interpreted as over-conservatism in design margin sizing. But design margin definition is more intricate problem that could be solved on the basis of probabilistic analysis of mechanical calculations. More other aspects both the operational and the design experience should be taken into consideration.

Carried out calculation enables to provide comparison of statistical estimation of mechanical FR parameters with deterministic one in investigated transients.

Conservative computation of the single maximum overrated in this transient fuel rod was carried out. It's necessary to note that selection of such kind of FR in huge file of neutron data is also probabilistic task. The calculations achieve the maximum cladding hoop stress of 225 MPa.

That could be interpreted as probability of realization of such stress level in core with value of

$$1/N_{pin}N_{FA} \approx 1.2\cdot10^{-4}$$

as minimum, where:

N_{FA} – number of FA in symmetry core sector,

N_{pin} – number of pins in FA.

Statistical estimation of probability for that level of stress is not exceed $3\cdot10^{-8}$. Thus use of statistical computations in design of fuel permits to decrease limitation in operational conditions on power units.

5. Conclusions

The method for statistical analysis of mechanical and physical characteristics of fuel rods by START-3 code was developed.

Wide set of input both technological parameters and operational parameters are possible to be random in statistical analysis.

Special tests and verifications based on PIE of VVER fuel were carry out to provide the minimum adequate quantity of calculations at which the distribution mode and the values of the distribution parameter estimations remain at an adequate level of accuracy.

Two series of the statistical demonstrative calculations were implemented with 100-50-100%

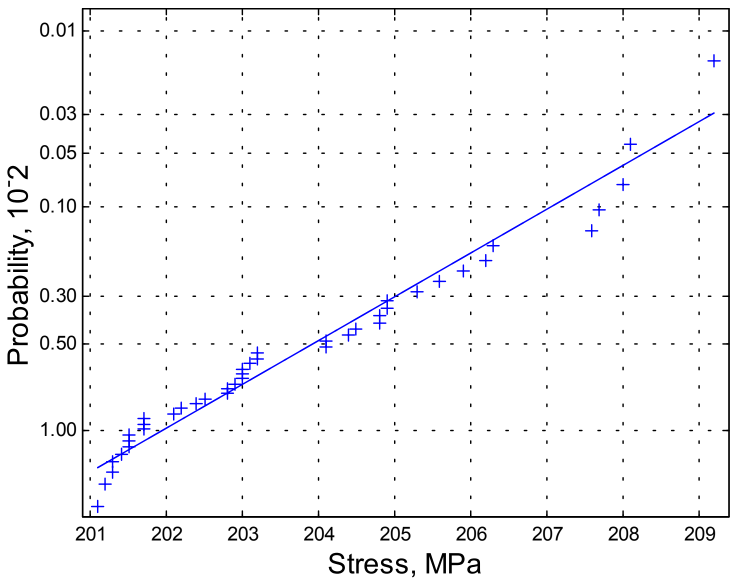

Figure 8. Probability paper of Gnedenko-Gumbel distribution

normal operation transient at the beginning (20 eff. days) and at the end (280 eff. days) of the WWER fuel cycle.

Analysis indicates that probability of SCC limits exceeds in investigated transient acquired by statistical and deterministic methods differ more then tens.

Statistical methods provide on the one hand reliability estimations possibility on the other hand – estimations of conservatism of deterministic analysis.

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