



DETERMINISTIC CHAOS IN RADON TIME VARIATIONS

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INTRODUCTION

Radon, ^{222}Rn , is an inert radioactive gas emitted ubiquitously by soil. The exhalation rate of the radon depends on the concentration of its parent (^{226}Ra) existing in the Earth's crust, the meteorological conditions, and the properties of the soil, like porosity and water content [1].

The atmosphere can be considered as a complex and non-linear dynamic system whose variables interact, some of them in a chaotic, but also deterministic way.

To examine the dynamics of radon concentrations, it is to measure radon in the air continuously over some period of time. The radon time series can be analyzed to determine the degree of chaotic behavior, to predict their fluctuations in the future using fractal methods and to investigate their correlations with meteorological parameters by comparing their fractal dimensions.

However, different chaos based measurements provide different measures of the degree of chaotic behavior [2].

METHODS

We applied fractal methods to radon time series with a few approaches; we examined the fractal properties of ^{222}Rn indoor and outdoor time series using Hurst's rescaled range analysis, so we calculated Hurst exponent (H); as well we calculated Lyapunov exponent (λ), then attractor fractal dimensions as capacity dimension (or box-counting dimension, D_b) correlation dimension (D_c) and Lyapunov dimension (or Kaplan-Yorke dimension, D_L).

The Hurst exponent lies in the region of 0 to 1 [3]; if the H is close to 0.5, the time series point out random and uncorrelated data, or successive steps are independent.

The Lyapunov exponent, λ , is a measure of the rate at which nearby trajectories in phase space diverge. If λ is positive, then we say the behavior is chaotic [4]. The larger the positive exponent, the more chaotic is the system and, conversely, the shorter is time scale of the system's predictability.

The embedding procedure uses a recorded series of X values for some dynamical system and forms the series of values X_1, X_2, \dots, X_d , so the embedding dimension, d , is the dimension of the embedding space [4].

In calculating the mentioned fractal dimensions, we used the programs of the Chaos Data Analyzer (CDA), Dataplore (DP) and Visual Recurrence Analysis (VRA) [6].

MEASUREMENTS AND RESULTS

Radon concentration measurements were made at three different locations (site A: Tiborjanci, indoors, living room; site B: Valpovo, outdoors; site C: Valpovo, indoors, basement) with an AlphaGuard PQ 2000, as well as the barometric pressure and temperature of the air. The measurements were carried out in 10 minutes intervals in May (site A; 4332 intervals), June (B; 4038) and September 2002 (C; 2269), and the mean radon concentrations of 31.1, 13.6 and 121.2 Bq/m³, with the standard deviations of 19.8, 11.4 and 98.0 Bq/m³, respectively, were obtained.

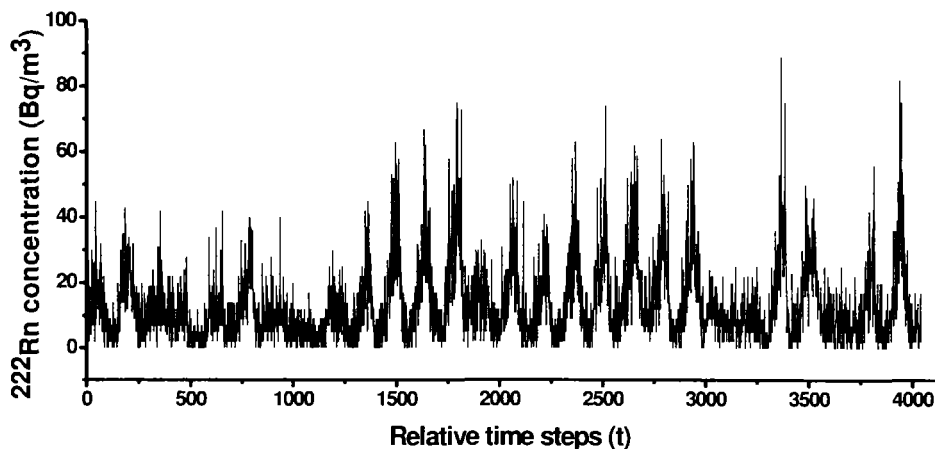


Figure 1. Outdoor radon variation (site B) versus number of 10 minutes intervals.

The radon concentration variation (site B) against number of 10 minutes intervals is presented at Figure 1; one can see the radon variation is not quite periodically, or it seems irregular, but if it is chaotic, or if the radon changes are random and they behavior as a noise, that should we examine.

Calculation of the embedding dimension for the outdoor radon variation by means of the VRA program and the False Nearest Neighbors (FNN) method shown clearly the dimension of 5 (a minimum position of the curve at 47 percent of false neighbors; otherwise the FNN calculations for the uniform noise gave near 50 percent of false neighbors for all embedding dimensions).

Well, considering the FNN method, the outdoor radon variation shown characteristics of deterministic chaos.

The application of the fractal methods to three radon time series (at the sites A, B, C), as well as to the barometric pressure and temperature ones, by means of the CDA program, except the values of the λ_h , D_L and d , which were calculated by the DP program and the VRA one, respectively, gave the results presented in Table 1.

We can see for radon, from Table 1, the λ is positive at all the measurement sites, that points out a chaotic regime of the radon concentration in the air. The outdoor radon measurement had the highest value of the Lyapunov exponent ($\lambda_B = 0.83$), afterwards followed the radon in the living room ($\lambda_A = 0.72$) and in the basement ($\lambda_C = 0.50$), that spoken, the most chaotic atmosphere was outdoors; this was to expect considering influence of more outdoor meteorological parameters to the radon.

The Hurst exponent for radon was small and less than 0.5, that indicated anti-persistent behavior, i.e. an increasing trend in the past implied a probable decreasing trend in the future, and conversely; this behavior was the most expressed by the outdoor radon, that had the least value of $H (= 0.15)$.

According to the attractor's non-integer or fractal dimension for radon in the living room and outdoors ($D_b \cong 2.3$), the lower bound on the number of dependent variables, required to describe the time evolution of the radon concentration in air, is 3; for radon in the basement ($D_b \cong 1.7$), the number of required variables is 2.

Of course, the considering variables, in the mentioned experiments with radon, were meteorological parameters like barometric pressure, temperature and wind, or ventilation, and others (e.g. diffusion parameters).

Table 1. The values of the Hurst exponent (H), Lyapunov exponent (λ , λ_h), capacity dimension (D_b), correlation dimension (D_c), Lyapunov dimension (D_L) and embedding dimension (d), calculated for radon, barometric pressure and temperature, at the sites A (living room), B (outdoors) and C (basement), for N data or measurements in 10 minutes intervals; the values of the λ_h and D_L were obtained for N/6 data or measurements in 1 hour intervals.

Radon:

Site	N	H	λ	D_b	D_c	λ_h	D_L	d
A	4332	0.16	0.72 ± 0.02	2.34 ± 0.25	4.50 ± 0.17	0.29	1.44	8
B	4038	0.15	0.83 ± 0.36	2.27 ± 0.25	4.99 ± 0.47	-	-	5
C	2269	0.23	0.50 ± 0.03	1.69 ± 0.25	3.92 ± 0.25	0.15	1.17	9

Barometric pressure:

Site	N	H	λ	D_b	D_c	λ_h	D_L	d
A	4332	0.74	0.11 ± 0.02	2.59 ± 0.46	1.22 ± 0.13	0.17	1.25	8
B	4038	0.71	0.10 ± 0.02	2.49 ± 0.43	1.09 ± 0.12	0.09	1.18	12
C	2269	0.79	0.15 ± 0.03	2.44 ± 0.62	1.35 ± 0.20	0.06	1.14	27

Temperature:

Site	N	H	λ	D_b	D_c	λ_h	D_L	d
A	4332	0.61	0.13 ± 0.05	1.63 ± 0.19	1.37 ± 0.15	0.05	1.08	10
B	4038	0.86	0.16 ± 0.03	2.60 ± 0.86	1.29 ± 0.14	0.15	1.52	9
C	2269	0.53	0.05 ± 0.06	2.13 ± 0.25	0.50 ± 0.07	0.11	1.18	5

The parameters of the fractal analysis for barometric pressure and temperature, from Table 1, also shown a chaotic behavior, like above for radon time series, although the positive λ values were lower than the ones for radon; it spoke on less grade of the deterministic chaos by temperature and pressure, and particularly by the temperature time series in the basement, where λ was 0.05 ± 0.06 only, and $H = 0.53$. Herby one can speak on absent of deterministic chaos (λ near zero); also the Hurst exponent ($H \cong 0.5$) indicates random and uncorrelated data of the temperature time series in the basement.

The capacity dimension, D_b , of pressure and temperature had similar values like the radon (Table 1), that spoke on a multiple correlation between time variation of radon, barometric pressure and temperature; namely, similar D_b values, as well as the D_L ones, indicated the same chaotic way of the mentioned variables. The high values of the Hurst exponent ($0.5 < H < 1$)

for pressure and temperature indicated persistent behavior, or an increasing trend in the past implied on the average a continued increase in the future, and conversely for decreasing trend.

The DP program gave all the Lyapunov exponents, among them one was positive and the second one negative for all the time series; only the positive λ_h are presented in Table 1. Since the reciprocal of the λ_h , or the entropy, is roughly the time over which meaningful prediction is possible, the value of $1/\lambda_h$ (so-called Lyapunov time) in the living room is $1/0.29 \text{ h}^{-1} = 3.4 \text{ h}$ and $1/0.05 \text{ h}^{-1} = 20.0 \text{ h}$ for radon and temperature, respectively.

CONCLUSION

The application of the fractal methods to three radon time series (in the living room, outdoors and in the basement), as well as to the barometric pressure and temperature ones, gave the values of the Hurst exponent (H), Lyapunov exponent (λ), capacity dimension (D_b), correlation dimension (D_c), Lyapunov dimension (D_L) and embedding dimension (d).

The positive values of λ for all the time series of radon, barometric pressure and temperature indicated their chaotic behavior; to be sure, the radon measurements shown higher λ (and higher grade of deterministic chaos) than the one of pressure and temperature; still the temperature measurements in the basement shown the Lyapunov exponent near zero; hereby one can speak on absent of deterministic chaos. Also the respective Hurst exponent ($H \cong 0.5$) indicated random and uncorrelated data of the temperature time series in the basement.

The Hurst exponent for radon was small and less than 0.5, that indicated anti-persistent behavior, i.e. an increasing trend in the past implied a probable decreasing trend in the future, and conversely; this behavior was the most expressed by the outdoor radon, that had the least value of H ($= 0.15$). Meanwhile, the high values of the Hurst exponent ($0.5 < H < 1$) for pressure and temperature indicated persistent behavior, or an increasing trend in the past implied on the average a continued increase in the future, and conversely for decreasing trend.

According to the attractor's non-integer or fractal dimension for radon in the living room and outdoors ($D_b \cong 2.3$), the lower bound on the number of dependent variables required to describe the time evolution of the radon concentration in air is 3; for radon in the basement ($D_b \cong 1.7$), the number of required variables is 2. The considering variables in the mentioned experiments with radon were meteorological parameters like barometric

pressure, temperature and wind, or ventilation indoors, and others (e.g. diffusion parameters).

The capacity dimension, D_b , of pressure and temperature had similar values like the radon, that spoke on a multiple correlation between time variation of radon, barometric pressure and temperature; namely, similar D_b values, as well as the D_L ones, indicated the same chaotic way of the mentioned variables.

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ABSTRACT

Radon concentrations were continuously measured outdoors, in living room and basement in 10-minute intervals for a month. The radon time series were analyzed by comparing algorithms to extract phase-space dynamical information. The application of fractal methods enabled to explore the chaotic nature of radon in the atmosphere. The computed fractal dimensions, such as Hurst exponent (H) from the rescaled range analysis, Lyapunov exponent (λ) and attractor dimension, provided estimates of the degree of chaotic behavior. The obtained low values of the Hurst exponent ($0 < H < 0.5$) indicated anti-persistent behavior (non random changes) of the time series, but the positive values of the λ pointed out the great sensitivity on initial conditions and appearing deterministic chaos by radon time variations. The calculated fractal dimensions of attractors indicated more influencing (meteorological) parameters on radon in the atmosphere.