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ANGULAR CRITERION FOR DISTINGUISHING BETWEEN FRAUNHOFER AND FRESNEL DIFFRACTION

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THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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ABSTRACT

The distinction between Fresnel and Fraunhofer diffraction is a crucial condition for the accurate analysis of diffracting structures. In this paper we propose a criterion based on the angle subtended by the first zero of the diffraction pattern from the center of the diffracting aperture. The determination of the zero of the diffraction pattern is the crucial point for assuring the precision of the criterion. It mainly depends on the dynamical range of the detector. Therefore, the applicability of adequate thresholds for different detector types is discussed. The criterion is also generalized by expressing it in terms of the number of Fresnel zones delimited by the aperture. Simulations are reported for illustrating the feasibility of the criterion.

1. INTRODUCTION

The distinction between Fresnel and Fraunhofer diffraction is a crucial condition for the accurate analysis of diffracting structures. Conventional textbooks for optics propose distinguishing criteria, which are based on conditions with different requirement levels, from the relative weak ones usually related to visual conditions, to the very strong ones that involve the performance of very sensitivity instruments. Furthermore, most of them are restricted to the incidence of plane waves onto the diffracting structure.

In this paper we propose a criterion based on the angle subtended by the first zero of the diffraction pattern from the center of the diffracting aperture, which does not require illumination with plane waves. Specifically, it determines the distance from the aperture up that this angle remains invariant to the infinity, assuming the infinity as the distance up that Fraunhofer diffraction holds under illumination with plane waves. This definition of "optical infinity" is also assumed by most the optics textbooks independently of the criteria they propose for distinguishing Fresnel and Fraunhofer diffraction.

This criterion is generalized to many equivalent situations by expressing it in terms of the number of Fresnel zones delimited by the aperture in each specific set up. The determination of the zero of the diffraction pattern is the crucial point for assuring the precision of the criterion. It mainly depends on the dynamical range of the detector. Therefore, we will discuss the applicability of adequate thresholds for visual detection and for a conventional CCD.

Simulation results are reported for illustrating the application of the criterion.

2. A DIFFRACTION CRITERION

Let us consider a diffracting aperture whose maximum and minimum dimensions are of similar length. So, its area approaches to the area of the circular aperture that circumscribes it. Then, we can compare the diffraction patterns produced by both apertures in order to establish the criterion for distinguishing between Fraunhofer and Fresnel diffraction.

The ideal Fraunhofer diffraction pattern produced by the circular aperture is observed on a plane located at an infinite distance from the aperture, when the aperture is illuminated by a plane wave. Its intensity is distributed over a central disc, usually called the *Airy disc*, surrounded by rings of increasing radius. The maximum intensity is obtained at the centre of the Airy disc. There are also secondary maxima of intensity in the rings, but their values are much smaller than the maximum of the Airy disc and diminish monotonically. In fact, about 84% of the total intensity of the pattern is encircled by the Airy disc, and about 95% will be encircled by the Airy disc and the first ring.

Circumferences of zero intensity delimit the Airy disc and the surrounding rings, so that the angle that subtends the first circumference of zero intensity of the pattern from the centre of the aperture will be a characteristic descriptor of this diffraction pattern. This descriptor allows us to compare the ideal Fraunhofer pattern with the diffraction pattern provided by the probe aperture. From this point of view, the following statement is reasonable:

Fraunhofer diffraction occurs if the angle that subtends the first zero of the diffraction pattern from the centre of the probe aperture is equal (or very similar under a tolerance condition previously assumed) to that of the Airy pattern provided by the circular aperture that circumscribes it.

Most of the textbooks for optics implicitly include this statement in the discussion of the Fraunhofer diffraction.

So, for distinguishing between Fraunhofer and Fresnel diffraction it should be enough to compare the angle that subtend the first zero of the observed diffraction pattern, θ_0, to the angle that subtends the first zero or the Airy pattern, θ_A, taking into account the changes in the value of the angle due to changes in the parameters that define the geometry of the diffraction setup. The percent of this difference can be expressed as $\Delta\theta_{\%} = \frac{\theta_0 - \theta_A}{\theta_A} \times 100\%$, with $\theta_A = 0.61(\lambda / R)$. We regard $\Delta\theta_{\%} \leq 5\%$ as permissible for Fraunhofer diffraction, otherwise Fresnel diffraction must be considered.

The number of Fresnel zones inscribed by the aperture is other interesting quantitative descriptor for diffraction. It is more general that the angle because its value is the same for all diffraction set-ups that provide diffraction patterns with intensity distributions, which only differ on scale factors or orientation. In parabolic approach, the number of Fresnel zones N will be given by [12]

$$N = \frac{R^2}{\lambda}\left(\frac{1}{r_0} + \frac{1}{r}\right), \tag{1}$$

with λ the wavelength. The parameters of this expression are sketched in the set-up in Figure 1. The line $\overline{SP}$ determines the optical axis of the set-up. It is normal to the aperture plane and intersects it at the aperture centre. The radius of the circular aperture will be denoted as R and the pupil function of the aperture is assumed to be constant. A point source is located at S, so that r_0 is the radius of the spherical wave front emitted by the source at the aperture centre. The point P, located on the optical axis at a distance r from the aperture centre, will be the diffraction pattern centre. Equation (1) holds under the conditions $R \ll r_0$ and $R \ll r$, which determine the *paraxial approach.*

Now, Table 1 shows the intensity profiles recorded for different values of N. For $N < 0.5$ the first minimum of the diffraction pattern closely approaches to zero and the difference between the value of the corresponding angle and the angle of the Airy pattern will be not greater than the 5%. So, these patterns are closely approached to the ideal Fraunhofer pattern.

For visual observations this error is not relevant because, in a vicinity of a given maximum of intensity, the eye perceives as null the intensity values smaller than its 20%. All the set-ups in Figure 3 can accomplish this condition by properly choosing the geometrical parameters of equation (1). Therefore, as visual criterion Fresnel diffraction appears for $N \geq 0.5$, condition that can be realized by the set-ups in Figure 4 too.

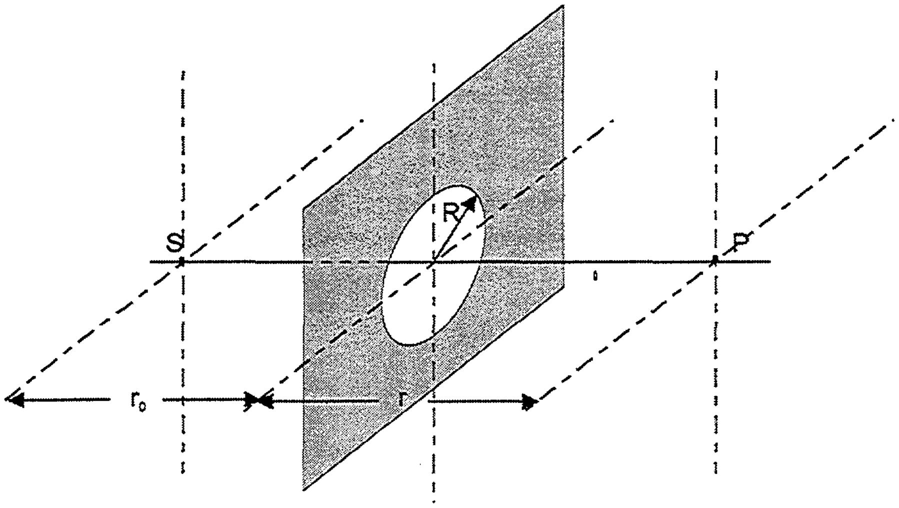


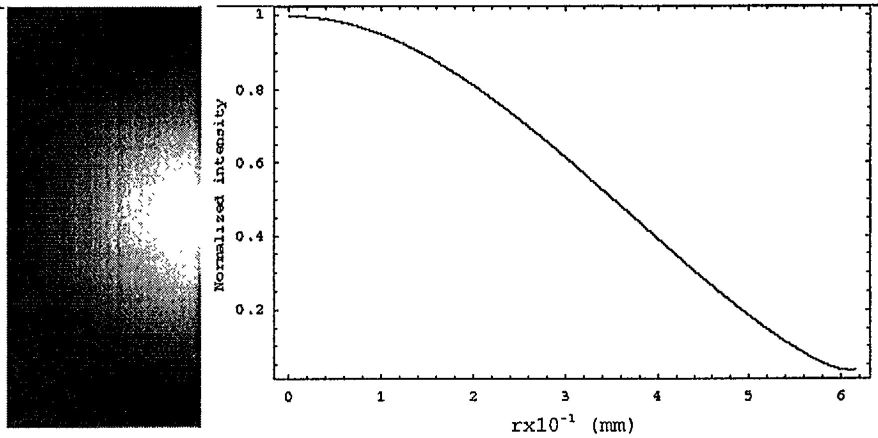
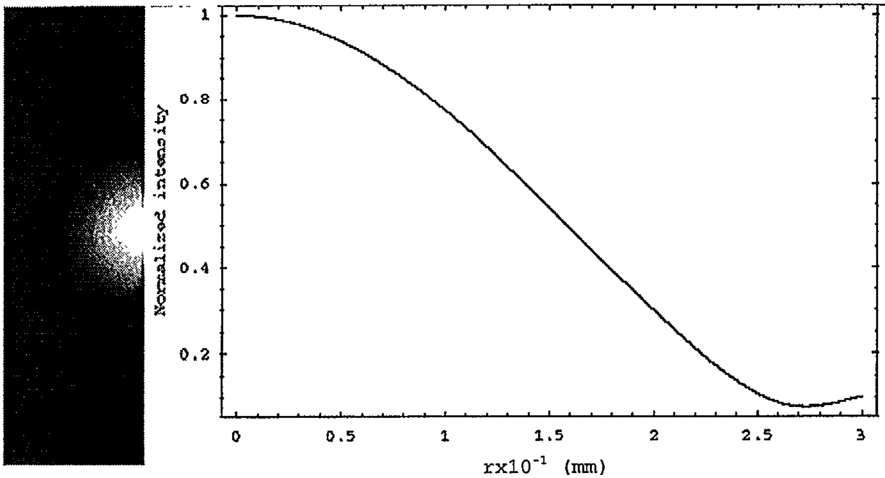
Fig. 1: Diffraction set-up.
***S*= point source, *R*= radius of the circular aperture, *P*= centre of the observed diffraction pattern.**

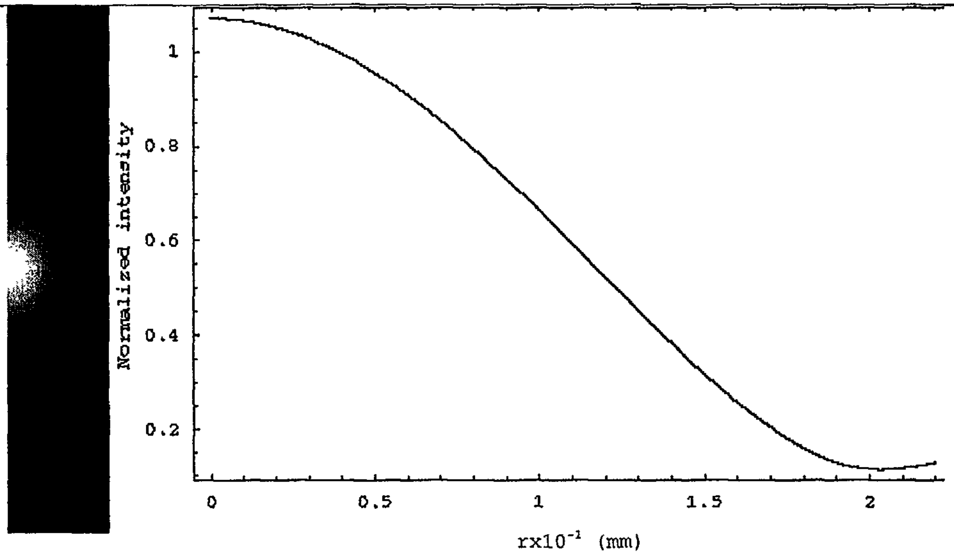
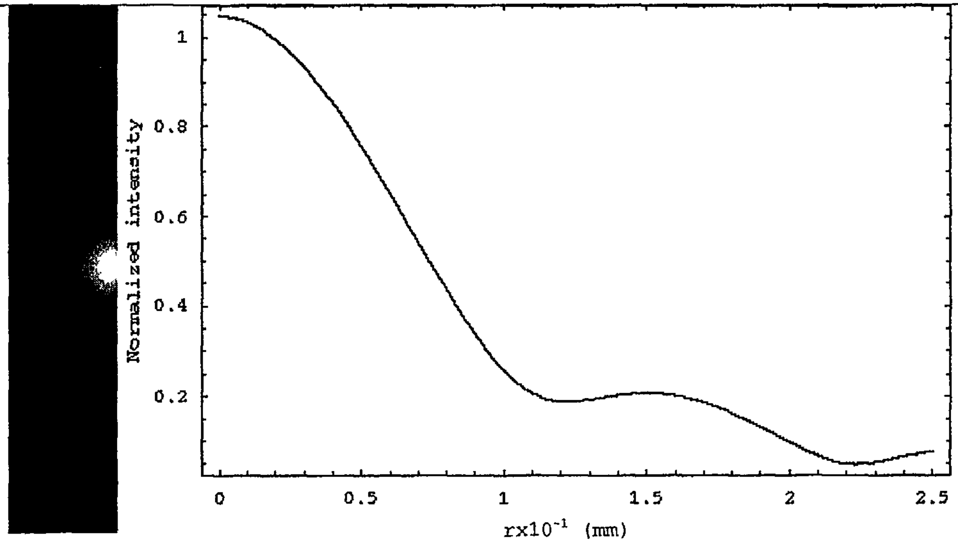
By N=0.5 the value of the first minimum and the first near maximum grow and approach to a plateau. Consequently, the first detectable zero of the diffraction pattern will subtend an angle greater than the angle that subtends the first zero of the Airy diffraction pattern in about 5.18%.

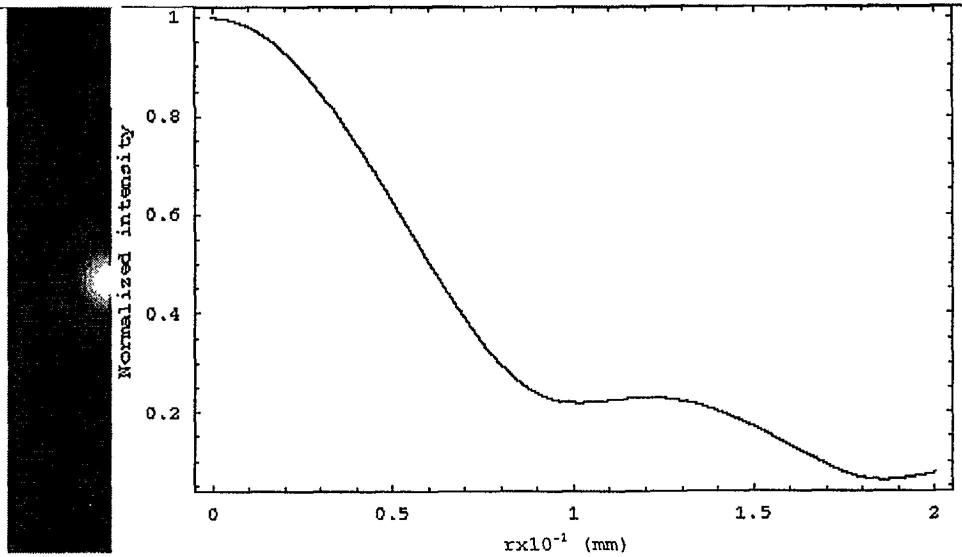
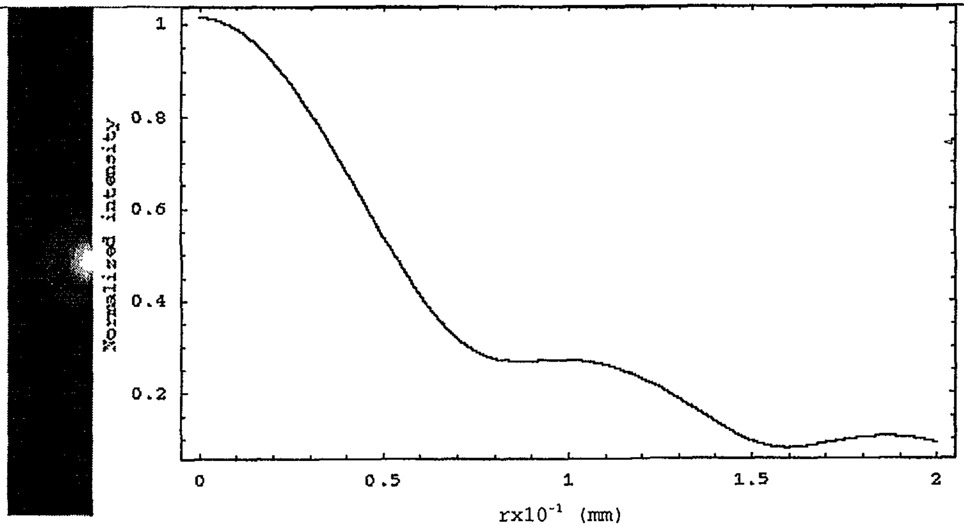
For $N \geq 0.5$ more plateaus will be produced in the diffraction pattern because of Fresnel diffraction effects. Consequently, $\Delta\theta_{\%}$ will grow as shown in Table 1.

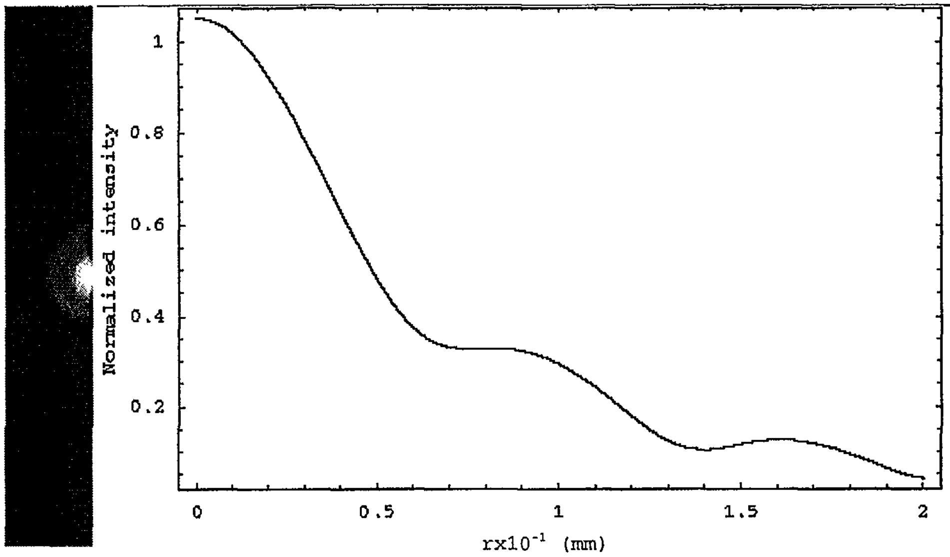
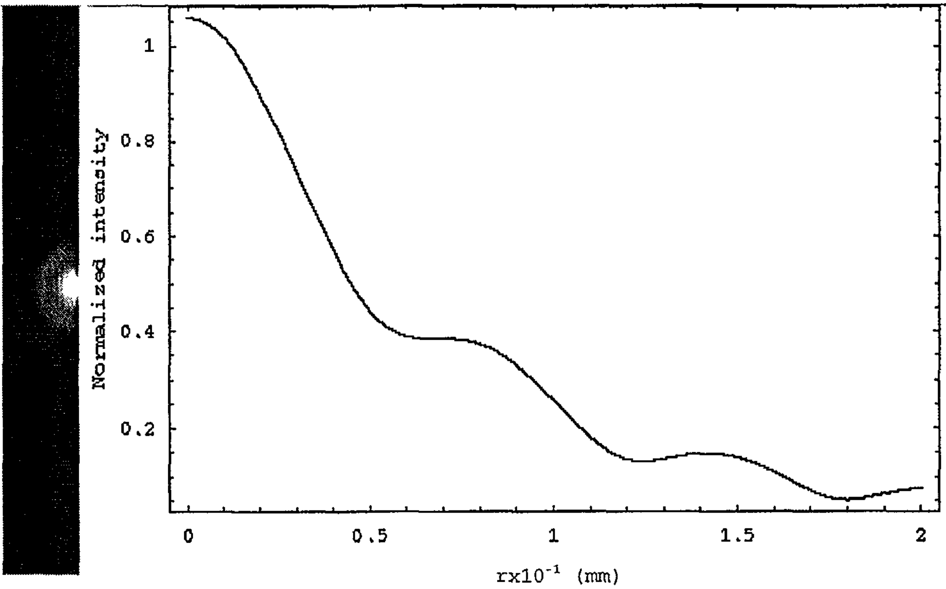
However, this precision can be improved by using detectors, whose dynamical range is greater than that of the eye. For example with a conventional CCD cameras, the distinction can be performed for N>0.2232 [16]. Nevertheless, the improvement requirements result from the trade off between costs and experimental specifications, because in some applications it is not necessary to impose strong restriction in precision.

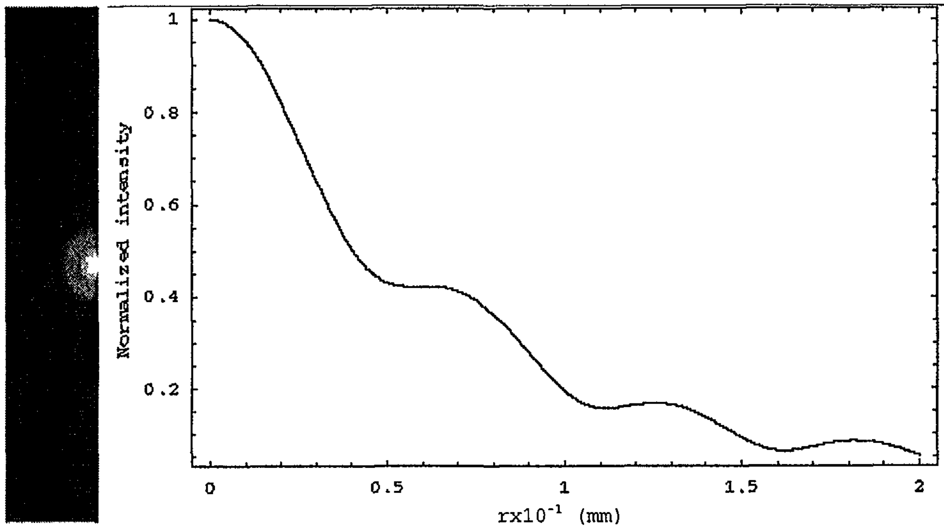
TABLE 1: Intensity profiles of the diffraction pattern of a plane wave (λ= 632.8nm) through a circular aperture of radius R=80 µm for different values of N, so that $\theta_A = 0.61(\lambda / R) = 0.0048251$ rad.

N	Normalized Intensity profile	Angular position of the first zero in radians θ_0	$\Delta\theta_{\%}$
0.1		0.004970	3.01
0.2232		0.004995	3.53

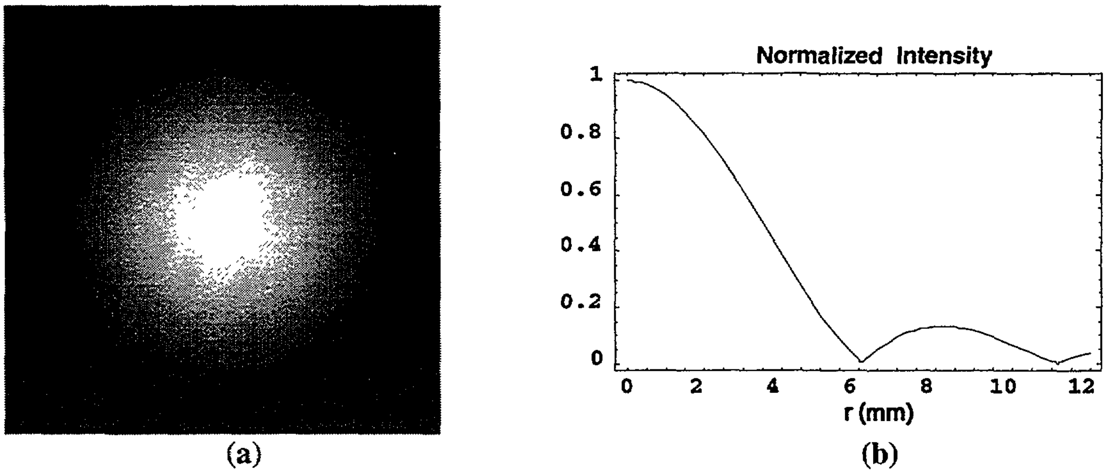
0.3		0.005022	4.07
0.5		0.005075	5.18

0.6		0.008602	78.27
0.7		0.008775	81.87

0.8		0.011604	140.49
0.9		0.011746	143.44

1.0		0.012656	162.3

From the above results, it is apparent that the Fraunhofer diffraction pattern will be observed on a plane located at an infinite distance from the aperture, if the aperture is illuminated by a plane wave, as expected (Figure 2). The incident plane wave can be produced by attaching the point source at an infinite distance from the aperture or at the frontal focal plane of a converging lens, with the aperture behind but near the lens.



(a) (b)

Fig. 2: (a) Intensity distribution and (b) its profile along a diameter of an ideal Fraunhofer diffraction pattern provided by a circular aperture, i.e. the Airy pattern.
Angular positions of the zeroes of the Airy pattern: 0.0244 rad., 0.04464 rad., 0.06476 rad respectively

A distinctive feature of Fraunhofer diffraction is the invariance of the angle that subtends the central maximum of the pattern from the centre of the diffracting aperture, i.e. the invariance of the ratio between the Airy radii and the distance between the aperture and the corresponding observation planes. The Fraunhofer diffraction pattern can be also observed at the rear focal plane of a converging lens, if the aperture is located al its frontal focal plane. Goodman [7] discusses the optical equivalence between the observation plane at infinity and the rear focal plane of the converging lens.

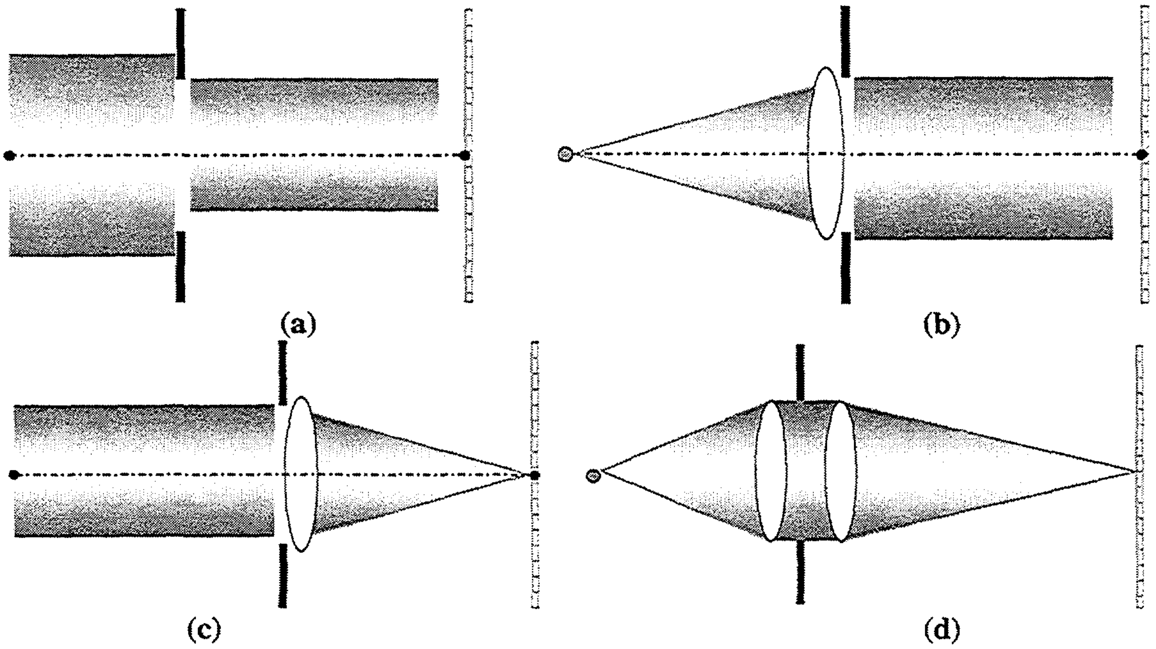


Fig. 3: Set-ups for ideal Fraunhofer diffraction

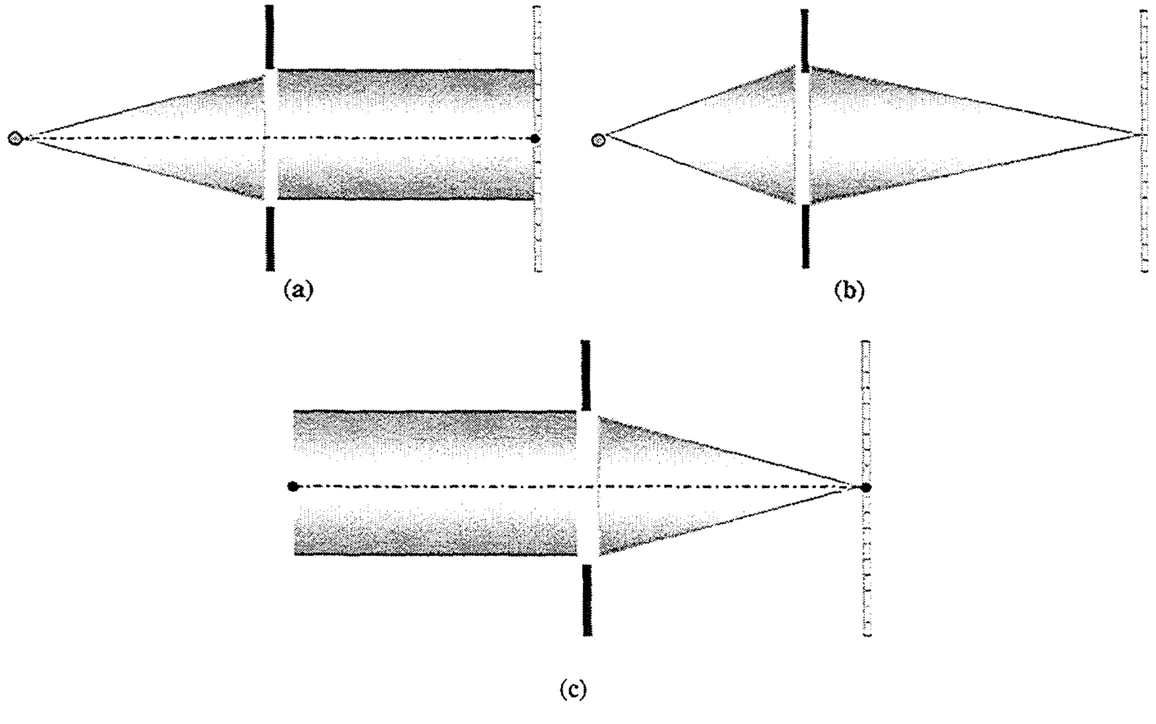


Fig. 4: Set-ups for both Fresnel and Fraunhofer diffraction. (a) Source at finite distance and observation plane at infinite distance from the aperture. (b) Both source and observation plane at finite distance from the aperture. (c) Source at infinite distance and observation plane at finite distance from the aperture.

3. DIFFRACTION CRITERIA DISCUSSED IN OPTICS TEXTBOOKS

In the following we will review the diffraction criteria discussed in most the optics textbooks. For comparison purposes, we classify the criteria into four types, refer all of them to the set-up in Fig.1 and express them in terms of the number of Fresnel zones *N*.

First type criteria [1-5]

They are based on the assumption that Fraunhofer diffraction only occurs for infinite distances from the aperture to the source and to the observation plane respectively (Fig. 2). The infinite distances can be achieve by properly placing converging lenses between the aperture and the source and/or between the aperture and the observation plane, as advertised in many textbooks. Otherwise, i.e. if any or both of these distances are finite, Fresnel diffraction is obtained (Fig. 3).

It is well known that all experimental set-ups, which accomplish the first part of this assumption, provide Fraunhofer diffraction. However, the second part of the assumption excludes the possibility of Fraunhofer diffraction for finite distances, which were yet theoretically predicted and experimentally proved as reported in Ref.14 by example.

Second type criteria [6,7].

Their basic assumption is that Fraunhofer diffraction occurs when the number of Fresnel zones within the aperture is:

- a small fraction of the first Fresnel zone, without specifying its numerical value, Rossi [6].
- $N << 0.16$ by illuminating the aperture only with plane waves, Goodman [7]. Similarly as Rossi, Goodman does not clarify how much smaller than 0.16 must be *N* for assuring the distinction between Fresnel and Fraunhofer diffraction. Furthermore, the condition on the illumination of the aperture is very restrictive. In fact, Fraunhofer diffraction under illumination with wave-fronts of different geometry has been reported.

Third type criteria [8,9].

They assume Fraunhofer diffraction if $N << 2$, without giving an specific value for *N*. In the case of Gori [9], an incident plane wave on the aperture is required.

Fourth type criteria (independents) [10-15].

Françon [10] agrees to the first type criteria except for the following situation: at a finite distance from the aperture to the observation plane, determined by attaching the observation plane at the focal plane of a converging lens, he assumes Fraunhofer diffraction even if the source is attached at a finite distance from the aperture. This situation is considered as Fresnel diffraction in the first type criteria.

Sears [11] points out the importance of the aperture dimensions and the distances from the aperture as basic parameters for diffraction criteria, but he does not propose a quantitative criterion. His analysis is restricted to a slit and an incident plane wave, as done by Goodman [7], Gori [9], Iizuka [13] and Hecht & Zajac [12].

So, he concludes that Fraunhofer diffraction occurs when the screen (observation plane) is sufficiently far away or the slit is sufficiently narrow, or if a lens is placed just beyond the slit; otherwise, Fresnel diffraction is achieved. Nevertheless, this qualitative conclusion is not useful for designing or analyzing experimental set-ups for practical applications, because it is not supported by quantitative conditions on the set-up geometry.

Hecht and Zajac [12] consider a small aperture illuminated by a plane wave from a point source located far away from the aperture. There is no explicit quantitative condition on the aperture size. The criterion is based on a visual comparison between the diffraction figure and the geometrical shape of the aperture, which is clearly subjective. Therefore, its application is not reliable because different observers can give contradictory results.

Iizuka [13] proposes Fraunhofer diffraction for $N<=0.5$ and Fresnel diffraction otherwise by illuminating the aperture with a plane wave. As discussed in sec.2, we agree with this criterion after removing the restriction on the illumination.

Born M. and E. Wolf [14] are very precise in establishing the phenomenological characteristics of both Fresnel and Fraunhofer diffraction included the virtual diffraction patterns, but they do not propose a quantitative criterion for distinguishing them.

Lipson and Lipson [15] assume the condition $N < 1$ for Fraunhofer diffraction and Fresnel diffraction otherwise, but under illumination with a plane wave.

Limitations of the Fresnel zones based criteria

Some of the above criteria are supported by conditions of the form $N \leq \alpha$ for Fraunhofer diffraction, with $\alpha > 0.5$, and Fresnel diffraction otherwise. A close approach of the actual diffraction pattern to the ideal Airy pattern is expected for $N \leq \alpha$. However, sometimes the position of the locus for the first minimum (or zero) value of intensity of the pattern differs from that predicted for Fraunhofer diffraction under such a criteria, as it is shown in the Table 1.

For example, let us consider diffraction through a circular aperture and the distinguishing criterion $N \leq 1$. The locus of the first minimum value of intensity coincides with third zero of the Fraunhofer pattern. In this case, the effective error is equal to 162.3%.

This kind of mistakes is not only of academic interest: they play a crucial role in practical applications based on diffraction, such as roughness estimation and diameter measurement of small particles.

4. CONCLUSION

Ideal Fraunhofer diffraction patterns are mathematically described ad the Fourier transform of the wavefront that emerges from the aperture. If the aperture is a circular hole that is illuminated by a uniform plane wave, the Fraunhofer diffraction pattern is called the *Airy pattern*.

These patterns can be characterized by the angle that subtends the first zero of their intensity distribution from the center of the aperture. So, it is possible to determine if any experimental diffraction pattern should be considered as a Fraunhofer one or not (i.e. a Fresnel diffraction pattern) by comparing the angle that subtends the first zero of its intensity distribution to the corresponding angle for the ideal Fraunhofer pattern.

Fresnel effects on diffraction can be represented by the number of Fresnel zones inscribed within the aperture, which is determined by the experimental setup. This is a global parameter that takes into account the shape and size of the aperture and the distance between the aperture and the observation plane. Thus, a specific number of Fresnel zones refers to different setups that produce the same diffraction pattern, except for a scaling factor.

Taking into account the performance of the eye, the condition $N < 0.5$ can be adopted as visual criterion for characterizing Fraunhofer diffraction. Thus, Fresnel diffraction should be considered for $N \geq 0.5$. However, the threshold of any distinguishing criterion depends on the dynamical range of the detector. By using conventional CCD cameras by example, this threshold can be fixed by $N = 0.2232$.

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