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**SELF-INTERACTING DARK MATTER  
AND HIGGS BOSONS IN THE  
 $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$  MODEL  
WITH RIGHT-HANDED NEUTRINOS**

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WITH RIGHT-HANDED NEUTRINOS**

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**Abstract**

We show that the  $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$  (3-3-1) model with right-handed neutrinos can provide candidates for self-interacting dark matter, namely they are the  $CP$ -even and odd Higgs bosons. These dark matters are stable without imposing of new symmetry and should be weak-interacting.

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It is an amazing fact that even as our understanding of cosmology progresses by leaps and bounds, we remain almost completely ignorant about the nature of most of the matter in the universe [1]. Cosmological models with a mixture of roughly 35% collisionless cold dark matter such as axions, WIMPs, or any other candidate interacting through the weak and gravitational forces only, and 65% vacuum energy or quintessence match observation of the cosmic microwave background and large scale structure on extra-galactic scales with remarkable accuracy [2, 3]. It is known that only a fraction of the dark matter can be made of ordinary baryons and its enormous amount has unknown, nonbaryonic origin [4].

Until a few years ago, the more satisfactory cosmological scenarios were those ones composed of ordinary matter, cold dark matter and a contribution associated with the cosmological constant. To be consistent with inflationary cosmology, the spectrum of density fluctuations would be nearly scale-invariant and adiabatic. However, in recent years it has been pointed out that the conventional models of collisionless cold dark matter lead to problems with regard to galactic structures. They were only able to fit the observations on large scales ( $\gg 1$  Mpc). Also,  $N$ -body simulations in these models result in a central singularity of the galactic halos [5] with a large number of sub-halos [6], which are in conflict with astronomical observations. A number of other inconsistencies are discussed in Refs. [7, 8]. Thus, the cold dark matter model is not able to explain observations on scales smaller than a few Mpc.

However, it has recently been shown that an elegant way to avoid these problems is to assume the so-called *self-interacting dark matter* [9]. One should notice that, in spite of all, self-interacting models lead to spherical halo centers in clusters, which is not in agreement with ellipsoidal centers indicated by strong gravitational lensing observations [10] and by Chandra observations [11]. However, self-interacting dark matter models are self-motivated as alternative models. It is a well-accepted fact that the plausible candidates for dark matter are elementary particles. The key property of these particles is that, they must have a large scattering cross-section and negligible annihilation or dissipation. The Spergel-Steinhard model has motivated many follow-up studies [4, 12, 13]. Several authors have proposed models in which a specific scalar singlet that satisfies the self-interacting dark matter properties is introduced in the standard model (SM) in an *ad hoc* way [4, 13].

The SM offers no options for dark matter. The first gauge model for SIDM were found by Fregolente and Tonasse [14] in the 3-3-1 model. It is to be noted that in the model considered in [14] to keep the Higgs sector with three triplets one has to propose an existence of *exotic leptons*. The 3-3-1 models were proposed with an independent motivation [15]. These models have the following intriguing features such as the models are anomaly free only if the number of families  $N$  is a multiple of three. If further one adds the condition of QCD asymptotic freedom, which is valid only if the number of families of quarks is to be less than five, it follows that  $N$  is equal to 3.

A subject that has not been given much attention by particle physicists in the past, could

prove to be a remarkable powerful and precise probe of the properties of dark matter.

The aim of this paper is to show that the 3-3-1 model with right-handed (RH) neutrinos [16] contains such self-interacting dark matter.

To frame the context, it is appropriate to recall briefly some relevant features of the 3-3-1 model with RH neutrinos [16]. In this model the leptons are in triplets, and the third member is a RH neutrino:

$$f_L^a = (\nu_L^a, e_L^a, (\nu_L^c)^a)^T \sim (1, 3, -1/3), e_R^a \sim (1, 1, -1). \quad (1)$$

The first two generations of quarks are in antitriplets while the third one is in a triplet:

$$Q_{iL} = (d_{iL}, -u_{iL}, D_{iL})^T \sim (3, \bar{3}, 0), \quad (2)$$

$$u_{iR} \sim (3, 1, 2/3), d_{iR} \sim (3, 1, -1/3), D_{iR} \sim (3, 1, -1/3), \quad i = 1, 2,$$

$$Q_{3L} = (u_{3L}, d_{3L}, T_L)^T \sim (3, 3, 1/3), \quad (3)$$

$$u_{3R} \sim (3, 1, 2/3), d_{3R} \sim (3, 1, -1/3), T_R \sim (3, 1, 2/3).$$

The charged gauge bosons are defined as

$$\begin{aligned} \sqrt{2} W_\mu^+ &= W_\mu^1 - iW_\mu^2, \sqrt{2} Y_\mu^- = W_\mu^6 - iW_\mu^7, \\ \sqrt{2} X_\mu^0 &= W_\mu^4 - iW_\mu^5. \end{aligned} \quad (4)$$

The *physical* neutral gauge bosons are again related to  $Z, Z'$  through the mixing angle  $\phi$ . The symmetry breaking can be achieved with just three  $SU(3)_L$  triplets

$$\begin{aligned} \chi &= (\chi^0, \chi^-, \chi'^0)^T \sim (1, 3, -1/3), \\ \rho &= (\rho^+, \rho^0, \rho'^+)^T \sim (1, 3, 2/3), \\ \eta &= (\eta^0, \eta^-, \eta'^0)^T \sim (1, 3, -1/3), \end{aligned} \quad (5)$$

The necessary VEVs are

$$\langle \chi \rangle = (0, 0, \omega/\sqrt{2})^T, \langle \rho \rangle = (0, u/\sqrt{2}, 0)^T, \langle \eta \rangle = (v/\sqrt{2}, 0, 0)^T. \quad (6)$$

After symmetry breaking the gauge bosons gain masses

$$m_W^2 = \frac{1}{4}g^2(u^2 + v^2), \quad M_Y^2 = \frac{1}{4}g^2(v^2 + \omega^2), \quad M_X^2 = \frac{1}{4}g^2(u^2 + \omega^2). \quad (7)$$

Eqn.(7) gives us a relation

$$v_W^2 = u^2 + v^2 = 246^2 \text{ GeV}^2. \quad (8)$$

In order to be consistent with the low energy phenomenology we have to assume that  $\langle \chi \rangle \gg \langle \rho \rangle, \langle \eta \rangle$  such that  $m_W \ll M_X, M_Y$ .

The symmetry-breaking hierarchy gives us splitting on the bilepton masses [17]

$$|M_X^2 - M_Y^2| \leq m_W^2. \quad (9)$$

Our aim in this paper is to show that the 3-3-1 model with RH neutrinos furnishes a good candidate for (self-interacting) dark matter. The main properties that a good dark matter candidate must satisfy are stability and neutrality. Therefore, we go to the scalar sector of the model, more specifically to the neutral scalars, and we examine whether any of them can be stable and in addition whether they can satisfy the self-interacting dark matter criterions [9]. In addition, one should notice that such dark matter particle must not overpopulate the Universe. On the other hand, since our dark matter particle is not imposed arbitrarily to solve this specific problem, we must check that the necessary values of the parameters do not spoil the other bounds of the model.

Under the assumption of the discrete symmetry  $\chi \rightarrow -\chi$ , the most general potential can then be written in the following form [18]

$$\begin{aligned}
V(\eta, \rho, \chi) &= \mu_1^2 \eta^+ \eta + \mu_2^2 \rho^+ \rho + \mu_3^2 \chi^+ \chi + \lambda_1 (\eta^+ \eta)^2 + \lambda_2 (\rho^+ \rho)^2 + \lambda_3 (\chi^+ \chi)^2 \\
&+ (\eta^+ \eta) [\lambda_4 (\rho^+ \rho) + \lambda_5 (\chi^+ \chi)] + \lambda_6 (\rho^+ \rho) (\chi^+ \chi) + \lambda_7 (\rho^+ \eta) (\eta^+ \rho) \\
&+ \lambda_8 (\chi^+ \eta) (\eta^+ \chi) + \lambda_9 (\rho^+ \chi) (\chi^+ \rho) + \lambda_{10} (\chi^+ \eta + \eta^+ \chi)^2.
\end{aligned} \tag{10}$$

We rewrite the expansion of the scalar fields which acquire a VEV:

$$\eta^o = \frac{1}{\sqrt{2}} (v + \xi_\eta + i\zeta_\eta); \quad \rho^o = \frac{1}{\sqrt{2}} (u + \xi_\rho + i\zeta_\rho); \quad \chi^o = \frac{1}{\sqrt{2}} (w + \xi_\chi + i\zeta_\chi). \tag{11}$$

For the prime neutral fields which do not have VEV, we get analogously:

$$\eta'^o = \frac{1}{\sqrt{2}} (\xi'_\eta + i\zeta'_\eta); \quad \chi'^o = \frac{1}{\sqrt{2}} (\xi'_\chi + i\zeta'_\chi). \tag{12}$$

Requiring that in the shifted potential  $V$ , the linear terms in fields must be absent, we get in the tree level approximation, the following constraint equations:

$$\begin{aligned}
\mu_1^2 + \lambda_1 v^2 + \frac{1}{2} \lambda_4 u^2 + \frac{1}{2} \lambda_5 w^2 &= 0, \\
\mu_2^2 + \lambda_2 u^2 + \frac{1}{2} \lambda_4 v^2 + \frac{1}{2} \lambda_6 w^2 &= 0, \\
\mu_3^2 + \lambda_3 w^2 + \frac{1}{2} \lambda_5 v^2 + \frac{1}{2} \lambda_6 u^2 &= 0.
\end{aligned} \tag{13}$$

Since dark matter has to be neutral, we then consider only the neutral Higgs sector. In the  $\xi_\eta, \xi_\rho, \xi_\chi, \xi'_\eta, \xi'_\chi$  basis the square mass matrix, after imposing of the constraints (13) has a quasi-diagonal form as follows:

$$M_H^2 = \begin{pmatrix} M_{3H}^2 & 0 \\ 0 & M_{2H}^2 \end{pmatrix}, \tag{14}$$

where

$$M_{3H}^2 = \frac{1}{2} \begin{pmatrix} 2\lambda_1 v^2 & \lambda_4 v u & \lambda_5 v w \\ \lambda_4 v u & 2\lambda_2 u^2 & \lambda_6 u w \\ \lambda_5 v w & \lambda_6 u w & 2\lambda_3 w^2 \end{pmatrix}, \tag{15}$$

and

$$M_{2H}^2 = \left( \frac{\lambda_8}{4} + \lambda_{10} \right) \begin{pmatrix} w^2 & vw \\ vw & v^2 \end{pmatrix}. \quad (16)$$

The above mass matrix shows that the prime fields mix themselves but do not mix with others. In the limit

$$\lambda_1 v, \lambda_2 u, \lambda_4 u \ll \lambda_5 w, \lambda_6 w, \quad (17)$$

we obtained physical eigenstates  $H_1(x)$  and  $\sigma(x)$

$$\begin{pmatrix} H_1(x) \\ \sigma(x) \end{pmatrix} = \frac{1}{(\lambda_5^2 v^2 + \lambda_6^2 u^2)^{1/2}} \begin{pmatrix} \lambda_6 u & -\lambda_5 v \\ \lambda_5 v & \lambda_6 u \end{pmatrix} \begin{pmatrix} \xi_\eta \\ \xi_\rho \end{pmatrix}, \quad (18)$$

with masses [18]

$$m_{H_1}^2 \approx \frac{v^2}{4\lambda_6} (2\lambda_1 \lambda_6 - \lambda_4 \lambda_5) \approx \frac{u^2}{4\lambda_5} (2\lambda_2 \lambda_5 - \lambda_4 \lambda_6), \quad (19)$$

$$m_\sigma^2 \approx \frac{1}{2} \lambda_1 v^2 + \frac{\lambda_4 \lambda_6 u^2}{4\lambda_5} \approx \frac{1}{2} \lambda_2 u^2 + \frac{\lambda_4 \lambda_5 v^2}{4\lambda_6}. \quad (20)$$

Eqs. (19) and (20) also give us relations among coupling constants and VEVs. Another massive physical state  $H_3$  with mass:

$$m_{H_3}^2 \approx -\lambda_3 w^2. \quad (21)$$

The scalar  $\sigma(x)$  is the one that we can identify with the SM Higgs boson [18].

In the approximation  $w \gg v$ , mass matrix  $M_{2H}^2$  gives us one Goldstone  $\xi'_\chi$  and one physical massive field  $\xi'_\eta$  with mass

$$m_{\xi'_\eta}^2 = - \left( \frac{\lambda_8}{4} + \lambda_{10} \right) w^2. \quad (22)$$

In the pseudoscalar sector, we have three Goldstone bosons which can be identified as follows:  $G_2 \equiv \zeta_\eta$ ,  $G_3 \equiv \zeta_\rho$ ,  $G_4 \equiv \zeta_\chi$  and in the  $\zeta_\eta^{i\prime o}, \zeta_\chi^{i\prime o}$  basis

$$M_{2A}^2 = \left( \frac{\lambda_8}{4} + \lambda_{10} \right) \begin{pmatrix} w^2 & vw \\ vw & v^2 \end{pmatrix}. \quad (23)$$

We easily get one Goldstone  $G'_5$  and one massive pseudoscalar boson  $\zeta'_\eta$  with mass

$$m_{\zeta'_\eta}^2 = - \left( \frac{\lambda_8}{4} + \lambda_{10} \right) w^2. \quad (24)$$

It is to be emphasized that, both  $\xi'_\eta$  and  $\zeta'_\eta$  are in a singlet of the  $SU(2)$ . Therefore they do not interact with the SM gauge bosons  $W^\pm, Z^0$  and  $\gamma$ . Unlike the 3-3-1 model considered in [14], here we have two fields which can be considered as dark matter.

To get the interaction of dark matter to the SM Higgs boson, we consider the following relevant parts

$$\begin{aligned}
L_{int}(\sigma, \zeta_\eta) &= \frac{1}{4}\lambda_1 \left[ v^2 + 2v\xi_\eta + \xi_\eta^2 + \zeta_\eta^2 + \xi_\eta'^2 + \zeta_\eta'^2 + 2\eta^+\eta^- \right]^2 \\
&+ \frac{1}{4}\lambda_4 \left[ v^2 + 2v\xi_\eta + \xi_\eta^2 + \zeta_\eta^2 + \xi_\eta'^2 + \zeta_\eta'^2 + 2\eta^+\eta^- \right] \\
&\times \left[ u^2 + 2u\xi_\rho + \xi_\rho^2 + \zeta_\rho^2 + 2\rho^+\rho^- + 2\rho'^-\rho'^+ \right]
\end{aligned} \tag{25}$$

Substituting (18) we get couplings of SIDM with the SM Higgs boson  $\sigma$

$$\begin{aligned}
L(\sigma, \zeta_\eta) &= \left[ \frac{\sigma(x)}{\sqrt{\lambda_5^2 v^2 + \lambda_6^2 u^2}} \left( \lambda_1 \lambda_5 v^2 + \frac{\lambda_4 \lambda_6}{2} u^2 \right) + \frac{H_1(x)\sigma(x)}{(\lambda_5^2 v^2 + \lambda_6^2 u^2)} \left( \lambda_1 - \frac{\lambda_4}{2} \right) \lambda_5 \lambda_6 u v \right. \\
&+ \left. \frac{\sigma^2(x)}{2(\lambda_5^2 v^2 + \lambda_6^2 u^2)} \left( \lambda_5^2 v^2 + \frac{\lambda_6^2}{2} u^2 \right) \right] (\xi_\eta'^2 + \zeta_\eta'^2).
\end{aligned} \tag{26}$$

From Yukawa couplings, we see that our candidates do not interact with ordinary leptons and quarks [19].

$$\begin{aligned}
\mathcal{L}_{Yuk}^\eta &= \lambda_{3a} \bar{Q}_{3L} u_{aR} \eta + \lambda_{4ia} \bar{Q}_{iL} d_{aR} \eta^* + \text{h.c.} \\
&= \lambda_{3a} (\bar{u}_{3L} \eta^0 + \bar{d}_{3L} \eta^- + \bar{T}_L \eta^0) u_{aR} + \lambda_{4ia} (\bar{d}_{iL} \eta^{0*} - \bar{u}_{iL} \eta^+ + \bar{d}'_{iL} \eta^{0*}) d_{aR} + \text{h.c.}
\end{aligned}$$

We see that the candidates for dark matter in this model have no couplings with all the SM particles except for the Higgs boson.

For stability of DM, we have to put a mass of the SM Higgs boson which is the twice bigger mass of the candidate

$$\frac{1}{2}\lambda_1 v^2 + \frac{\lambda_4 \lambda_6 u^2}{4\lambda_5} \approx \frac{1}{2}\lambda_2 u^2 + \frac{\lambda_4 \lambda_5 v^2}{4\lambda_6} \geq - \left( \frac{\lambda_8}{4} + \lambda_{10} \right) w^2. \tag{27}$$

To avoid the interaction of DM with Goldstone boson, we have

$$\lambda_1 = \frac{\lambda_4}{2} \tag{28}$$

The *wrong* muon decay ( $\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu$ ) gives a lower limit for singly charged bilepton  $M_Y \sim 230$  GeV

Combining Eqns. (7, 8) with (9) we obtain the following relation:  $u \sim v \approx 100 - 200$  GeV and  $w \approx (500 - 1000)$  GeV.

The cross section for  $hh \rightarrow hh$  with quarc interaction is  $\sigma = \lambda_1^2 / 64\pi m_h^2$ . The requirement on the mass  $m_h$  and self-interaction scattering cross section  $\sigma$  of the SIDM particles is that [9, 20]

$$2.05 \times 10^3 \text{ GeV}^{-1} \leq -\frac{\lambda_1^2}{\pi(\lambda_8 + 4\lambda_{10})^{3/2} w^3} \leq 2.57 \times 10^4 \text{ GeV}^{-1} \tag{29}$$

Taking  $w = 700$  GeV we get  $7 \times 10^9 \text{ GeV}^2 \leq -\frac{\lambda_1^2}{\pi(\lambda_8 + 4\lambda_{10})^{3/2}} \leq 8.8 \times 10^{12} \text{ GeV}^2$

This means that couplings constants of DM condidate  $\lambda_1 \sim \lambda_4$  are much small than that of non-dark matter ones.

In conclusion, we have shown in this paper that the 3-3-1 model with RH neutrinos provides two Higgs bosons: one is scalar or  $CP$ -even and another is pseudoscalar or  $CP$ -odd particle having properties of candidates for dark matter. A difference with the previous candidate which was introduced by hand, our self-interacting dark matter arises without imposing new properties to keep it stabilized. Recently, astronomical observations suggest that 70% of the total energy of the universe can be associated to the cosmological constant [21]. Thus, the contribution of an exotic particle to dark matter would be about 30%. Estimation has shown that SIDM should be weak.

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