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QUASI-RAYLEIGH WAVES IN TRANSVERSELY ISOTROPIC HALF-SPACE WITH INCLINED AXIS OF SYMMETRY

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Abstract

A method for determination of characteristics of quasi-Rayleigh (qR) wave in a transversely isotropic homogeneous half-space with inclined axis of symmetry is outlined. The solution is obtained as a superposition of qP, qSV and qSH waves, and surface wave velocity is determined from the boundary conditions at the free surface and at infinity, as in the case of Rayleigh wave in isotropic half-space. Though the theory is simple enough, a numerical procedure for the calculation of surface wave velocity presents some difficulties. The difficulty is conditioned by necessity to calculate complex roots of a non-linear equation, which in turn contains functions determined as roots of non-linear equations with complex coefficients. Numerical analysis shows that roots of the equation corresponding to the boundary conditions do not exist in the whole domain of azimuths and inclinations of the symmetry axis. The domain of existence of qR wave depends on the ratio of the elastic parameters: for some strongly anisotropic models the wave cannot exist at all. For some angles of inclination qR wave velocities deviate from those calculated on the basis of the perturbation method valid for weak anisotropy, though they have the same tendency of variation with azimuth. The phase of qR wave varies with depth unlike Rayleigh wave in isotropic half-space. Unlike Rayleigh wave in isotropic half-space, qR wave has three components – vertical, radial and transverse. Particle motion in horizontal plane is elliptic. Direction of the major axis of the ellipsis coincide with the direction of propagation only in azimuths 0°(180°) and 90°(270°).

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1.Introduction

Different seismological observations indicate that most parts of the Earth are anisotropic. The effect of anisotropy on seismic surface waves is manifested in two phenomena: (1) Rayleigh-Love wave discrepancy – impossibility to explain Rayleigh and Love wave dispersion by the same isotropic model (*Anderson, 1961; Mc.Evilly, 1964*); (2) azimutal anisotropy derived from azimuthal variation of phase and group velocities (*Forsyth, 1975; Tanimoto and Anderson, 1985; Suetsugu and Nakanishi, 1987, Nishimura and Forsyth, 1989, etc.*) as well as from anomalous polarization of surface waves (*Yu and Park, 1994; Laske and Masters, 1998; Pettersen and Maupin, 2002*). The first kind of the phenomena is explained in the framework of a transversely isotropic model with vertical axis of symmetry, usually referred to as radial anisotropy. The second one (azimuthal anisotropy) is considered as a result of the most general case of anisotropy, but due to mathematical and computational difficulties it is treated approximately, only for weak anisotropy (*Smith and Dahlen, 1973, 1975; Maupin, 1989*) by the perturbation technique. This technique allows to express a variation of phase velocity with azimuth ϕ as a composition of trigonometric functions of 2ϕ and 4ϕ. Such representation of phase velocity is commonly used in different seismological studies (*Tanimoto and Anderson, 1985; Montagner and Nataf, 1986; Montagner and Tanimoto, 1990; Park, 1996*) *Crampin (1970)* extended the Tomson-Haskell matrix method for a multilayered half-space with anisotropic layers of general type. However the procedure for construction of the dispersion equation was described in a general form, and due to computational difficulties it was impossible to conclude on a behavior of the surface wave field in any specific structure. *Farnell (1970)* has presented numerical solutions for a homogeneous anisotropic half-space with one of crystal axes directed perpendicular to the free surface. The existence of Rayleigh wave in such a model with velocity less than minimum of three body wave velocities was proved by *Lothe and Barnett (1976)*. However, these results cannot be applied to a case, when one of the axes (symmetry axis for transversely isotropic medium) is not perpendicular to the free surface.

In the present paper we use the approach similar to that proposed by Crampin for a simple type of anisotropy – transversely isotropic medium with inclined axis of symmetry and for the simplest model of a homogeneous half-space. Analytical and numerical analysis allowed us to display some properties of the wave field, and to verify the approximate solution for weak anisotropy.

2. Quasi-Rayleigh wave in homogeneous anisotropic half-space

2.1 Formulation of the problem

We consider a homogeneous half-space $z>0$. The material of the half-space is transversely isotropic, but the symmetry axis z' does not coincide with vertical axis z: the angle between z and z' is θ (Fig.1a). Hereafter we use the conventional notation for the elastic parameters A,C,F,N,L. We shall consider two Cartesian coordinate systems $-xyz$ related to the half-space, and $x'y'z'$ related to the symmetry axis. The axes x and x' are placed in the zz' plane, the axes y and y' coincide and are orthogonal to the plane zz'.

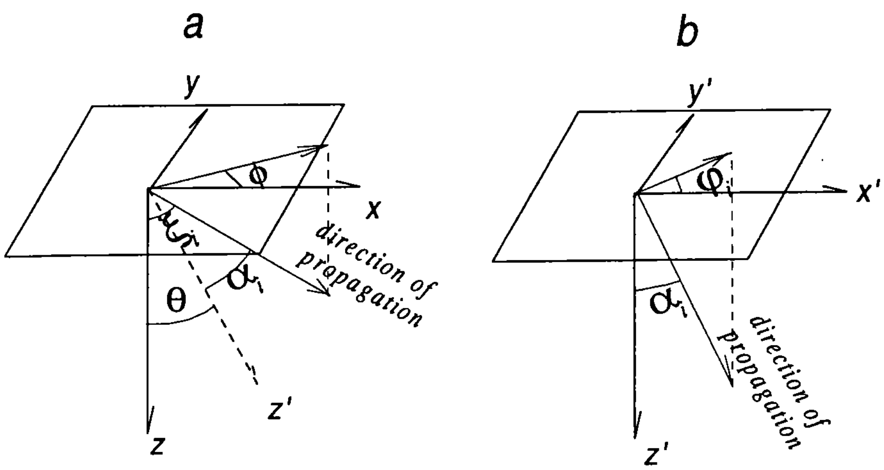

Fig.1. Geometry of model.

The stress-strain relation in the system xyz may be written in the form

$$\tau_i = C_{ij}\varepsilon_j \qquad (1)$$

where the indices i and j vary from 1 to 6 in concordance with the following rule: 1 →(11), 2 →(22), 3 →(33), 4→(23), 5→(13), 6→(12) , i.e. $\varepsilon_1 = \varepsilon_{11}, \varepsilon_2 = \varepsilon_{22}, ..., \varepsilon_6 = \varepsilon_{12}$.
Expressions for elements of the matrix **C** are given in the Appendix.

As in an isotropic case, we shall look for the solution of the elastodynamic equation as a superposition of plane waves qP,qSV and qSH propagating in a horizontal direction with the same velocity. The velocity is determined from the boundary condition at the free surface z=0.

As in an isotropic case, the field of surface wave propagating along xy plane at the azimuth ϕ (Fig.1a) may be represented as a superposition of inhomogeneous plane waves qP, qSH and qSV:

$$\mathbf{U} = \sum_{i=1}^{3} \mathbf{U}_i \exp\left(i\omega \left[t - \frac{(x\cos\phi + y\sin\phi)\sin\xi_i}{c_i(\xi_i)} - \frac{z\cos\xi_i}{c_i(\xi_i)} \right] \right) \qquad (2)$$

where we stand i=1 for qP, i=2 for qSV and i=3 for qSH. For inhomogeneous wave $\cos\xi_i$ (and ξ_i) should be complex-valued. Unlike the isotropic case the velocities c_i now depend on the angles ξ_i. These angles are related by

$$\frac{1}{V} = \frac{\sin\xi_1}{c_1(\xi_1)} = \frac{\sin\xi_2}{c_2(\xi_2)} = \frac{\sin\xi_3}{c_3(\xi_3)}$$

where V is the horizontal velocity, which should be obtained from the boundary conditions. For a better understanding the angle ξ_i is shown in Fig.1a for homogeneous wave.

2.2. Plane waves in the two coordinate systems

To determine polarization of each wave in (2) it is convenient to write the equation of motion in the coordinates x′y′z′. A plane wave propagating at the angle α_i respectively z′-axis and in the azimuth φ_i respectively to x′-axis (Fig.1b) may be written as

$$(U_i,V_i,W_i)\exp\left[i\omega\left(t-\frac{x'\sin\alpha_i\cos\varphi_i+y'\sin\alpha_i\sin\varphi_i+z'\cos\alpha_i}{c_i}\right)\right] \tag{3}$$

In a transversely isotropic medium, it is possible to find analytically the velocities and polarization of the three body waves propagating in any direction. The velocities are:

$$c_i^2=\frac{A\sin^2\alpha_i+C\cos^2\alpha_i+L\pm\sqrt{\left((A-L)\sin^2\alpha_i-(C-L)\cos^2\alpha_i\right)^2+4(F+L)^2\sin^2\alpha_i\cos^2\alpha_i}}{2\rho}\quad (i=1,2) \tag{4a}$$

$$c_3^2=\frac{N\sin^2\alpha+L\cos^2\alpha}{\rho} \tag{4b}$$

Vertical (Z) and radial (R) components of the displacement vector in qP (i=1) and qSV (i=2) waves are expressed as

$$R_i=a_i\frac{F+L}{c_i^2}\sin\alpha_i\cos\alpha_i$$
$$Z_i=a_i\left(\rho-\frac{A\sin^2\alpha_i+L\cos^2\alpha_i}{c_i^2}\right) \tag{5}$$

where a_i is a scaling factor. The wave qSH has only transverse component $T_3=a_3$. Thus x′,y′,z′ – components of the waves can be written as

$$\begin{pmatrix}U_{x'}\\U_{y'}\\U_{z'}\end{pmatrix}_i=\mathbf{G}_i a_i\quad\text{where }\mathbf{G}_i=\begin{pmatrix}\frac{F+L}{c_i^2}\sin\alpha_i\cos\alpha_i\cos\varphi_i\\ \frac{F+L}{c_i^2}\sin\alpha_i\cos\alpha_i\sin\varphi_i\\ \left(\rho-\frac{A\sin^2\alpha_i+L\cos^2\alpha_i}{c_i^2}\right)\end{pmatrix}(i=1,2),\quad \mathbf{G}_3=\begin{pmatrix}\sin\varphi_3\\-\cos\varphi_3\\0\end{pmatrix}$$

Then the components of the waves in the coordinate system x,y,z are obtained by the use of the matrix

$$\mathbf{Q}=\begin{pmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{pmatrix},$$

i.e. $\begin{pmatrix}U_x\\U_y\\U_z\end{pmatrix}_i=\mathbf{U}_i=\mathbf{QG}_i a_i$.

The direction of propagation in the x,y,z coordinates is determined from (2), so that

$$\cos\alpha_i=\sin\theta\sin\xi_i\cos\phi+\cos\xi_i\cos\theta \tag{6}$$

To express the angle φ_i in terms of the angles ξ_i,ϕ and θ it is sufficient to compare the unit vectors in the direction of propagation in both coordinate systems:

$$\mathbf{k}_i=\sin\alpha_i\cos\varphi_i\mathbf{e}_{x'}+\sin\alpha_i\sin\varphi_i\mathbf{e}_{y'}+\cos\alpha_i\mathbf{e}_{z'}=$$
$$=\mathbf{e}_x(\sin\alpha_i\cos\varphi_i\cos\theta+\cos\alpha_i\sin\theta)+\mathbf{e}_y\sin\alpha_i\sin\varphi_i+\mathbf{e}_z(\cos\alpha_i\cos\theta-\sin\alpha_i\cos\varphi_i\sin\theta) \tag{7}$$

Comparing (7) with (2) we obtain

$$\sin\varphi_i = \frac{\sin\xi_i \sin\phi}{\sin\alpha_i}$$

$$\cos\varphi_i = \frac{\cos\theta\sin\xi_i\cos\phi - \cos\xi_i\sin\theta}{\sin\alpha_i}$$

2.3. Equation for quasi-Rayleigh wave velocity

The quasi-Rayleigh wave (*qR*) is formed by a superposition of qP,qSV and qSH waves propagating with the same horizontal velocity. The velocity is determined from the boundary condition at the free surface τ_z=0 and from vanishing the displacement at $z\to\infty$. The latter condition means that the imaginary part of $\frac{\cos\xi_i}{c_i(\xi_i)}$ should be negative. Another constraint to ξ_i follows from the condition that the horizontal velocity *V* should be real: otherwise the *qR* wave would decay or grow infinitely along the direction of propagation. An obvious case, when these conditions are satisfied, is $\phi=\pi/2$, $\sin\xi_i > 1$ is real, $\cos\xi_i$ is imaginary, and radicand in (4a) is positive. In all the other cases $\sin\xi_i$, as well as $\cos\xi_i$ and $c_i(\xi_i)$, are complex which is clear from (6): $\cos\alpha_i$, and consequently c_i are complex. Thus, theoretically *V* should also be complex. But notwithstanding that they are determined as zeros of a complex function derived from the boundary conditions, they turn out to be real (see section 3.1) . So the above constraint to *V* is satisfied automatically.

According to (1), taking into account for the expressions for strains, we can write the stress components for each wave at the surface z=0 as follows:

$$\boldsymbol{\tau}_{zi} = -i\omega\mathbf{S}_i\mathbf{U}_i \quad (i=1,2,3).$$

Let us define

$$p = \frac{\sin\xi_i}{c_i(\xi_i)} \qquad \gamma_i = \frac{\cos\xi_i}{c_i(\xi_i)}$$

Here *p* has no index *i*, because it is the same for all the waves. Then

$$\mathbf{S}_i = \begin{pmatrix} C_{15}p\cos\phi + \gamma_i C_{25} & C_{25}p\sin\phi & C_{35}\gamma_i + C_{25}p\cos\phi \\ C_{44}p\sin\phi & C_{44}p\cos\phi + C_{66}\gamma_i & C_{66}p\sin\phi \\ C_{12}p\cos\phi + \gamma_i C_{35} & C_{33}p\sin\phi & C_{23}\gamma_i + C_{35}p\cos\phi \end{pmatrix}$$

and the boundary condition τ_z=0 can be finally written as

$$\sum_i \mathbf{S}_i\mathbf{Q}\mathbf{G}_i a_i = 0$$

Otherwise this equation can be written in the form

$$\mathbf{Za} = 0$$

where the matrix **Z** is formed by vector columns $\mathbf{S}_i\mathbf{Q}\mathbf{G}_i$, (*i*=1,2,3), and $\mathbf{a}^T = (a_1, a_2, a_3)$. Thus the equation for determining the unknown horizontal slowness *p* is

$$\det(\mathbf{Z}(p))=0. \qquad (8)$$

2.4 Algorithm for calculating velocity and polarization

Calculation of surface wave velocity as a function of θ and ϕ involves determination of roots of complex functions. This procedure consists of two stages. At first for a given p (in general complex), complex values ξ_i $(i = 1,2,3)$ are determined from the equations

$$\frac{\sin \xi_i}{c_i(\xi_i)} = p. \tag{9}$$

Then a value of p satisfying the equation

$$\det(\mathbf{Z}(p, \xi_1(p), \xi_2(p), \xi_3(p)) = 0 \tag{10}$$

is determined.

As soon as the slowness p is found, it is easy to calculate the components of the vector **U** up to an arbitrary multiplier.

However, Eq.(9) for a given p may have no roots for qP or qSV waves for some values of θ, ϕ. Due to this, the *qR* wave cannot exist for all values of θ and ϕ, even in the case of weak anisotropy. For strong anisotropy a domain of existence of qR wave is rather narrow, even the wave cannot exist in the whole range of θ and ϕ. This can be easily explained for ϕ=90°. In this case $\cos\alpha = \cos\theta \cos\xi$ (formula (6)), and the equations (9) can be reduced to a simple quadratic equation.

Equation (9) is equivalent to

$$\frac{c_i^2(\alpha(\xi_i))}{\sin^2 \xi_i} = p^{-2} \tag{11}$$

For qP and qSV waves (11) yields:

$A\sin^2\alpha + C\cos^2\alpha + L \pm \sqrt{((A-L)\sin^2\alpha - (C-L)\cos^2\alpha)^2 + 4(F+L)^2 \sin^2\alpha \cos^2\alpha} = 2\rho p^{-2} \sin^2 \xi$ or,

replacing cosα by cosξcosθ, and denoting $x = \sin^2 \xi$, we obtain the following equation for x:

$$\begin{aligned} &\pm\sqrt{((A+C-2L)(x\cos^2\theta + \sin^2\theta) - (C-L))^2 + 4(F+L)^2 x\cos^2\theta(x\cos^2\theta + \sin^2\theta)} = \\ &= 2\rho p^{-2} x - (A-C)(x\cos^2\theta + \sin^2\theta) - (C+L) \end{aligned} \tag{12}$$

"+' corresponds to qP wave, "-" to qSV wave.

Squaring both sides we obtain a quadratic equation, however its roots may not coincide with the roots of (11). For example, this occurs, if the roots are such that the right-hand side of (12) turns out to be negative for qP wave, or positive for qSV wave. This means that equation (11) has no solution for the corresponding wave. For instance, such a case may arise for qP wave in some range of θ if A>C. It is evident that a similar situation can also arise for azimuths ϕ different from 90°. Thus in some ranges of θ and ϕ qR wave does not exist.

Another reason for non-existence of qR wave is the vanishing of an imaginary part of $\cos\xi_i$. This can be in the case when $\sin\xi_i$ is real and less than unit. Such a case may arise for qSH wave if N<L, or for qSV wave if L<N.

Calculations of the qR wave velocity for different models show that when p fitting equation (8) can be found, it is always real. This proves the existence of qR waves propagating without attenuation along the surface. However, unlike the isotropic case, the terms $\frac{\cos\xi_i}{c_i(\xi_i)}$ are not purely imaginary indicating that the wave not only decays but also has a phase shift along a vertical direction. This was also mentioned by *Crampin (1970)*.

Since the qR wave in the case $\theta\neq 0$ is formed by superposition of three waves – qP,qSV and qSH, its polarization in the horizontal plane is not linear: it has both radial and transverse components (*Crampin and Taylor, 1971; Crampin, 1975*).

3. Numerical modeling

Variation of qR wave velocity and polarization with the azimuth ϕ and the angle θ are presented for two models 1 and 2 with elastic parameters shown in Table 1. For these models, which may be considered as "weakly anisotropic", a domain of existence of qR wave is rather wide. Also a domain of existence of qR wave in "strongly anisotropic" models 3 and 4 is considered.

Table 1
Parameters of the models

	A, GPa	C, GPa	F, GPa	L, GPa	N, GPa	ρ, g/cm³
Model 1	**60**	**70**	**15**	**25**	**20**	**3**
Model 2	**65**	**60**	**25**	**20**	**25**	**3**
Model 3	**75**	**60**	**20**	**15**	**25**	**3**
Model 4	**75**	**60**	**20**	**25**	**15**	**3**

3.1 Domain of existence of qR wave

Fig.2 shows domains of θ and ϕ, where qR wave can exist, for the models 1,2,3. In the model 4 qR wave cannot exist for any θ and ϕ due to the reasons mentioned above. In the model 1 N<L, consequently $c_{qSH} < c_{qSV}$, and this results in occurrence of a region, where $\sin\xi_{qSH} < 1$. Since in this model A<C, the situation when equation (9) has no root for qP wave, does not arise. Model 2 has the opposite correlation between A and C, and in this case the roots of (9) for qP wave do not exist for all azimuths when θ<28°, and also for large θ and azimuths near 0°(180°). Domain of existence for model 3 is extremely small: non-existence of qR wave in a wide range of θ and ϕ is conditioned by the absence of the roots of (9) for qP and/or qSV wave, and/or by realization of the condition $\sin\xi_{qSV} < 1$. Model 4 differs from model 3 only by the interchange of N and L, and this turns out to be sufficient for non-existence of qR wave for any values of θ and ϕ.

3.2. Velocity

Velocity variations with azimuth in the models 1 and 2 for different values of θ are shown in Fig.3. The curves are displayed for the parameters corresponding to the domain of existence of qR wave (see Fig.2). It is interesting to compare the velocity variations with an approximate solution calculated on the basis of the perturbation technique valid for weak anisotropy (*Smith and Dahlen, 1973, 1975*). It is evident that we cannot expect a good coincidence of exact and approximate solutions, because in particular, the exact solution exists in a limited range of θ and ϕ, while an approximate solution can be calculated for any values of these parameters.

Variations of qR wave velocity with azimuth in models 1 and 2 calculated by the exact and approximate methods are compared in Fig.4. A tendency of velocity variation is similar for the exact and approximate solutions, but the values are

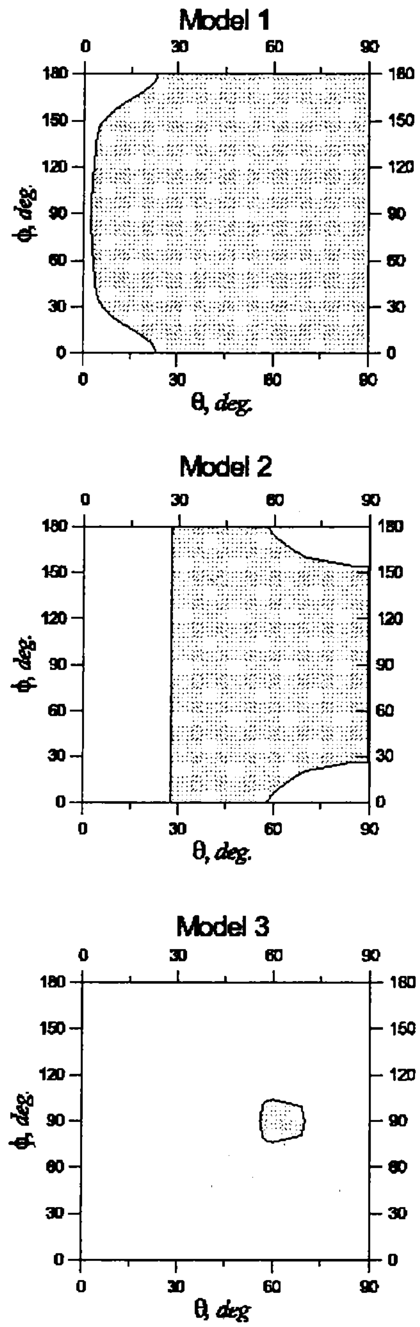

Fig.2. Domains of existence of qR waves in the models 1-3 shown by shaded areas.

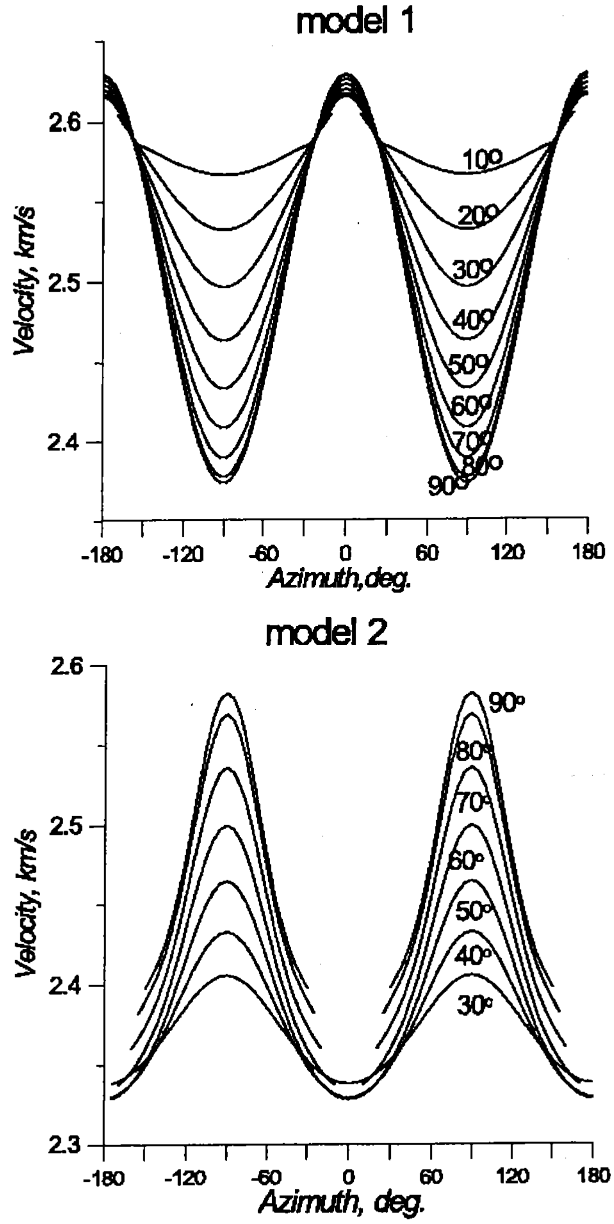

Fig.3. Velocity variation with azimuth for different inclination θ of the symmetry axis in the models 1 and 2. Values of θ are indicated at the curves.

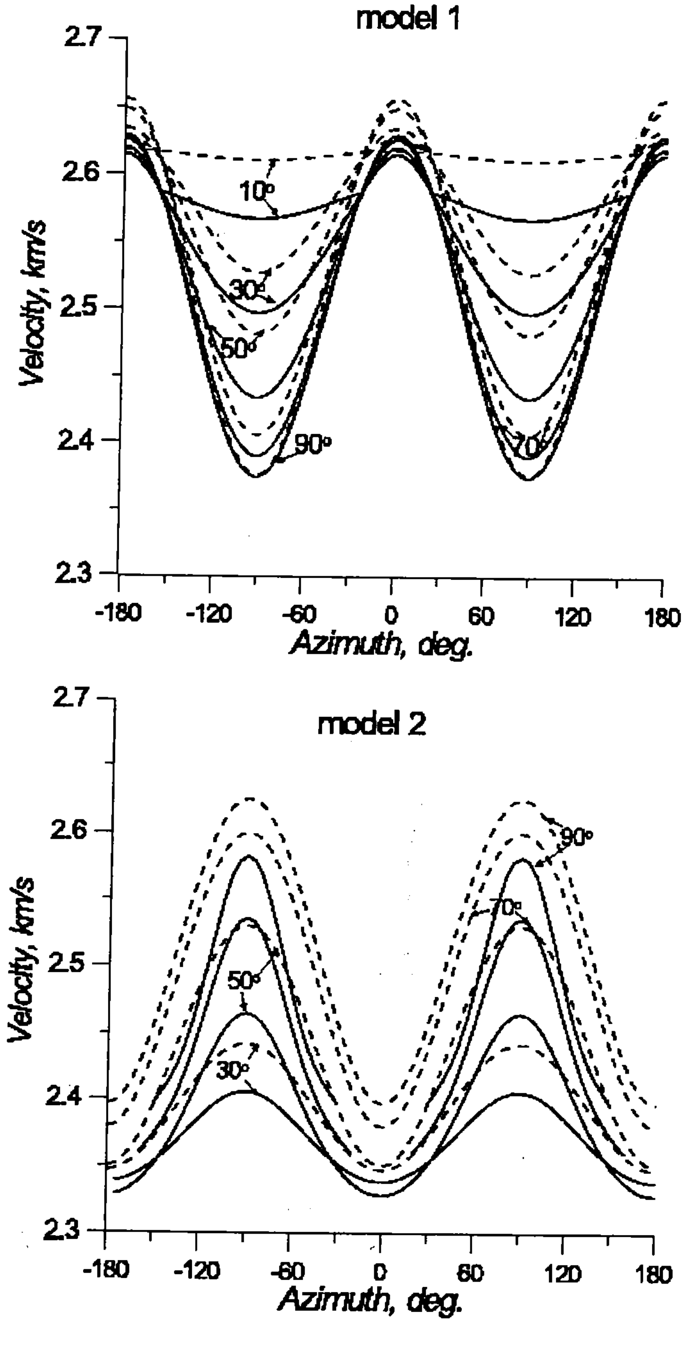

Fig.4. Velocity variation with azimuth calculated by the exact method (bold lines) and by the perturbation method (dashed lines).

different. The biggest difference for model 1 is observed for small values of θ, whereas for $\theta = 90°$ the discrepancy is negligible. For model 2 the approximate values are higher than the exact ones for all values of θ (for this model we cannot compare velocities for $\theta = 10°$ because for such a value qR wave does not exist, see Fig.2).

One may suppose that the discrepancy between the exact and the approximate solutions follows from that anisotropy is not sufficiently weak in the models considered above. However, a similar discrepancy is observed even in the models with much weaker anisotropy for the angles $\theta \sim 20$-$60°$ and for the azimuths around 90°, while for $\theta \sim 0$ and 90° the curves are in good agreement. A possible explanation of this phenomenon may be that at such angles θ and ϕ there is a strong effect of qSH wave, which is not taken into account in the approximate solution obtained by the ordinary perturbation method. In derivation of the approximate solution it is assumed that the starting model is isotropic, and Rayleigh wave is formed by superposition of only P and SV waves. As we shall see in the next section, polarization of qR wave in horizontal plane is elliptic which indicates a contribution of qSH wave.

3.3. Polarizarion of qR waves

Since qR wave is a superposition of qP, qSV and qSH waves, it has all three components – along x, y and z. From x and y components we can calculate particle motion in qR wave in horizontal plane. The particle motion for $\theta = 45°$ for models 1 and 2 is shown in Fig.5. This angle of inclination of the symmetry axis was chosen because in this case qR wave exists for all azimuths in both models (see Fig.2). It is seen that for all azimuths except $\phi = 0°$ and 180° the particle motion in horizontal plane is elliptic, unlike a transversely isotropic case with vertical symmetry axis, where it is linear. Arrows indicate the direction of particle motion. The particle motion in the azimuth range 180° – 360° is opposite: if in the range 0°-180° it is clockwise, then in the range 180° – 360° it is counterclockwise, and vice versa. The major axes of the ellipses do not coincide with the direction of propagation. So if the 'radial' component in qR wave is defined as directed along the major axis, azimuthal anomalies should be observed in the azimuths different from 0 and 90°.

As was mentioned above, the wave acquires a phase shift with depth. The phase shift is found to vanish in azimuth $\phi = 90°$, and it has a maximum between 0° and 90° (and between 90° and 180°). The phase shift is varying with depth non-linearly that means that the wave propagates along z direction with varying velocity. Examples of the phase shift of vertical component versus depth normalized to a wavelength are shown in Fig.6 .

Similar features of Rayleigh waves - strong ellipticity in horizontal plane and variation of phase with depth, have also been obtained by Maupin (2001) for some multilayered anisotropic models, who used a method for numerical modelling developed by Thomson (1997).

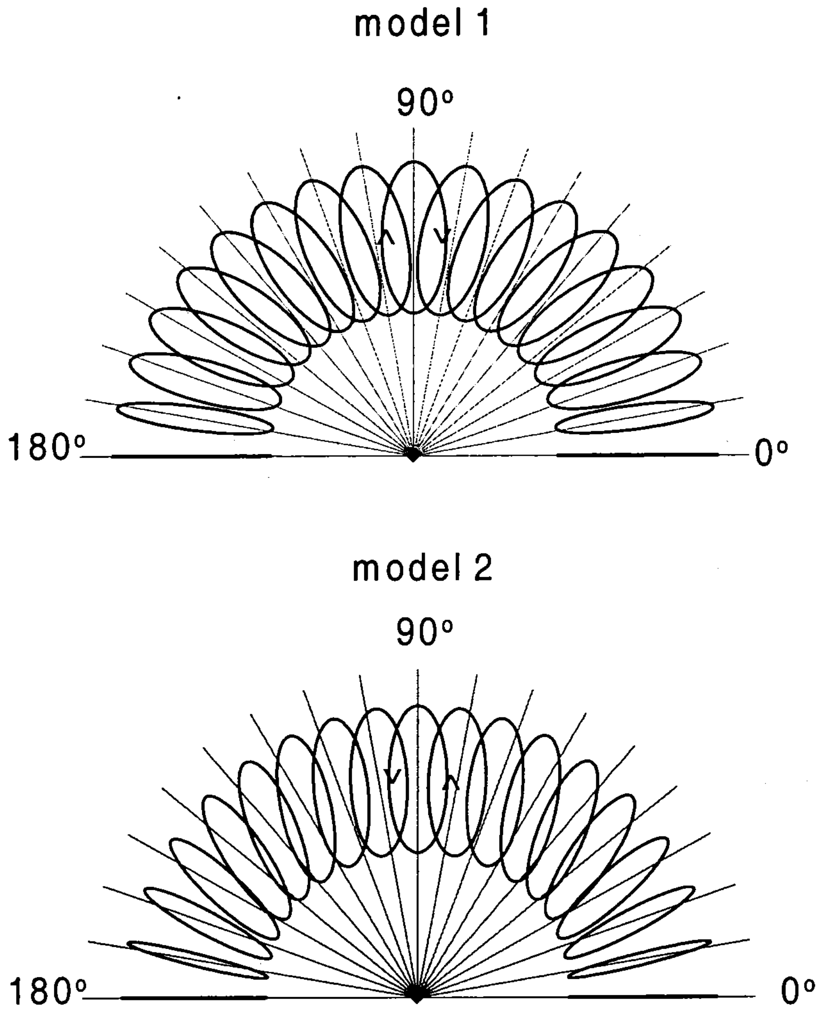

Fig.5. Particle motion in horizontal plane for different azimuths. Arrows show direction of motion.

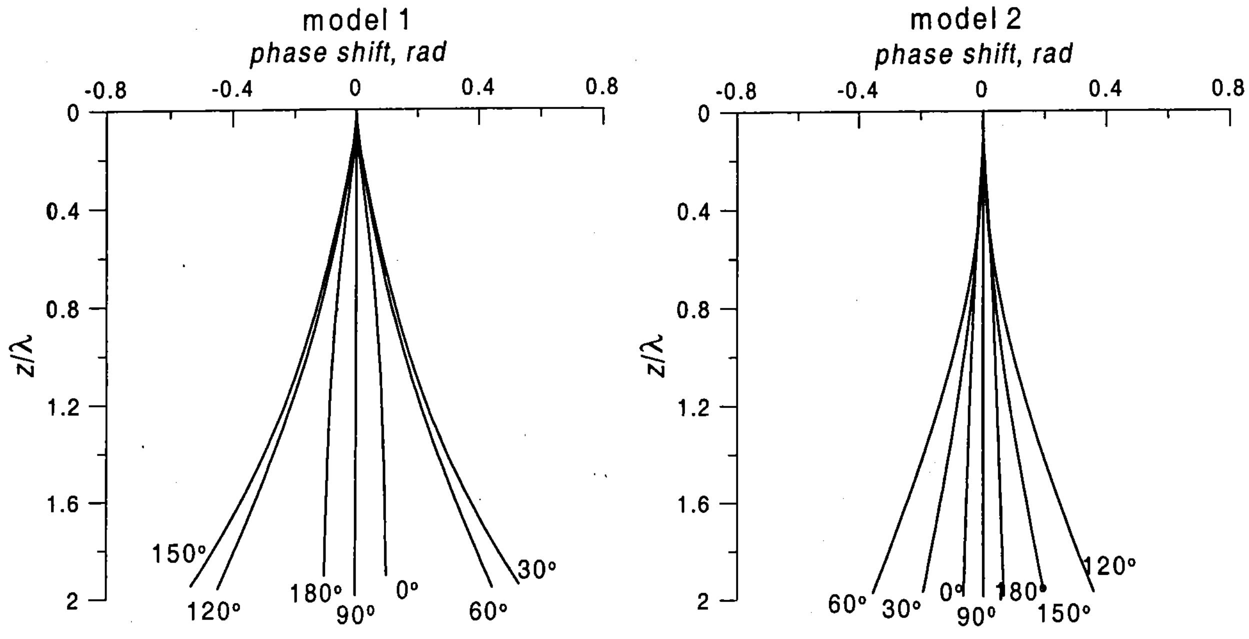

Fig.6. Phase shift versus depth/wave length calculated for θ=45° for different azimuths. Values of the azimuths are indicated by the corresponding curves.

4. Conclusion

Numerical procedure based on the exact solution is developed for the calculation of quasi-Rayleigh wave field in a transversely isotropic half-space with inclined symmetry axis. Some features unexplained by the ordinary perturbation method for weak anisotropy based on the starting isotropic model were discovered from the results of numerical modeling.

The main peculiarity is that qR wave cannot exist for all values of inclination of the symmetry axis and of the azimuths. Domain of existence of qR wave depends on the ratio of elastic parameters. For some angles of inclination qR velocities deviate from those calculated on the basis on the perturbation method, though they have the same tendency of variation with azimuth. The only azimuth, in which the wave has the same phase at all depths, is 90°: at all other azimuths the phase is varying with depth, being positive in one quadrant and negative in the other. This may be interpreted as the propagation of the wave in vertical direction, in addition to the propagation in horizontal direction.

Unlike Rayleigh wave in transversely isotropic half-space with vertical symmetry axis, qR wave has three components – vertical, radial and transverse. Radial and transverse components have a phase shift, so that particle motion in the horizontal plane is elliptic. The direction of the major axis of the ellipsis coincides with the direction of propagation only in the azimuths 0°(180°) and 90°(270°).

Though all results are obtained for the simplest model of a homogeneous half-space, we may suppose that in multilayered inhomogeneous model of the real Earth some discovered properties of qR wave may have a place. At least some difference of the exact field and that calculated by the ordinary perturbation method for weak anisotropy is to be expected. Therefore it is important to analyze numerically surface wave fields, both quasi-Rayleigh and quasi-Love, in realistic models of the Earth on the basis of the exact theory. This would also answer the topical question: is the non-existence of quasi-Rayleigh wave specific to homogeneous half-space, or is it a more general feature valid for models with vertical variation of the elastic parameters? It seems that in a layered half-space at very high and at very low frequencies for the fundamental mode the feature should be the same as in a homogeneous half-space. Probably it will be not so for higher modes, because their generation is similar to that of Love waves, which do not exist in homogeneous half-space but exist in media with velocity increasing with depth.

Appendix

$$C_{11} = A\cos^4\theta + C\sin^4\theta + 2(2L+F)\sin^2\theta\cos^2\theta$$
$$C_{12} = C_{21} = F\sin^2\theta + (A-2N)\cos^2\theta$$
$$C_{13} = C_{31} = (A+C-2F-4L)\sin^2\theta\cos^2\theta + F$$
$$C_{14} = C_{41} = 0$$
$$C_{15} = C_{51} = \sin\theta\cos\theta\left(-A\cos^2\theta + C\sin^2\theta + (F+2L)(\cos^2\theta - \sin^2\theta)\right)$$
$$C_{16} = C_{61} = 0$$
$$C_{22} = A$$
$$C_{23} = C_{32} = F\cos^2\theta + (A-2N)\sin^2\theta$$
$$C_{24} = C_{42} = 0$$
$$C_{25} = C_{52} = (F-A+2N)\sin\theta\cos\theta$$
$$C_{26} = C_{62} = 0$$
$$C_{33} = A\sin^4\theta + C\cos^4\theta + 2(2L+F)\sin^2\theta\cos^2\theta$$
$$C_{34} = C_{43} = 0$$
$$C_{35} = C_{\cdot 53} = \sin\theta\cos\theta\left(-A\sin^2\theta + C\cos^2\theta - (F+2L)(\cos^2\theta - \sin^2\theta)\right)$$
$$C_{36} = C_{63} = 0$$
$$C_{44} = (N\sin^2\theta + L\cos^2\theta)$$
$$C_{45} = C_{54} = 0$$
$$C_{46} = C_{64} = (L-N)\sin\theta\cos\theta$$
$$C_{55} = (A+C-2F-4L)\sin^2\theta\cos^2\theta + L$$
$$C_{56} = C_{65} = 0$$
$$C_{66} = (N\cos^2\theta + L\sin^2\theta)$$

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