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CORRECTIONS TO THE LEADING EIKONAL AMPLITUDE FOR HIGH-ENERGY SCATTERING AND QUASIPOTENTIAL APPROACH

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Abstract

Asymptotic behavior of the scattering amplitude for two scalar particle at high energy and fixed momentum transfers is reconsidered in quantum field theory. In the framework of the quasipotential approach and the modified perturbation theory a systematic scheme of finding the leading eikonal scattering amplitudes and its corrections is developed and constructed. The connection between the solutions obtained by quasipotential and functional approaches is also discussed.

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1 Introduction

Eikonal scattering amplitude for the high energy of the two particles in the limit of high energies and fixed momentum transfers is found by many authors in quantum field theory [1-9], including the quantum gravity in recent years [9-20]. Comparison of the results obtained by means of the different approaches for this problem has shown that they all coincide in the leading order approximation, while the corrections (non-leading terms) provided by them are rather different [15,17,20,21,22,23]. Determination of these corrections to gravitational scattering are now open problems [10-14]. These corrections play a crucial role in such problems as strong gravitational forces near black hole, string modification of theory of gravity and some other effects of quantum gravity [9-20].

The purpose of the present paper is to develop a systematic scheme based on modified perturbation theory to find the correction terms to the leading eikonal amplitude for high-energy scattering by means of solving the Logunov-Tavkhelidze quasipotential equation [24-27]. In spite of the lack of a clear relativistic covariance, the quasipotential method keeps all information about properties of scattering amplitude which could be received starting from general principle of quantum field theory [25]. Therefore, at high energies one can investigate analytical properties of the scattering, its asymptotic behavior and some regularities of a potential scattering etc. Exactly, as it has been done in the usual S-matrix theory [24]. The choice of this approach is dictated also by the following reasons: 1. in the framework of the quasipotential approach the eikonal amplitude has a rigorous justification in quantum field theory [4]; 2. in the case of smooth potentials, it was shown that a relativistic quasipotential and the Schrodinger equations lead to qualitatively identical results [28,29].

The outline of the paper is as follows. In the second section the Logunov-Tavkhelidze quasipotential equation is written in the operator form. In the third section the solution of this equation is presented in the exponent form which is favorable to modify the perturbation theory. The asymptotic behavior scattering amplitude at high energies and fixed momentum transfers is considered in the fourth section. The lowest-order approximation of the modified theory is the leading eikonal scattering amplitude. Corrections to leading eikonal amplitude are also calculated. In the fifth section the solution of quasipotential equation is presented in the form of a functional path integral. The connection between the solutions obtained by quasipotential and functional is also discussed. It is shown that the approximations used are similar and the expressions for correction to the leading eikonal amplitude are found identical. Finally, we draw our conclusions.

2 Two particle quasipotential equation

Let us consider the elastic scattering of two scalar particles with the model of the interaction Lagrangian $L_{int} = g\varphi^2(x)\phi(x)$. Following Ref. [23] for two scalar particle amplitude the quasipotential equation with local quasipotential has the form:

$$T(\mathbf{p},\mathbf{p}';s) = gU(\mathbf{p}-\mathbf{p}';s) + g\int d\mathbf{q}U(\mathbf{p}-\mathbf{q};s)K(\mathbf{q}^2,s)T(\mathbf{q},\mathbf{p}';s), \tag{2.1}$$

where $s = 4(\mathbf{p}^2 + m^2) = 4(\mathbf{p}' + m^2)$ is the energy and $\mathbf{p}$, $\mathbf{p}'$ and are the relative momenta of particles in the centre of mass system in the initial , final states respectively. Equation (2.1) is one of the possible generalizations of the Lippman-Schwinger equation for the case of relativistic quantum field theory. The quasipotential U is a complex function of the energy and the relative momenta. The quasipotential equation simplifies considerably if U is a function of only the difference of the relative momenta and the total energy, i.e. if the quasipotential is local [1]. The existence of a local quasipotential has been proved rigorously in the weak coupling case /27/ and a method has been specified for constructing it. The local potential constructed in this manner gives a solution of Eq. (2.1), being equal to the physical amplitude on the mass shell [24-26] .

Making the following Fourier transformations

$$U(\mathbf{p}-\mathbf{p}';s) = \frac{1}{(2\pi)^3}\int d\mathbf{r}e^{i(\mathbf{p}-\mathbf{p}')\mathbf{r}}U(\mathbf{r};s), \tag{2.2}$$

$$T(\mathbf{p},\mathbf{p}';s) = \int d\mathbf{r}d\mathbf{r}'e^{i(\mathbf{p}\mathbf{r}-\mathbf{p}'\mathbf{r}')}T(\mathbf{r},\mathbf{r}';s), \tag{2.3}$$

and introducing the representation

$$T(\mathbf{r},\mathbf{r}';s) = \frac{g}{(2\pi)^3}U(\mathbf{r};s)F(\mathbf{r},\mathbf{r}';s), \tag{2.4}$$

we transform (2.1) into the coordinate representation

$$F(\mathbf{r},\mathbf{r}';s) = \delta^{(3)}(\mathbf{r}-\mathbf{r}') + \frac{g}{(2\pi)^3}\int d\mathbf{q}K(\mathbf{q}^2;s)e^{-i\mathbf{q}\mathbf{r}}\times$$

$$\times\int d\mathbf{r}''e^{i\mathbf{q}\mathbf{r}''}U(\mathbf{r}'';s)F(\mathbf{r}'',\mathbf{r}';s). \tag{2.5}$$

Defining the pseudodifferential operator

$$\widehat{L_r} = K(-\nabla_{\mathbf{r}}{}^2;s), \tag{2.6}$$

then

$$K(\mathbf{r};s) = \int d\mathbf{q}K(\mathbf{q}^2;s)e^{-i\mathbf{q}\mathbf{r}} = \widehat{L_r}(2\pi)^3\delta^{(3)}(\mathbf{r}). \tag{2.7}$$

[1]Since the total energy s appears as an external parameter of the equation, the term "local" here has direct meaning and it can appear in a three-dimensional δ-function in the quasipotential in the coordinate representation

With allowance for (2.6) and (2.7) Eq. (2.5) can be rewritten in the symbolic form:

$$F(\mathbf{r},\mathbf{r}';s) = \delta^{(3)}(\mathbf{r}-\mathbf{r}') + g\widehat{L_r}[U(\mathbf{r},s)F(\mathbf{r},\mathbf{r}',s)]. \qquad (2.8)$$

3 Modified perturbation theory

In the framework of the quasipotential equation the potential is defined as an infinite power series in the coupling constant which corresponds to the perturbation expansion of the scattering amplitude on the mass shell. The approximate equation has been obtained only in the lowest order of the quasipotential. Using this approximation the relativistic eikonal expression of elastic scattering amplitude was first found in quantum field theory for large energies and fixed momentum transfers [22]. In this paper we follow a somewhat different approach based on the idea of the modified perturbation theory proposed by Fradkin [30][2]The solution of equation (2.8) can be found in the form

$$F(\mathbf{r},\mathbf{r}';s) = \frac{1}{(2\pi)^3}\int d\mathbf{k} \exp\Big[W(\mathbf{r};\mathbf{k};s)\Big] e^{-i\mathbf{k}(\mathbf{r}-\mathbf{r}')}. \qquad (3.1)$$

Substituting (3.1) into (2.8) we obtained an equation for the function $W(\mathbf{r};\mathbf{k};s)$

$$\exp W(\mathbf{r};\mathbf{k};s) = 1 + g\widehat{L_r}\Big\{U(\mathbf{r},s)\exp[W(\mathbf{r};\mathbf{k};s) - i\mathbf{k}\mathbf{r}]\Big\}e^{i\mathbf{k}\mathbf{r}}. \qquad (3.2)$$

The function $W(\mathbf{r};\mathbf{k};s)$ in exponent (3.1) can now be written as an expansion in series in the coupling constant g:

$$W(\mathbf{r};\mathbf{k};s) = \sum_{n=1}^{\infty} g^n W_n(\mathbf{r};\mathbf{k};s). \qquad (3.3)$$

Then from Eq. (3.2) we obtain the following expressions for the functions $W_n(\mathbf{r};\mathbf{k};s)$

$$W_1(\mathbf{r};\mathbf{k};s) = \int d\mathbf{q} U(\mathbf{q};s)K[(\mathbf{k}+\mathbf{q})^2;s]e^{-i\mathbf{q}\mathbf{r}}; \qquad (3.4)$$

$$W_2(\mathbf{r};\mathbf{k};s) = -\frac{W_1^2(\mathbf{r};\mathbf{k};s)}{2!} +$$

$$\frac{1}{2}\int d\mathbf{q}_1 d\mathbf{q}_2 U(\mathbf{q}_1;s)U(\mathbf{q}_2;s)K[(\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2)^2;s]\times$$

$$\times[K(\mathbf{k}+\mathbf{q}_1;s) + K[\mathbf{k}+\mathbf{q}_2;s]]e^{-i\mathbf{q}_1\mathbf{r}-i\mathbf{q}_2\mathbf{r}}; \qquad (3.5)$$

$$W_3(\mathbf{r};\mathbf{k};s) = -\frac{W_1^2(\mathbf{r};\mathbf{k};s)}{3!} + \int d\mathbf{q}_1 d\mathbf{q}_2 d\mathbf{q}_3 U(\mathbf{q}_1;s)U(\mathbf{q}_2;s)U(\mathbf{q}_3;s)K[(\mathbf{k}+\mathbf{q}_1)^2;s]\times$$

[2]The interpretation of the perturbation theory from the view-point of the diagrammatic technique is as follows. The typical Feynman denominator of the standard perturbation theory is of the form (A): $(p+\sum q)^2 + m^2 - i\varepsilon = p^2 + m^2 + 2p\sum q + (\sum q)^2$, where p is the external momentum of the scalar (spinor) particle, and the $q's$ are virtual momenta of radiation quanta. The lowest order approximation (A) of modified theory is equivalent to summing all Feynman diagrams with the replacement: $(\sum q)^2 = \sum (q)^2$ in each denominator (A). The modified perturbation theory thus corresponds to a small correlation of the radiation quanta: $\mathbf{q}_i\mathbf{q}_j = \mathbf{0}$ and is often called the $\mathbf{q}_i\mathbf{q}_j$-approximation. In the framework of functional integration this approximation is called the straight-line path approximation i.e. high-energy particles move along Feynman paths, which are practically rectilinear [18,19].

$$\times K[(\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2)^2;s]K[(\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3)^2;s]e^{-i\mathbf{q}_1\mathbf{r}-i\mathbf{q}_2\mathbf{r}-i\mathbf{q}_3\mathbf{r}}. \tag{3.6}$$

Oversleeves by W_1 only we obtain from Eqs (3.1), (3.3) and (2.3) the approximate expression for the scattering amplitude [22]

$$T_1(\mathbf{p},\mathbf{p}';s) = \frac{g}{(2\pi)^3}\int d\mathbf{r} e^{i(\mathbf{p}-\mathbf{p}')\mathbf{r}}U(\mathbf{r},s)e^{gW_1(\mathbf{r},\mathbf{p},s)}. \tag{3.7}$$

To establish the meaning of this approximation , we expand T_1 in a series in g:

$$T_1^{(n+1)}(\mathbf{p},\mathbf{p}';s) = \frac{g^{n+1}}{n!}\int d\mathbf{q}_1...d\mathbf{q}_n U(\mathbf{q}_1;s)....U(\mathbf{q}_n;s)$$

$$\times U(\mathbf{p}-\mathbf{p}'-\sum_{i=1}^{n} q_i;s)\prod_{i=0}^{n} K[(\mathbf{q}_i+\mathbf{p}')^2;s]. \tag{3.8}$$

Let us compare Eq. (3.8) with the $(n+1)-th$ iteration term of exact Eq. (2.1)

$$T^{(n+1)}(\mathbf{p},\mathbf{p}';s) = \int d\mathbf{q}_1...d\mathbf{q}_n U(\mathbf{q}_1;s)...U(\mathbf{q}_n;s)U(\mathbf{p}-\mathbf{p}'-\sum_{i=1}^{n} q_i;s)\times$$

$$\sum_p K[(\mathbf{q}_1+\mathbf{p}')^2;s]K[(\mathbf{q}_1+\mathbf{q}_2+\mathbf{p}')^2;s]...K[(\sum_{i=1}\mathbf{q}_i+\mathbf{p}')^2;s], \tag{3.9}$$

where $\sum_p$ is the sum over the permutations of the momenta $\mathbf{q}_1$,$\mathbf{p}_2$... $\mathbf{p}_n$. It is readily seen from (3.8) and (3.9) that our approximation in the case of the Lippmann-Schwinger equation is identical with the $\mathbf{q}_i\mathbf{q}_j$ approximation.

4 Asymptotic behavior of the scattering amplitude at high energies

In this section the solution of the Logunov-Tavkhelidze quasipotential equation obtained in the previous section for the scattering amplitude can be used to find the asymptotic behavior as $s \to \infty$ for fixed t. In the asymptotic expressions we shall retain both the principal term and the following term, using the formula

$$e^{W(\mathbf{r},\mathbf{p}';s)} = e^{W_1(\mathbf{r},\mathbf{p}';s)}\Big[1+g^2W_2(\mathbf{r},\mathbf{p}';s)+...\Big], \tag{4.1}$$

where W_1 and W_2 are given by (3.4) and (3.5).

We take the z axis along the vector $(\mathbf{p}+\mathbf{p}')$ then

$$\mathbf{p}-\mathbf{p}' = \nabla_\perp; \quad \mathbf{\Delta}_\perp\mathbf{n_z} = 0; \quad t = -\mathbf{\Delta}_\perp^2. \tag{4.2}$$

Noting

$$K(\mathbf{p}+\mathbf{p}';s) = \frac{1}{\sqrt{(\mathbf{p}+\mathbf{p}')^2+m^2}}\frac{1}{(\mathbf{p}+\mathbf{p}')^2-\frac{s}{4}+m^2-i\varepsilon} =$$

$$= \frac{2}{s(q_z^2 - i\epsilon)}[1 - \frac{3q_z^2 + \mathbf{q}_\perp{}^2 + \mathbf{q}_\perp\Delta_\perp}{\sqrt{s}(q_z - i\epsilon)}] + O(\frac{1}{s^2}), \tag{4.3}$$

and using Eqs (3.4) (3.5) and (3.6) we obtain

$$W_1 = \left(\frac{W_{10}}{s}\right) + \left(\frac{W_{11}}{s\sqrt{s}}\right) + \left(\frac{1}{s^2}\right); \tag{4.4}$$

$$W_2 = \left(\frac{W_{20}}{s^2\sqrt{s}}\right) + O\left(\frac{1}{s^3}\right), \tag{4.5}$$

where

$$W_{10} = 2\int d\mathbf{q} U(\mathbf{q};s)\frac{e^{i\mathbf{q}\mathbf{r}}}{(q_z^2 - i\varepsilon)^2} = 2i\int_{-\infty}^{z} dz' U(\sqrt{\mathbf{q}_\perp{}^2 + z'^2};s); \tag{4.6}$$

$$W_{11} = -2\int d\mathbf{q} U(\mathbf{q};s)e^{-i\mathbf{q}\mathbf{r}}\frac{3q_z^2 + \mathbf{q}_\perp{}^2 + \mathbf{q}_\perp\Delta_\perp}{(q_z - i\epsilon)^2} =$$

$$= -6U(\sqrt{\mathbf{q}_\perp{}^2 + z'^2};s) + 2(-\nabla_\perp{}^2 - i\mathbf{q}_\perp\nabla_\perp)\int_{-\infty}^{\mathbf{z}} \mathbf{dz'U}(\sqrt{\mathbf{q}_\perp{}^2 + \mathbf{z'^2}};\mathbf{s}); \tag{4.7}$$

$$W_{20} = -4\int d\mathbf{q}_1 d\mathbf{q}_2 e^{-i(\mathbf{q}_1+\mathbf{q}_2)\mathbf{r}} U(\mathbf{q}_1;s)U(\mathbf{q}_2;s)\frac{3q_{1z}q_{2z} + \mathbf{q}_{1\perp}\mathbf{q}_{2\perp}}{(q_{1z} - i\varepsilon)(q_{2z} - i\varepsilon)(q_{1z} + q_{2z} - i\varepsilon)}$$

$$= -4i\{3\int_{-\infty}^{z} dz' U^2(\sqrt{\mathbf{q}_\perp{}^2 + z'^2};s) + [\nabla_\perp\int_{-\infty}^{z'} dz'' U^2(\sqrt{\mathbf{q}_\perp{}^2 + z''^2};s)]^2\}. \tag{4.8}$$

In the limit $s \to \infty$ and $(t/s) \to 0$ W_{10} makes the main contribution, and the remaining terms are corrections. Therefore, the function $\exp W$ can be represented by means of the expansion (4.1) where W_{10}, W_{11} and W_{20} are determined by Eqs. (4.6) – (4.8) respectively. The asymptotic behavior scattering amplitude can be written in the following form

$$T(\mathbf{p},\mathbf{p}';\mathbf{s}) = \frac{\mathbf{g}}{(2\pi)^3}\int \mathbf{d^2r_\perp dz e^{i\Delta_\perp r_\perp} U}(\sqrt{\mathbf{r^2 + z^2}};\mathbf{s})\times$$

$$\times\exp\left(g\frac{W_{10}}{s}\right)\left(1 + g\frac{W_{11}}{s\sqrt{s}} + g^2\frac{W_{20}}{s^2\sqrt{s}} + \ldots\right). \tag{4.9}$$

Substituting (4.6) – (4.8) into (4.9) and making calculations, at high energy $s \to \infty$ and fixed momentum transfers $(t/s) \to 0$ we finally obtain[22]

$$T(s,t) = \frac{g}{2i(2\pi)^3}\int d^2\mathbf{r}_\perp \mathbf{e^{i\Delta_\perp r_\perp}}\left\{\mathbf{e}^{\left[\frac{\mathbf{2ig}}{\mathbf{s}}\int_{-\infty}^{\infty}\mathbf{dzU}(\sqrt{\mathbf{r^2+z^2}};\mathbf{s})\right]} - \mathbf{1}\right\} -$$

$$-\frac{6g^2}{(2\pi)^3 s\sqrt{s}}\int d^2\mathbf{r}_\perp \mathbf{e^{i\Delta_\perp r_\perp}}\exp\left[\frac{\mathbf{2ig}}{\mathbf{s}}\int_{-\infty}^{\infty}\mathbf{dz'U}(\sqrt{\mathbf{r}_\perp^2 + \mathbf{z^2}};\mathbf{s})\right]\int_{-\infty}^{\infty}\mathbf{dzU}(\sqrt{\mathbf{r}_\perp^2 + \mathbf{z^2}};\mathbf{s}) -$$

$$-\frac{ig}{(2\pi)^3\sqrt{s}}\int d^2\mathbf{r}_\perp \mathbf{e^{i\Delta_\perp r_\perp}}\times$$

$$\times\int_z^{\infty} dz\left\{\exp\left[\frac{2ig}{s}\int_z^{\infty} dz' U(\sqrt{\mathbf{r}_\perp^2 + \mathbf{z}'^2};s)\right] - \exp\left[\frac{2ig}{s}\int_{-\infty}^{\infty} dz' U(\sqrt{\mathbf{r}_\perp^2 + \mathbf{z}'^2};s)\right]\right\}\times$$

$$\times\left\{\int_{z}^{\infty} dz'\nabla_{\perp}{}^{2}U(\sqrt{\mathbf{r}_{\perp}^{2}+\mathbf{z}'^{2}};s)-\frac{2ig}{s}\left[\int_{z}^{\infty} dz'\nabla_{\perp}U(\sqrt{\mathbf{r}_{\perp}^{2}+\mathbf{z}^{2}};s)\right]^{2}\right\}$$

$$-\frac{2ig}{(2\pi)^{3}s}\mathbf{\Delta}_{\perp}{}^{2}\int d^{2}\mathbf{r}_{\perp}\mathbf{U}(\sqrt{\mathbf{r}_{\perp}^{2}+\mathbf{z}'^{2}};\mathbf{s})]\mathrm{e}^{\mathrm{i}\mathbf{\Delta}_{\perp}\mathbf{r}_{\perp}}\times$$

$$\times\int_{-\infty}^{\infty} zdzU(\sqrt{\mathbf{r}_{\perp}^{2}+\mathbf{z}^{2}};s)\exp\left[\frac{2ig}{s}\int_{z}^{\infty} dz'U(\sqrt{\mathbf{r}_{\perp}^{2}+\mathbf{z}'^{2}};s)\right]. \tag{4.10}$$

In this expression (4.10) the first term describes the leading eikonal behavior of the scattering amplitude, while the remaining terms determine the corrections of relative magnitude $1/\sqrt{s}$ The similar result Eq.(4.10) is also found by means of the functional integration [20].

As is well known from the investigation of the scattering amplitude in the Feynman diagrammatic technique, the high-energy asymptotic behavior can contain only logarithms and integral powers of s. A similar effect is observed here, since integration of the expression (4.10) leads to the vanishing of the coefficients for half-integral powers of s. Nevertheless, allowance for the terms that contain the half-integral powers of s is needed for the calculations of the next corrections in the scattering amplitude, and leads to the appearance of the so-called retardation effects, which are absent in the principal asymptotic term.

To conclude this section it is important to note that: in the framework of standard field theory for the high-energy scattering, different methods have been developed to investigate the asymptotic behavior of individual Feynman diagrams and their subsequent summation. In different theories including quantum gravity the calculations of Feynman diagrams in the eikonal approximation is carried out in a similar way as the calculations in QED. Reliability of the eikonal approximation depends on spin of the exchanges field [5,6]. The eikonal captures the leading behavior of each order in perturbation theory, but the sum of leading terms is subdominant to the terms neglected by this approximation. The reliability of the eikonal amplitude for gravity is uncertain [14]. Instead of the diagram technique perturbation theory, our approach is based on the exact expression of the scattering amplitude and modified perturbation theory which in lowest order contains the leading eikonal amplitude and the next orders are its corrections.

5 Relationship between the operator and Feynman path methods

What actual physical picture may correspond to our result given by Eq. (4.10) ? To answer this question we establish the relationship between the operator and Feynman path methods in Ref. [31], which treats the quasipotential equation in the language of functional integrals. The solution of this equation can be written in the symbolic form:

$$\exp(W) = \frac{1}{1 - gK[(-i\nabla - \mathbf{k})^2]U(\mathbf{r})} \times 1 =$$

$$= -i \int_0^\infty d\tau \exp[i\tau(1 + i\varepsilon)] \exp\{-i\tau g K[(-i\nabla - \mathbf{k})^2]U(\mathbf{r})\} \times 1. \quad (5.1)$$

In accordance with the Feynman parametrization [31], we introduce an ordering index η and write Eq. (5.1) in the form

$$\exp(W) = -i \int_0^\infty d\tau e^{i\tau(1+i\varepsilon)} \exp\{-ig \int_0^\infty d\eta K[(-i\nabla_{\eta+\varepsilon} - \mathbf{k})^2]U(\mathbf{r}_\eta)\} \times 1. \quad (5.2)$$

Using Feynman transformation

$$F[P(\eta)] = \int D\mathbf{p} \int_{x(0)=0} \frac{D\mathbf{x}}{(2\pi)^3} \exp\{i \int_0^\tau d\eta \dot{\mathbf{r}}(\eta)[\mathbf{p}(\eta) - P(\eta)]\} F[\mathbf{p}(\eta)], \quad (5.3)$$

we write the solution of Eq. (2.8) in the form of the functional integral

$$\exp(W) = -i \int_0^\infty d\tau e^{i\tau(1+i\varepsilon)} \times$$

$$\times \int D\mathbf{p} \int_{x(0)=0} \frac{D\mathbf{x}}{(2\pi)^3} \exp\{i \int_0^\tau d\eta \dot{\mathbf{x}}(\eta)[\mathbf{p}(\eta) - P(\eta)]\} G(\mathbf{x}, \mathbf{p}; \tau) \times 1. \quad (5.4)$$

In Eq.(5.4) we enter the function G

$$G(\mathbf{x}, \mathbf{p}; \tau) = \exp\{-i \int_0^\tau d\eta \dot{\mathbf{x}}(\eta) \nabla_{\eta+\varepsilon}\} \times \exp\{-ig \int_0^\tau d\eta K[(\mathbf{p}(\eta) - \mathbf{k})^2]U(\mathbf{r}_\eta)\}, \quad (5.5)$$

which satisfies the equation

$$\frac{dG}{d\tau} = \{-igK[(\mathbf{p}(\tau) - \mathbf{k})^2]U(\mathbf{r} - \dot{\mathbf{x}}(\tau - \varepsilon))\nabla\}G;$$

$$G(\tau = 0) = 1. \quad (5.6)$$

Finding from Eq. (5.6) the operator function G and substituting it into Eq. (5.6) for W we obtained the following final expression:

$$\exp(W) = -i \int_0^\infty d\tau e^{i\tau(1+i\varepsilon)} \int D\mathbf{p} \frac{1}{(2\pi)^3} \int_{x(0)=0} \frac{D\mathbf{x}}{(2\pi)^3} \exp\{i \int_0^\tau d\eta \dot{\mathbf{x}}(\eta) p(\eta)\} \exp(g \prod), \quad (5.7)$$

where

$$\prod = -i \int_0^\infty d\tau K[(\mathbf{p}(\eta) - \mathbf{k})^2] U[\mathbf{r} - \int_0^\tau d\xi \dot{\mathbf{x}}(\xi) \vartheta(\xi - \eta + \varepsilon)]; \quad (5.8)$$

$$\prod^2 = - \int_0^{\tau_1} \int_0^{\tau_2} d\tau_1 d\tau_2 K[(\mathbf{p}(\eta_1) - \mathbf{k})^2] K[(\mathbf{p}(\eta_2) - (\mathbf{k}))^2] \times$$

$$\times U[\mathbf{r_1} - \int_0^{\tau_1} d\xi \dot{\mathbf{x}}(\xi) \vartheta(\xi - \eta + \varepsilon)] U[\mathbf{r_2} - \int_0^{\tau_2} d\xi \dot{\mathbf{x}}(\xi) \vartheta(\xi - \eta + \varepsilon)]. \quad (5.9)$$

Writing out the expansion [2,3]

$$\exp(W) = \overline{\exp(g \prod)} = \exp(g \overline{\prod}) \sum_{n=0}^{\infty} \frac{g^n}{n!} \overline{(\prod - \overline{\prod})^n}, \quad (5.10)$$

in which the sign of averaging denoted integration with respect to τ, $\mathbf{x}(\eta)$ and $\mathbf{p}(\eta)$ with the corresponding measure (see, for example Eq. (5.7)), and performing the calculations, we find

$$W_1 = \overline{\prod} = -i\int_0^{\infty} d\tau K[(\mathbf{p}(\eta) - \mathbf{k})^2] \exp[-\int_0^{\tau} d\xi \dot{\mathbf{x}}(\xi)\vartheta(\xi - \eta + \varepsilon)\nabla_{\eta}]U(\vec{r})$$
$$= \int d\mathbf{q} e^{-\mathbf{q}\mathbf{r}} K[(\mathbf{q}+\mathbf{k})^2]U(\mathbf{q}; s); \tag{5.11}$$

$$\overline{\prod^2} = K[(\nabla_{\mathbf{r_1}} + \nabla_{\mathbf{r_2}} + \mathbf{k})^2]\mathbf{K}[(\nabla_{\mathbf{r_1}} + \mathbf{k})^2]\mathbf{K}[(\nabla_{\mathbf{r_2}} + \mathbf{k})^2]\mathbf{U}(\mathbf{r_1};\mathbf{s})\mathbf{U}(\mathbf{r_2};\mathbf{s})$$

$$= \int d\mathbf{q_1}\int d\mathbf{q_2} e^{-i(\mathbf{q_1}+\mathbf{q_2})\mathbf{r}} K[(_1+\mathbf{q_2}+\mathbf{k})^2]\{K[(\mathbf{q_1}+\mathbf{k})^2]+K[(\mathbf{q_2}+\mathbf{k})^2]\}U(\mathbf{r_1};s)U(\mathbf{r_2};s), \tag{5.12}$$

$$W_2 = \frac{\overline{\prod^2} - \overline{\prod}^2}{2!} = -\frac{W_1^2}{2!} + \frac{1}{2}\int d\mathbf{q_1}d\mathbf{q_2}U(\mathbf{q_1})U(\mathbf{q_2})\{K[(\mathbf{q_1}+\mathbf{k})^2; s] + K[(\mathbf{q_2}+\mathbf{k})^2; s]\}. \tag{5.13}$$

$$W_3 = \frac{1}{3!}[\overline{\prod^3} - \overline{\prod}^3 - 3\overline{\prod}(\overline{\prod^2} - \overline{\prod}^2)] =$$

$$= -\frac{W_1^3}{3!} + \int d\mathbf{q}_1 d\mathbf{q}_2 d\mathbf{q}_3 U(\mathbf{q}_1; s)U(\mathbf{q}_2; s)U(\mathbf{q}_3; s)K[(\mathbf{k}+\mathbf{q}_1)^2; s]\times$$
$$\times K[(\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2)^2; s]K[(\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3)^2; s]e^{-i\mathbf{q}_1\mathbf{r}-i\mathbf{q}_2\mathbf{r}-i\mathbf{q}_3\mathbf{r}}. \tag{5.14}$$

i.e. the expressions (5.10) and (4.1) are identical.

Restricting ourselves in the expansion (5.10) to the first term ($n = 0$), we obtain the approximate expression (3.7) for the scattering amplitude, which corresponds to the allowance for the particle Feynman paths. These paths can be considered as a classical paths and coincide in the case of the scattering of high-energy particles through small angles to straight-line paths trajectories.

6 Conclusions

Asymptotic behavior of scattering amplitude for two scalar particles in the case of the interaction Lagrangian $L_{int} = g\varphi^2(x)\phi(x)$ at high energy and fixed momentum transfers was studied. In the framework of quasipotential approach and the modified perturbation theory the systematic scheme of finding the corrections to the principal asymptotic leading scattering amplitudes was developed. Results obtained by two different approaches (quasipotential and functional) for this problem, as it has shown that they are identical. Results obtained by us can be extended to the case of scalar particles of the field $\varphi(x)$ interacting with a gravitational field. However, in the latter case one must further study the physical effects related with corrections to the leading eikonal amplitude of high-energy scattering in quantum gravity.

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References

[1] H. D. I.Abarbanel and C. Itzykson, Phys. Rev. Lett. 23 (1969) 53.

[2] B. M. Barbashov, S. P. Kuleshov, V. A. Matveev, V. N. Pervushin, A. N. Sissakian and A. N. Tavkhelidze, Phys. Lett. 33B (1970) 484.

[3] B. M. Barbashov, D. I. Blokhintsev, V. V. Neterenko, and V. N. Pervushin, " Fiz. Elm. Chast. At. Yadra ", 2 (1973) 623.

[4] V. R. Garsevanishvili, V. A. Matveev, L. A. Slepchenko, and A. N. Tavkhelidze, Phys. Lett. 29B (1969) 191.

[5] G. Tikotopoulos and S. B.Treinman, Phys. Rev. 3D (1971) 1037.

[6] E.Eichen and R. Jackiw, Phys. Rev. D4 (1971) 439.

[7] M. Levy and J. Sucher, Phys. Rev. 186 (1969) 1656; D2 (1971) 1716.

[8] L. N. Lipatov, Phys. Lett. B116 (1992) 411; Nucl. Phys. B365 (1991) 614.

[9] G. 'tHooft, Phys. Lett. 198B (1987) 61.

[10] D. Amati, M. Ciafaloni and G. Veneziano, Phys. Lett. 197B (1987) 81.

[11] D.Amati, M. Ciafaloni and G. Veneziano, Int. J. Mod. Phys. A3 (1988) 1615.

[12] D. Amati, M. Ciafaloni and G. Veneziano, Nucl. Phys. B347 (1990) 550.

[13] E. Verlinde and H. Verlinde, Nucl. Phys. B371 (1992) 246.

[14] D. Kabat and M. Ortiz, Nucl. Phys. B388 (1992) 570.

[15] M. Fabbrichesi, R. Pettorino, G. Veneziano, G. A. Vilkovisky, Nucl. Phys. B419 (1994) 147.

[16] I.Muzinich and M. Soldate, Phys. Rev. D37 (1988) 353.

[17] R. Kirschner, Phys. Rev. D52, N4 (1995) 2333.

[18] Nguyen Suan Han and E. Ponna, Nuovo Cimento A, 110 (1997) 459.

[19] Nguyen Suan Han, Eur. Phys. J. C16 (2000) 547.

[20] Nguyen Suan Han, and Nguyen Nhu Xuan, Eur. Phys. J. C24 (2002) 643; gr-qc/0203054.

[21] V. R. Garsevanishvili, V. A. Matveev, and L. A. Slepchenko, "Fiz. Elm. Chast. At. Yadra" 1 (1970) 91.

[22] V. R. Garsevanishvili, V. A. Matveev, L. A. Slepchenko, and A. N. Tavkhelidze, Theor. Math. Fiz. 6(1971) 36.

[23] S. P. Kuleshov, V. A. Matveev, A. N. Sissakian, and M. A. Smodyrev, Theor. Math. Fiz. 14B (1973) 325.

[24] A. A. Logunov and A. N. Tavkhelidze. Nuovo Cim, 29 (1963) 380.

[25] A.A. Logunov, Nguyen Van Hieu and O.A. Khrustalev, Nucl Phys., 50 (1964) 295.

[26] I. T. Todorov, Phys. Rev.D, 3 (1971)2351.

[27] V. G. Kadyshevskii and A. N. Tavkhelidze, I n: Problems of Theoretical Physics (Delicated to N.N. Bogoliubov on occasion of his 60th birthday), Nauka, 1969.

[28] M. A. Mestvirshvili and G. L. Rcheulishvili, Theor. Math. Fiz. 8 (1970) 206.

[29] A. T. Filippov, Nucl Phys., " Fiz. Elm. Chast. At. Yadra" 10 (1970) 501.

[30] E. S. Fradkin, Acta Phys. Hung 19 (1965) 175; Nucl.Phys. 76 (1966) 588.

[31] V. N. Pervushin, Theor. Math. Fiz. 14(1972)332.