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DIRECT NUMERICAL SIMULATION OF HEAT TRANSFER IN A TURBULENT FLUME

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ABSTRACT

Direct Numerical Simulation (DNS) can be used for the description of turbulent heat transfer in the fluid at low Reynolds numbers. DNS means precise solving of Navier-Stoke's equations without any extra turbulent models. DNS should be able to describe all relevant length scales and time scales in observed turbulent flow. The largest length scale is actually dimension of system and the smallest length and time scale is equal to Kolmogorov scale.

In the present work simulations of fully developed turbulent velocity and temperature fields were performed in a turbulent flume (open channel) with pseudo-spectral approach at Reynolds number 2670 (friction Reynolds number 171) and constant Prandtl number 5.4, considering the fluid temperature as a passive scalar. Two ideal thermal boundary conditions were taken into account on the heated wall. The first one was an ideal isothermal boundary condition and the second one an ideal isoflux boundary condition. We observed different parameters like mean temperature and velocity, fluctuations of temperature and velocity, and auto-correlation functions.

1 INTRODUCTION

Heat transfer in the turbulent boundary layer is of great importance from the scientific and engineering points of view. A large number of experimental and numerical studies have been made to understand the near-wall turbulence momentum and scalar transport. The Direct Numerical Simulation (DNS) became an important research tool of the turbulent heat transfer in the last decade.

Infinite flat wall is heated by constant heat source and cooled by an incompressible flow above it. The buoyancy is neglected, which means that the temperature is a passive scalar and does not influence on the turbulence. Two ideal thermal boundary conditions were taken into account: isothermal boundary condition (zero temperature fluctuations near the wall) and isoflux boundary condition (nonzero temperature fluctuations near the wall). If a flow of air is heated by a thick metal wall the influence of the turbulence on the wall is practically negligible due to the low density, heat capacity and thermal conductivity of air in comparison with high density, heat capacity and conductivity of wall [1]. In such case the first ideal boundary condition (isothermal BC) is approached. If water flow is heated by a tiny metal foil the influence of the turbulence on the wall is strong because of the small wall thickness and relatively high density, heat capacity and thermal conductivity of water in comparison with density, thermal conductivity and heat capacity of the foil. The second boundary condition (isoflux BC) is almost reached in such case.

Turbulent movement is composed of vortices of different dimensions. The biggest vortices are determined by system's geometry and the smallest by the viscous forces. DNS should be able to describe vortices of various dimensions that can occur in a system at given Reynolds number. The biggest vortex has the system's dimensions; the dimensions of the smallest vortex can be estimated by the Kolmogorov theory [2]

$$\eta \approx \mathrm{Re}^{-3/4}\, l \tag{1}$$

where η stands for the smallest, and l for the biggest possible vortex scale. The distance between points of the discrete grid where the equations are solved must be almost equal to the dimension of the smallest vortices.

For turbulent heat transfer the dimensions of the smallest thermal structures are approximately [2]

$$\eta_T \approx \mathrm{Pr}^{-1/2}\, \eta \;. \tag{2}$$

In order to resolve all the thermal scales finer grid is required for simulations at Prandtl numbers higher than one.

2 EQUATIONS AND NUMERICAL PROCEDURE

The flow in the flume is assumed to be fully developed. Bottom wall of the flume is heated by constant heat source, while top surface of the flume is free, as shown in Fig. 1.

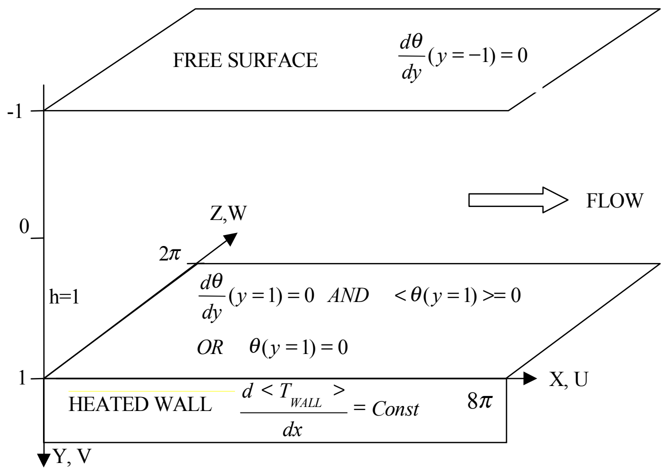

Figure 1. Computational domain and boundary conditions.

The dimensionless equations normalized with flume width h, friction velocity u_τ kinematic viscosity ν, and friction temperature $T_\tau = q_w / \left(u_\tau \rho_f c_{pf} \right)$ can be found in the papers of Kasagi [6] or Kawamura [7]:

$$\nabla \cdot \vec{u}^{+} = 0 \tag{3}$$

$$\frac{\partial \vec{u}^+}{\partial t} = -\nabla \cdot \left(\vec{u}^+ \vec{u}^+\right) + \frac{1}{\mathrm{Re}_\tau} \nabla^2 \vec{u}^+ - \nabla p + \vec{l}_x \tag{4}$$

$$\frac{\partial \theta^+}{\partial t} = -\nabla \cdot \left(\vec{u}^+ \theta^+\right) + \frac{1}{\mathrm{Re}_\tau \, \mathrm{Pr}} \nabla^2 \theta^+ + \frac{u_x^+}{2u_B^+} \tag{5}$$

$\mathrm{Re}_\tau = u_\tau h/\nu$ is the friction Reynolds number and Pr is Prandtl number. Terms $\vec{l}_x$ (unit vector in streamwise direction) and $u_x^+/2u_B^+$ appear in the equations (4) and (5) due to the numerical scheme that requires boundary conditions in streamwise and spanwise directions. Dimensionless wall temperature difference is defined as

$$\theta^+(x,y,z,t) = \left(\frac{\langle T_w \rangle - T(x,y,z,t)}{T_\tau} \right). \tag{6}$$

The top surface of the flume is an adiabatic free surface, with boundary conditions for wall-normal velocity $v_{\text{free surface}}=0$. Velocity boundary condition at the free surface is not physical since it does not allow surface waves. However, experiments of Hetsroni et. al. (1997, 1999) and DNS results (Li et. al. 1999) show that this is an acceptable approximation at low Reynolds numbers, where surface waves are negligible and do not affect near wall behavior.

As can be seen from Eqs. (3-5) temperature is assumed to be a passive scalar. This assumption introduces two approximations: 1) neglected buoyancy, 2) neglected temperature dependence of the material properties - especially viscosity and heat conductivity. Results of the present study are thus very accurate only for the systems, where the temperature differences are not too large, while some caution is required for the systems, where the temperature differences are not negligible.

The isothermal boundary condition at the free surface is $d\theta^+/dy = 0$. The isothermal boundary condition at the wall is

$$\theta^+ = 0, \tag{7}$$

while the isoflux boundary condition is imposed with mean dimensionless temperature at heated wall fixed to zero

$$\left\langle \theta^+(y=1) \right\rangle = 0 \tag{8}$$

and the

$$\frac{d\theta^+}{dy^+}(y=1) = 0. \tag{9}$$

The equations are solved with pseudo-spectral scheme using Fourier series in x and z directions and Chebyshev polynomials in the wall-normal y direction. Numerical procedure and the code of Gavrilakis [8] modified by Lam and Banerjee [9], and Lam [10] are used to solve the continuity and momentum equations. Equations (3-5) are periodic in streamwise (x) and spanwise (z) directions.

All our DNS simulations were performed at Re = 2670 (Re_τ = 171) and Pr = 5.4 using both isoflux and isothermal boundary conditions. The size of computational domain was 8π x 2π x 2 units in x, z and y directions or 2146, 537 and 171 wall units, respectively. Wall units are useful because turbulent flows with different Reynolds numbers can be easily compared. In wall units the height of the flume (open channel) is equal to the friction Reynolds number. In our case the height is equal 171 wall units, which is according to the Fig. 1 equal to 2 units of the computational box (units that are used in the computer code). Therefore, 1 unit of the computational box is equal to the 85.5 wall units. 256x128 Fourier modes and 129 Chebyshev polynomials were used in x, z and y direction. The applied resolution was $\Delta x^+ = 8.38$, $\Delta z^+ = 4.19$ and $\Delta y^+ = 0.05 – 2.10$. The averaging was performed over time interval t^+=2560 after the fully developed flow was achieved (100000 time steps). The simulation was performed on supercomputer SGI Origin and was running for approximately 5 days on 32 processors.

3 RESULTS AND DISCUSSION

The Fig. 2 represents instantaneous velocity field of the streamwise velocity in the y^+-z^+ plane. Such results are not very useful because such instantaneous pattern of turbulent flow can not be compared with any other numerical or experimental instantaneous picture. We have to make an average of the results according to time and/or planes, which are paralleled with the wall, so that we can get some kind of average picture in a longer interval and on entire plane. Only such results can be compared with other simulations and experiments.

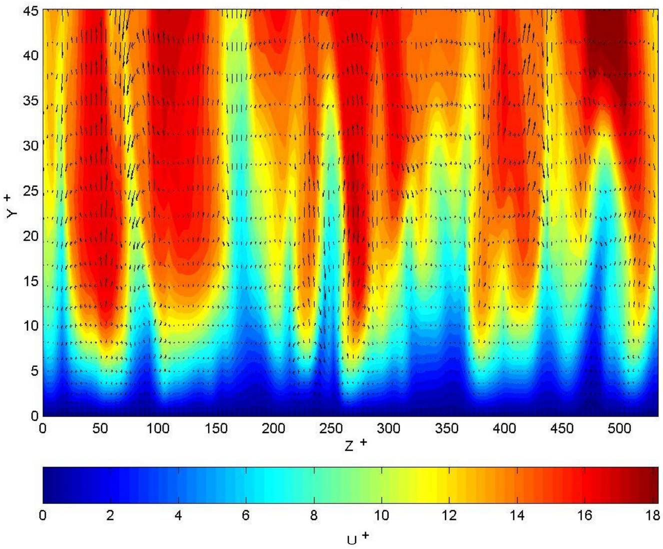

Figure 2: An example of instantaneous velocity field in y^+-z^+ plane for isoflux boundary condition.

Fig. 3 shows the temperatures and velocity averaged over the time and x^+-z^+ planes, which are paralleled to the heated wall. Temperature field and thermal boundary conditions do not affect the velocity field (temperature is a passive scalar). That means that we can perform DNS of a single velocity field and two temperature fields (one for each thermal boundary condition at the wall) at the same time. Therefore, a single mean velocity profile is plotted in Fig. 3b. As can be seen from Fig. 3a boundary conditions do not affect the mean temperature. It should be emphasized that temperature θ is actually negative dimensionless temperature, which means that the lowest temperature is at the wall, and the highest in the middle of the channel. The only reason of negative dimensionless temperature is similarity with velocity profile.

The RMS (root mean square) of the temperature and velocity fluctuations are shown in Figs. 4. The differences between isothermal and isoflux boundary conditions can be clearly seen in these diagrams. In the case of isoflux thermal boundary condition the θ^{+}_{RMS} remains approximately constant across the viscous sublayer ($y^{+} \leq 5$).

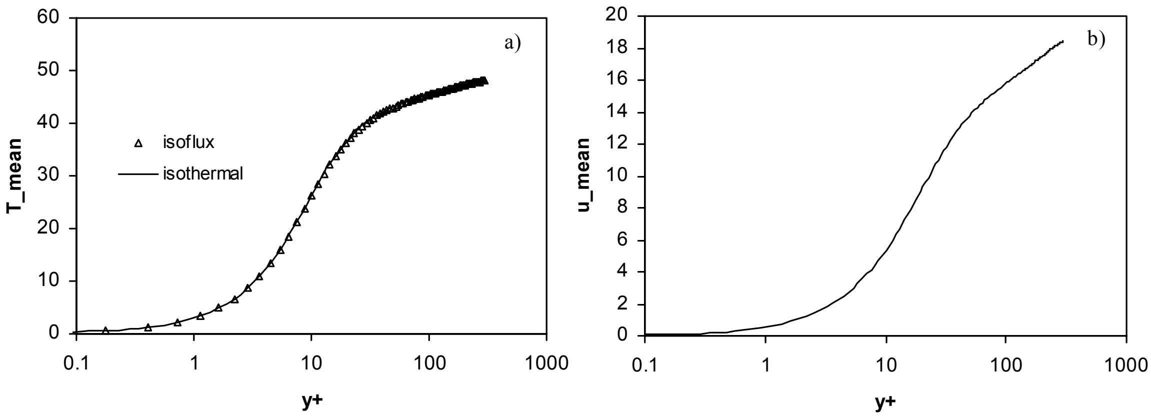

Figure 3: Profile of mean temperature with isothermal and isoflux boundary condition (a) and mean streamwise velocity (b).

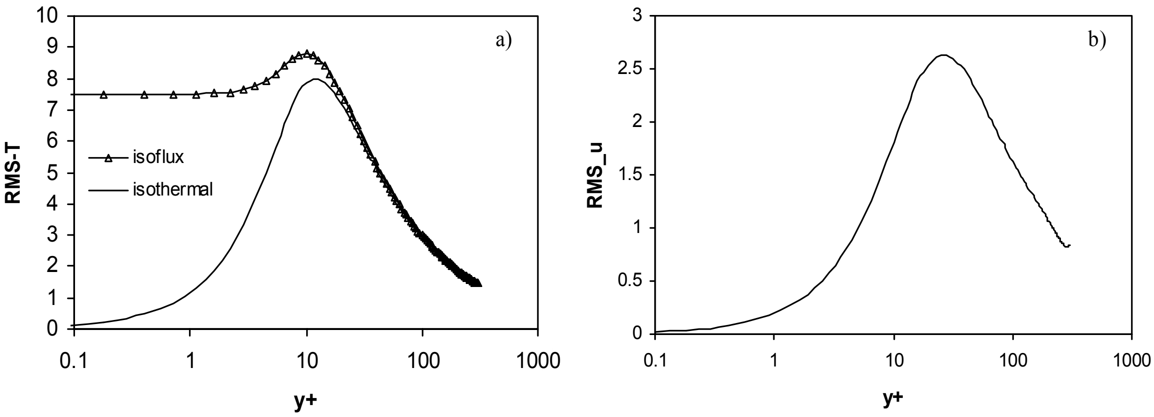

Figure 4: Profiles of RMS fluctuations: a) temperature fluctuations: isothermal and isoflux boundary condition, b) streamwise velocity fluctuations.

Figures 5 (a-d) show two-point auto-correlation functions of temperature (for two boundary conditions) and streamwise velocity in streamwise direction calculated at distance y^{+}=3.7 from the heated wall. Auto-correlation function in the numerical simulation is used in order to find out if the dimensions of the channel are suitable. Because of the periodical boundary conditions the fluid must be well mixed in the first half of the channel, which means that the situation in the middle of the channel has nothing to do with the situation at the entrance (or end) of the channel. The points at the end of the channel are neighbours to the points at the entrance of the channel due to the periodic boundary conditions. This is the reason why the auto-correlation functions are calculated only in the first half of the channel. An important difference between two different boundary conditions can be seen in Fig. 5a. It shows the streamwise auto-correlation function of temperature at two different boundary conditions. While the auto-correlation function for isothermal BC decays close to zero in the observed computational domain, the decay of the auto-correlation function for the isoflux BC is slower and remains well above zero. This could mean that the length of the computational domain for the isoflux BC should be longer in order to satisfy the generally accepted criteria for DNS about the sufficient length of the computational domain. These criteria state that the influence of the periodic boundary conditions on the DNS are avoided, when the periodicity length ensures that auto-correlation functions of all fields fall to zero in the directions in which the periodic boundary conditions are imposed.

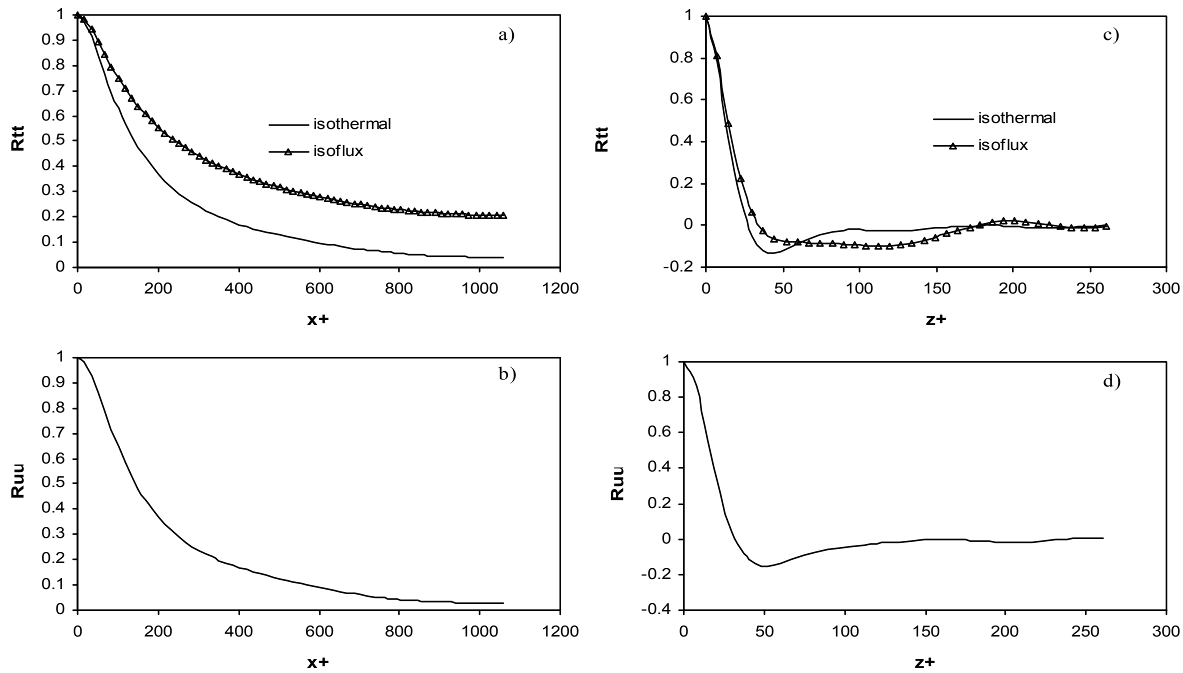

Figure 5: Two-point auto-correlation in streamwise (a, b) and spanwise direction (c, d) at $y^+=3.7$: a, c) temperature (isothermal and isoflux); b, d) streamwise velocity.

The physical background of the differences between the auto-correlation functions (Fig. 5a) at different thermal boundary conditions is not completely clear yet. We attempted to get a clearer physical picture with a new DNS study, which has been performed with roughly two times longer computational domain. The preliminary conclusions are:

1) The differences between the DNS in the "standard" length computational domain (approx. 2300 wall units) and the DNS in the extended computational domain (approx. 4600 wall units) did not show any differences larger than the statistical uncertainty for first-order statistics and also for the power spectra.

2) The periodicity length, which is long enough for the velocity field as an origin of the turbulence (Fig. 5b), is long enough also for the passive scalar fields, despite the behavior of the streamwise two-point correlation at isoflux BC.

Figures 5c and 5d show the spanwise two-point correlations for temperatures at two different BCs and for streamwise velocity. The appearance of the minimum in the spanwise two-point correlation shows the presence of the so-called “coherent structures” in the turbulent flow near the wall. These structures are the streamwise vortices that can be measured with the position of the first minimum. This minimum shows the average distance between the high speed and low speed regions. The vortices push the fast fluid from the turbulent sublayer towards the wall, and on the other side the slower fluid from the wall, which results in appearance of high-speed regions and low speed streaks. The position of the minimum in the streamwise velocity correlation (Fig. 5d) at approximately 50 wall units shows the presence of the low speed streaks with a typical spanwise distance of about 100 wall units between them. Minimums in Fig. 5c (two-point correlation of thermal fields) show that the positions of the minimums are slightly shifted comparing to the velocity auto-correlation minimum in Fig. 5d. In other words, the high temperature streaks do not exactly coincide with the low speed streaks. Fig. 5c also show that the thermal boundary condition affects the high temperature streaks (high temperatures appear in low speed streaks), when they are measured relatively close to the wall ($y^+ = 3.7$).

4 CONCLUSIONS

A direct numerical simulation of the fully developed flow in the flume has been performed at low Reynolds number Re = 2670, and Prandtl number Pr = 5.4 (water), considering the fluid temperature as a passive scalar.

Wall-normal profiles of velocity, temperatures, velocity fluctuations, and temperature fluctuations were shown. Two different thermal boundary conditions at the heated wall were used and it was shown that the mean temperature profile does not depend on the type of boundary condition. The main difference in the first order statistics was seen in temperature fluctuations near the wall. Temperature fluctuations retained a nonzero value on the wall for isoflux boundary condition, and zero for isothermal wall boundary condition.

Streamwise auto-correlation functions are used to verify if the length of the computational domain is long enough. If the auto-correlation function decays to zero, the box is long enough; otherwise it should be extended because of the periodic boundary conditions. On the other side, spanwise auto-correlation functions are used also to shows the presence of the “coherent structures” of the near wall flow. These structures are streamwise vortices with the characteristic diameter of approximately 100 wall units.

NOMENCLATURE

h	channel half height
k	wave number
L_1, L_3	streamwise and spanwise length of turbulent box
p	pressure
Pr	Prandtl number
q_w	wall-to-fluid heat flux
Re_τ	friction Reynolds number
Rtt	auto-correlation function for temperature
Ruu	auto-correlation function for streamwise velocity
t	time
$T_\tau = q_w/(u_\tau \rho_f c_{pf})$	friction temperature
u, v, w	velocity components in x, y and z directions
$u_\tau = \sqrt{\tau_w / \rho}$	friction velocity
u_B	bulk mean velocity
x	streamwise distance
y	distance from the wall
z	spanwise distance
Greek	
$\alpha = \lambda / \rho c_p$	thermal diffusivity
$\vartheta = (T_w - T)/T_\tau$	dimensionless temperature difference

λ	thermal conductivity
ν	kinematic viscosity
ρ	density
$\vec{1}_x$	unit vector in x direction (1,0,0)
Subscripts	
$(\,)_f$	fluid
Superscripts	
$(\,)^+$	normalized by u_τ, T_τ, ν

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