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Derivation of Simplified Basic Equations of Gas-Liquid Two-Phase Dispersed Flow Based on Two-Fluid Model

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ABSTRACT

The simplified and physically reasonable basic equations for the gas-liquid dispersed flow were developed based on some appropriate assumptions and the treatment of dispersed phase as isothermal rigid particles. Based on the local instant formulation of mass, momentum and energy conservation of the dispersed flow, time-averaged equations were obtained assuming that physical quantities in the dispersed phase are uniform. These assumptions are approximately valid when phase change rate and/or chemical reaction rate are not so large at gas-liquid interface and there is no heat generation in within the dispersed phase. Detailed discussions were made on the characteristics of obtained basic equations and physical meanings of terms consisting the basic equations. It is shown that, in the derived averaged momentum equation, the terms of pressure gradient and viscous momentum diffusion do not appear and, in the energy equation, the term of molecular thermal diffusion heat flux does not appear. These characteristics of the derived equations were shown to be very consistent concerning the physical interpretation of the gas-liquid dispersed flow. Furthermore, the obtained basic equations are consistent with experiments for the dispersed flow where most of averaged physical quantities are obtained assuming that the distributions of those are uniform within the dispersed phase.

Investigation was made on the problem whether the obtained basic equations are well-posed or ill-posed for the initial value problem. The eigenvalues of the simplified mass and momentum equations are calculated for basic equations obtained here and previous two-fluid basic equations with one pressure model. Well-posedness and ill-posedness are judged whether the eigenvalues are real or imaginary. The result indicated the newly developed basic equations always constitute the well-posed initial value problem while the previous two-fluid basic equations based on one pressure model constitutes ill-posed initial value problem except velocities of both phases being equal.

1.INTRODUCTION

In order to accurately analyze the thermohydrodynamic phenomena in gas-liquid two-phase flow, it is quite important to formulate rigorous basic equations for mass, momentum and energy conservations in gas-liquid two-phase flow. In recent years, such demands for the rigorous basic equations increase due to the remarkable developments of the numerical analyses using high speed computers Usually, in the analyses of gas-liquid two-phase flow, we need information of average two-phase flow behaviors. Therefore, what we need are the basic equations which are averaged over appropriate time and space domains.

There have been several researches on averaging methods for gas-liquid two-phase flow, and rigorous and reliable basic equations have been already obtained [1-3]. Presently, the basic equations based on the two-fluid model are most often used in the analyses of gas-liquid two-phase flow. This model treats gas and liquid phase separately and basic equations for each phase are formulated. Using this two-fluid model, it has become possible to analyze steady and transient behaviors of gas-liquid two-phase flow with certain accuracy.

The previous basic equations based two-fluid model are ,of course, are very rigorous in their original form, and can be applied to gas-liquid two-phase flow in any flow regime. However, when these basic equations are applied to dispersed flow such as bubbly flow and droplet flow, they are too rigorous to interpret the physical meaning of certain terms in the basic equations.

For example, in the momentum equation based on two-fluid model, the averaged pressure gradient terms appear in both phases. However, for dispersed phase such as bubbles and droplets, it is difficult to understand the physical meaning of averaged pressure gradient (of course its definition is possible) and body force acting on the bubbles and droplets due to the averaged pressure gradient.

Moreover, the averaged physical quantities (averaged velocity, averaged density etc.) for dispersed phase which appear in the previous basic equations based on two-fluid model, are averaged values considering the distribution within the bubble and droplet. However, it is possible to assume the uniformity of physical quantities (temperature, pressure, velocity) within dispersed phased for the dispersed flow in practical applications where phase change rate and chemical reaction rate are not so large at interface and there are no internal heat generation within the dispersed phase. In addition to this, in the experiments for velocity measurement of dispersed flow using electrical resistivity probe, anemometer and LDV, we usually measure the velocity at surface of bubble and droplet and obtain averaged velocity assuming that velocity within bubble and droplet is equal to the velocity at interface. Therefore, basic equations of dispersed phase we need in the practical analyses of gas-liquid dispersed flow are those averaged over dispersed phase assuming that bubble and droplet can be treated as a particle with constant distribution of physical quantities within it.

There are other problems in basic equations of gas-liquid two-phase flow based on the two-fuid model which are presently used with appropriate approximations and constitutive equations such as the problems of ill-posedness, difficulties in application to multidimensional analyses. In view of above mentioned discussions, in the present paper, more practical and simplified basic equations for gas-liquid dispersed flow were derived based on

the two-fluid model assuming that bubble and droplet can be treated as a particle with uniform physical quantities within it. Furthermore, investigation was made on the problem whether the obtained basic equations are well-posed or ill-posed as an initial value problem.

Finally, one of the application of present study is mentioned here. Recently, more detailed analyses of two-phase flow are required for the safety and performance of various industrial equipments. For this purpose, more accurate constitutive equations become needed. Along with experiments, numerical simulation for bubbles and droplets behavior becomes very useful method for obtaining such constitutive equations. Numerical simulation of each droplet or bubble in Lagrangian scheme become possible due to the progresses in numerical method and computer capacity. However, in the previous basic equations based on two-fluid model, dispersed phase is also treated as continuous fluid. Therefore, it is not easy to apply the constitutive equations for dispersed phase obtained by Lagrangian analyses to the two-fluid model basic equations. The present study is a part of such project for more detailed analyses of gas-liquid two-phase flow in corporation with Lagrangian simulation of bubbles and droplets.

2.DERIVATION OF AVERAGED BASIC EQUATIONS BASED ON TWO-FLUID MODEL

Here, averaged basic equations of mass, momentum and energy conservations for dispersed phase are considered. Averaged basic equations for continuous phase (liquid phase in bubbly flow, gas phase in droplet flow) are same as those obtained previously [1-3]. Averaged basic equations of gas-liquid two-phase flow are obtained by averaging local instant basic equations [2] which are given by
(mass)

$$\phi_d\{\frac{\partial \rho_d}{\partial t}+\mathrm{div}\,(\rho_d v_d)\}=0$$

(1)

(momentum)

$$\phi_d\{\frac{\partial}{\partial t}(\rho_d v_d)+\mathrm{div}\,(\rho_d v_d v_d)\}$$
$$=\phi_d(-\mathrm{grad}\,P_d+\mathrm{div}\,\tau_d+\rho_d g)$$

(2)

(energy)

$$\phi_d[\frac{\partial}{\partial t}\{\rho_d(E_d+\frac{1}{2}v_d^2)\}$$
$$+\mathrm{div}\,\{\rho_d(E_d+\frac{1}{2}v_d^2)v_d)\}]$$
$$=\phi_d\{-\mathrm{div}\,q_d-\mathrm{div}\,(P_d v_d)$$
$$+\mathrm{div}\,(\tau_d\cdot v_d)+\rho_d g\cdot v_d+Q_d\}$$

(3)

Here, suffix d denotes dispersed phases and ϕ_d is the characteristic function of dispersed phase which gives unity where dispersed phase exists and otherwise gives zero [2].

Averaging Eqs.(1) through (3) for the time interval T, we obtain time averaged basic equations for dispersed phase. In averaging, we assume that physical quantities are uniform within each dispersed phase (within each bubble and droplet). This can be given by following relations.

$$\mathrm{div}\,(\rho_d v_d)=0$$

(4)

$$\mathrm{div}\,(\rho_d v_d v_d)=0$$

(5)

$$\mathrm{div}\,\tau_d=0$$

(6)

$$\mathrm{div}\,\{\rho_d(E_d+(1/2)v_d^2)v_d\}=0$$

(7)

$$\mathrm{div}\,(P_d v_d)=0$$

(8)

$$\mathrm{div}\,(\tau_d\cdot v_d)=0$$

(9)

These assumptions are considered to be valid for gas-liquid dispersed flow where phase change rate and chemical reaction rate are not so large at interface and there are no internal heat generation within the dispersed phase. For pressure field within the dispersed phase, we consider the force at the gas-liquid interface and assume uniform pressure within each dispersed phase. This is expressed by the following equation (see Appendix).

$$-\mathrm{grad}\,P_d=-P_{di}\,n_{di}a_i$$

(10)

Similarly, for heat flux field within the dispersed phase, we consider the heat flux at gas-liquid interface and assume uniform heat flux within each dispersed phase which is given by

$$-\mathrm{div}\,(q_d)=-q_{di}\cdot n_{di}a_i$$

(11)

Here, a_i is local instant interfacial area concentration [2] and n_{di} is outward unit normal vector at interface. Suffix i denotes the value at gas-liquid interface.

Averaging Eqs.(1) through (3) for time interval T using above mentioned relations (Eqs.(4) through (11)) and averaging method for differential terms including gas-liquid interfaces within time interval T [2], we obtain time averaged basic equations for dispersed phase for practical use. We denote time averaging for time interval T by — hereafter. Furthermore, we define phasic averaging of each phase which is given by

$$\overline{\overline{A_k}}=\overline{\phi_k A_k}/\alpha_k \qquad \text{(Phasic averaging of phase)}$$

(12)

Here, α_k is volumetric fraction of each phase given by

$$\alpha_k=\overline{\phi_k}$$

(13)

All these relations mentioned above, we finally obtain the time averaged basic equations for dispersed phase which are given by
(mass)

$$\frac{\partial}{\partial t}(\alpha_d\overline{\overline{\rho_d}})+\mathrm{div}\,(\alpha_d\overline{\overline{\rho_d}}v_{dm})$$
$$=\frac{1}{T}\sum_{j=1}^{N}\frac{\rho_{dij}n_{dij}\cdot(v_{ij}-v_{dij})}{|n_{dij}\cdot v_{ij}|}$$

(14)

(momentum)

$$\frac{\partial}{\partial t}(\alpha_d\overline{\overline{\rho_d}}v_{dm})+\mathrm{div}\,(\alpha_d\overline{\overline{\rho_d}}v_{dm}v_{dm})$$
$$=-\mathrm{div}\,(\alpha_d\overline{\overline{\rho_d v'_{dm}v'_{dm}}})$$
$$+\alpha_d\overline{\overline{\rho_d}}g-\frac{1}{T}\sum_{j=1}^{N}\frac{P_{dij}n_{dij}}{|n_{dij}\cdot v_{ij}|}$$
$$+\frac{1}{T}\sum_{j=1}^{N}\frac{\rho_{dij}n_{dij}\cdot(v_{ij}-v_{dij})v_{dij}}{|n_{dij}\cdot v_{ij}|}$$

(15)

(energy)

$$\frac{\partial}{\partial t}(\alpha_d\overline{\overline{\rho_d}}E_{dm})+\mathrm{div}\,(\alpha_d\overline{\overline{\rho_d}}E_{dm}v_{dm})$$
$$=-\mathrm{div}\,(\alpha_d\overline{\overline{\rho_d E'_{dm}v'_{dm}}})$$
$$-\frac{1}{T}\sum_{j=1}^{N}\frac{q_{dij}\cdot n_{dij}}{|n_{dij}\cdot v_{ij}|}$$
$$+\frac{1}{T}\sum_{j=1}^{N}\frac{\rho_{dij}n_{dij}\cdot(v_{ij}-v_{dij})E_{dij}}{|n_{dij}\cdot v_{ij}|}$$
$$+\alpha_d\overline{\overline{Q_d}}$$

(16)

Here, N denotes the number of interfaces passing a measuring point

within time interval T and suffix j denotes j th interface. v_i is velocity of interface. Mass averaged quantities and fluctuating quantities are defined by

$$v_{dm} = \overline{\overline{\rho_d v_d}} / \overline{\rho_d} \tag{17}$$

$$\overline{\overline{\rho_d v_d' v_d'}} = \overline{\overline{\rho_d (v_d - v_{dm})(v_d - v_{dm})}} \tag{18}$$

$$E_{dm} = \overline{\overline{\rho_d E_d}} / \overline{\rho_d} \tag{19}$$

$$\overline{\overline{\rho_d E_d' v_d'}} = \overline{\overline{\rho_d (E_d - E_{dm})(v_d - v_{dm})}} \tag{20}$$

In obtaining Eq.(16), we assumed that kinetic energy is negligible compared with thermal energy.

The right hand side of Eq.(14) corresponds to interfacial mass transfer term (phase change rate) which is rewritten by

$$\frac{1}{T} \sum_{j=1}^{N} \frac{\rho_{dij} n_{dij} \cdot (v_{ij} - v_{dij})}{|n_{dij} \cdot v_{ij}|} = \dot{m}_d \overline{a_i} \tag{21}$$

where $\dot{m}_d$ is phase change rate of dispersed phase (evaporation, condensation) per unit interfacial area.   The third term in the right hand side of Eq.(15) represents interfacial momentum transport term due to the pressure distribution around bubble and droplet. More concretely, it corresponds to buoyancy force, drag force, transverse lift force (such as Magnus force etc,) and virtual mass force.   It can be rewritten by

$$-\frac{1}{T} \sum_{j=1}^{N} \frac{P_{dij} n_{dij}}{|n_{dij} \cdot v_{ij}|}$$

$$= -\alpha_d (\alpha_d \overline{\rho_d} + \alpha_c \overline{\rho_c}) g$$

$$-C_D (1/2) \overline{\overline{\rho_c}} (v_{cm} - v_{dm})^2 \overline{a_i}$$

$$-C_T \overline{\overline{\rho_c}} (v_{cm} - v_{dm}) \text{grad } v_{cm}$$

$$+C_{VM} \overline{\overline{\rho_c}} \{ \frac{D_d}{Dt} (v_{dm} - v_{cm}) \} \tag{22}$$

The fourth term in the right hand side of Eq.(15) corresponds to the interfacial momentum transport due to phase change, that is

$$\frac{1}{T} \sum_{j=1}^{N} \frac{\rho_{dij} n_{dij} \cdot (v_{ij} - v_{dij}) v_{dij}}{|n_{dij} \cdot v_{ij}|} = \dot{m}_d v_{di} \overline{a_i} \tag{23}$$

The second term in the right hand side of energy conservation equation (Eq.(16)) represents the interfacial energy transport term due to the temperature gradient at interface and given in the form of interfacial heat transfer term which is given by

$$-\frac{1}{T} \sum_{j=1}^{N} \frac{q_{dij} \cdot n_{dij}}{|n_{dij} \cdot v_{ij}|} = -h_i (T_{cm} - T_{dm}) \overline{a_i} \tag{24}$$

where h_i is interfacial heat transfer coefficient and T_{cm} and T_{dm} are averaged temperatures of continuous and dispersed phase respectively. The third term in the right hand side of Eq.(16) corresponds to interfacial energy transport term due to phase change which is rewritten by

$$\frac{1}{T} \sum_{j=1}^{N} \frac{\rho_{dij} n_{dij} \cdot (v_{ij} - v_{dij}) E_{dij}}{|n_{dij} \cdot v_{ij}|} = \dot{m}_d E_{di} \overline{a_i} \tag{25}$$

On the other hand, basic equations for continuous phase are given by following equations based on the previous researches [1-3].

$$\frac{\partial}{\partial t}(\alpha_c \overline{\rho_c}) + \text{div}(\alpha_c \overline{\rho_c} v_{cm}) = \dot{m}_c \overline{a_i} \tag{26}$$

$$\frac{\partial}{\partial t}(\alpha_c \overline{\rho_c} v_{cm}) + \text{div}(\alpha_c \overline{\rho_c} v_{cm} v_{cm})$$

$$= -\text{grad}(\alpha_c P) - \text{div}(\alpha_c \overline{\rho_c} \overline{v_{cm}' v_{cm}'})$$

$$+\text{div}(\alpha_c \overline{\overline{\tau_c}}) + \alpha_c \overline{\rho_c} g + \dot{m}_c v_{ci} \overline{a_i}$$

$$-C_D (1/2) \overline{\overline{\rho_c}} (v_{dm} - v_{cm})^2 \overline{a_i}$$

$$-C_T \overline{\rho_c} (v_{dm} - v_{cm}) \text{grad } v_{cm}$$

$$+C_{VM} \overline{\rho_c} \{ \frac{D_d}{Dt} (v_{cm} - v_{dm}) \} \tag{27}$$

$$\frac{\partial}{\partial t}(\alpha_c \overline{\rho_c} E_{cm}) + \text{div}(\alpha_c \overline{\rho_c} E_{cm} v_{cm})$$

$$= -\text{div}(\alpha_c \overline{\rho_c E_{cm}' v_{cm}'}) + \text{div}(\alpha_c \overline{\overline{q_c}})$$

$$+\dot{m}_c E_{ci} \overline{a_i} - h_i (T_{dm} - T_{cm}) \overline{a_i} + \alpha_d \overline{\overline{Q_d}} \tag{28}$$

3. CHARACTERISTICS OF BASIC EQUATIONS FOR DISPERSED PHASE

The basic equations of dispersed phase in gas-liquid two-phase flow based on simplified assumptions obtained in previous sections have some characteristics in the physical interpretation of their terms which are described below.

The averaged physical quantities of dispersed phase in the present equations are defined using the values at the interface due to the assumption of uniform distribution in each dispersed phase. For example, some averaged quantities are defined by following equations.

$$\alpha_d = \frac{1}{T} \sum_{j=1}^{N/2} (t_{2j} - t_{2j-1}) \tag{29}$$

$$\overline{\alpha_d \rho_d} = \frac{1}{T} \sum_{j=1}^{N/2} (t_{2j} - t_{2j-1}) \rho_{d,2j-1} \tag{30}$$

$$\overline{\alpha_d \rho_d v_{dm}} = \frac{1}{T} \sum_{j=1}^{N/2} (t_{2j} - t_{2j-1}) \rho_{di,2j-1} v_{di,2j-1} \tag{31}$$

$$\overline{\alpha_d \rho_d E_{dm}} = \frac{1}{T} \sum_{j=1}^{N/2} (t_{2j} - t_{2j-1}) \rho_{di,2j-1} E_{di,2j-1} \tag{32}$$

Here, t_{2j-1} and t_{2j} are the times when the j th dispersed phase arrives and departs a measuring point, respectively as shown in Fig.1. For example, speaking of velocity, the averaged velocity defined Eq.(32) is given by only the interfacial velocities when the bubbles and droplets passes the measuring probe.   This averaged value well corresponds to the actual velocity measurements in the experiments. In the practical measurements of velocity of dispersed phase, we usually use electrical resisitvity probe, anemometer, and laser doppler anemometer.   What these probe ususally measure for dispersed phase is interfacial velocity of dispersed phase and velocity wihtin the dispersed phase is assumed to be equal to the interfacial velocity.   Therefore, the measured averaged velocity (and other properties) of dispersed phase is well represented by Eqs.(29) through (32).   Therefore, when we compare the experiment with analysis in dispersed gas-liquid two-phase flow in detail, the present basic equations are much more appropriate in regard to consistency between measured and predicted values.

Fig.1 Situation of Passing of Interface through Measuring Point

In the momentum equation of dispersed phase, Eq.(15), the averaged pressure gradient term does not appear which is one of the most characteristic difference from previous equations. Although dispersed phase (bubble and droplet) consists of continuous fluid, each dispersed phase (bubble and droplet) does not connected each other. Therefore, it is difficult to interpret the force exerted on dispersed phase due to the averaged pressure gradient of dispersed phase unlike continuous phase. Of course, there exists an interaction between dispersed phases. However, it is considered that dispersed phases interact each other not directly but through the continuous phase. In gas-liquid two-phase flow where the particle concentration is not so high, it is reported that the similar basic equations to the present ones was obtained in which pressure gradient term does not appear in the momentum equation of dispersed phase (solid particle phase) [4,5].

Such feature of momentum equation of dispersed phase (no averaged pressure gradient term) makes averaged pressure modeling in two-fluid model much easier. In the previous two-fluid model where dispersed and continuous phases are treated as continuous fluid, there appear averaged pressure gradient terms in momentum equations of both phases. Therefore, in order to get the closure relation, we need modeling of the relation between averaged pressures of both phases. Usually, the averaged pressures are assumed to be equal. However, this assumption causes the ill-posed problem which is still serious and unresolved problem in two-fluid model [6]. On the other hand, in the present equations, the averaged pressure gradient term does not appear in the dispersed phase momentum equation but appears only in the continuous phase equation. Therefore, there needs no modeling for the averaged pressures of both phases. Furthermore, as shown in the following section, the present equations do not cause ill-posed problem unlike the previous equations.

Another characteristic feature of momentum equation of dispersed phase is that there are no divergence term of viscous stress unlike the continuous phase. Viscous stress is proportional to the velocity gradient in Newtonian fluid. This force is reasonable in continuous phase. However, in the dispersed phase, it is difficult to interpret physically the averaged viscous stress in the dispersed phase since each dispersed phase (bubble and droplet) does not connected each other. Therefore, this feature is considered to be reasonable for dispersed phase.

Similar discussion is possible regarding to molecular heat diffusion term in the energy equation of dispersed phase, Eq.(16). In the equation, there are no divergence term of heat flux due to the molecular heat diffusion. In the continuous phase, it is natural to consider the heat flux due to the molecular heat diffusion proportional to the averaged temperature gradient. However, in the dispersed phase, each dispersed phase is not connected each other. Therefore, it is quite difficult to consider physically the heat flux due to the molecular heat diffusion from one dispersed phase to

another dispersed phase. Of course, there are some energy transfer between dispersed phases. However, it is not transferred from one dispersed phase to another dispersed phase directly but it is transferred through continuous phase. In this regards, the present equation of dispersed phase energy conservation is reasonable in physical interpretation of molecular heat diffusion term.

As mentioned in Introduction, the present basic equations can be applied to the more detailed analyses of dispersed gas-liquid two-phase flow in corperation with numerical simulation of bubble or droplet in Lagrangean scheme. Averaging method in the present basic equations is based on the assumption that bubble and droplet are taken as a particle. Therefore, the present equations are consistent with the averaged basic equations in Lagrangian analyses. As an example, we will show in below that for droplet flow, averaged momentum equation in Lagrangian simulation of droplets is identical to the modmetum equation in the present basic equation

In Lagrangian simulation of droplet, the eqaution of motion for each droplet is given by

$$\rho_d V_d \frac{d v_d}{d t} = -C_D{}' \frac{1}{2} \rho_c (v_d - v_c)^2 A_d + g \rho_d V_d \qquad (33)$$

where, V_d and A_d are volume and projected area of each droplet and $C_D{}'$ is drag coefficient of single droplet. Here, for simplicity, the main interacting force is assumed to be the drag force acting on the droplet.

Now, we consider averaging Eq.(33) in appropriate control volume containing N droplet. The droplet population balance in the control volume is given by

$$\frac{\partial N}{\partial t} + d i v (N v_d) = 0 \qquad (34)$$

Averaging Eq.(33) for N droplets in the control volume under the condition of Eq.(34), we approximately obtain

$$\frac{\partial}{\partial t} (\alpha_d \overline{\rho}_d \overline{v}_{dm}) + d i v (\alpha_d \overline{\rho}_d \overline{v_{dm} v_{dm}}) = -d i v (\alpha_d \overline{\rho_d v_{dm}' v_{dm}'})$$

$$- C_D{}' K \frac{1}{2} \overline{\rho_c} (v_d - v_c)^2 \overline{a_i} + \alpha_d \overline{\rho}_d g$$

$$(35)$$

Here, we used the relations for void fraction and interfacial area which are given by

$$\alpha_d = N V_d \qquad (36)$$
$$a_i = K N A_d \qquad (37)$$

where K is the coefficient related to the ratio between surface area and projected area of droplet.

Comparing Eq.(35) with Eq.(15), the momentum equation of present basic equation is consistent with averaged momentum eqaution in Lagrangian simulation of each droplet, in particular, with respect that there are no gradient term of averaged pressure of dispersed phase in both equations.

As discussed above, the present basic equations of dispersed phase are very reasonable and consistent in physical interpretation, although they are derived based on several simplifications and assumptions.

4. WELL-POSEDNESS OF BASIC EQUATIONS

As already discussed in the previous section, previous basic equations based on two-fluid one-pressure model are ill-posed as a

initial value problem when we assume averaged pressures of both phases are equal [5,6]. On the other hand, in the present basic equations, the averaged pressure gradient term does not appear in dispersed phase momentum equation. The detailed discussion about the well-posedness of various two-phase flow systems (gas-liquid, gas-solid and liquid-solid flows) was carried out [5]. The result showed that the basic equations of gas-solid flow [4,5] which is very similar to the present basic equations are always well-posed [5]. Therefore, the problem of ill-posedness is expected to be avoidable in the present basic equations. In order to confirm this, the mathematical features of previous and present basic equations are analyzed in this section.

The problem of ill-posedness is mainly related to the averaged pressure gradient term. Therefore, when we analyze this problem, it is sufficient to treat the conservation equations of mass and momentum for both phase. Furthermore, the ill-posed problem of the equation without diffusion term is related only to the differential terms in the basic equations. In view of these, here, we treat the basic equations of mass, and momentum equations, Eqs.(14), (15), (26) and (27) neglecting nondifferential terms in these equations. Moreover, for simplicity, we consider the one dimensional case (x direction as flow direction) with the assumption of both phases being incompressible. The basic equations under these assumptions are given by

(mass)
$$\frac{\partial \alpha_d}{\partial t} + v_d \frac{\partial \alpha_d}{\partial x} + \alpha_d \frac{\partial v_d}{\partial x} = 0 \tag{38}$$

$$\frac{\partial \alpha_c}{\partial t} + v_c \frac{\partial \alpha_c}{\partial x} + \alpha_c \frac{\partial v_c}{\partial x} = 0 \tag{39}$$

(momentum)
$$\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} = -\frac{\xi}{\rho_d} \frac{\partial P}{\partial x} \tag{40}$$

$$\frac{\partial v_c}{\partial t} + v_c \frac{\partial v_c}{\partial x} = -\frac{1}{\rho_c} \frac{\partial P}{\partial x} \tag{41}$$

When we put $\xi=0$ in eq.(41), Eqs.(38) through (41) correspond to the present basic equations. On the other hand, when we put $\xi=1$ in eq.(41), Eqs.(38) through (41) correspond to the previous basic equations. Considering

$$\alpha_d + \alpha_c = 1 \tag{42}$$

Eqs.(38) through (41) can be rewritten in term of $f=(\alpha_d, P, v_d, v_c)^{\mathsf{T}}$ by

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \frac{\partial f}{\partial t} + \begin{pmatrix} v_d & 0 & \alpha_d & 0 \\ -v_c & 0 & 0 & \alpha_c \\ 0 & \xi/\rho_d & v_d & 0 \\ 0 & 1/\rho_c & 0 & v_c \end{pmatrix} \frac{\partial f}{\partial x} = 0 \tag{43}$$

When we denote the first matrix by A and the second by B, Eq.(43) can be given by

$$A \frac{\partial f}{\partial t} + B \frac{\partial f}{\partial x} = 0 \tag{44}$$

Equations (38) through (41) become hyperbolic or well-posed when the eigenvalue of Eq.(44) which is denoted by λ is real and become elliptic or ill-posed when λ is imaginary. λ is given as a root of equation given by

$$\det(A\lambda + B) = 0 \tag{45}$$

Here, it should be noted that the matrix A in Eq.(44) is a singular matrix and therefore the characteristic equation Eq.(45) is reduced to quadratic equation in regard to λ. This equation is expressed more concretely by

$$\rho_d \alpha_c (\lambda + v_d)^2 + \xi \rho_c \alpha_d (\lambda + v_c)^2 = 0 \tag{46}$$

In the previous basic equations based on one pressure two-fluid model, the discriminant for Eq.(46) is given by following equation putting $\xi=1$ in Eq.(46).

$$D = -\rho_d \rho_c \alpha_d \alpha_c (v_d - v_c)^2 \tag{47}$$

This equation indicates that the eigenvalue λ becomes real only when

$$v_d = v_c \tag{48}$$

Therefore, previous basic equations based on one pressure two-fluid model are ill-posed for ordinary two-phase flow where velocities of both phases are different.

On the other hand, in the basic equations for dispersed flow derived here, putting $\xi=0$ in Eq.(46) we obtain

$$\rho_d \alpha_c (\lambda + v_d)^2 = 0 \tag{49}$$

this equation gives the eigenvalue by

$$\lambda = -v_d \tag{50}$$

As shown in this equation, the eigenvalue for the present basic equations is always real which means the basic equations are always well-posed as an initial value problem.

As shown in above discussions, the present basic equations where the dispersed phase is treated as a particle constitute mathematically reasonable equation system and considered to be more appropriate for the analyses of dispersed flow such as bubbly and droplet flow compared with previous basic equations based on one pressure two-fluid model.

5. CONCLUSIONS

Averaged basic equations of mass, momentum and energy conservations were derived for gas-liquid dispersed two-phase flow based on the simplified and reasonable assumptions. The obtained equations are shown to have the physically and mathematically reasonable characteristics.

Starting from the local instant basic equations for gas-liquid two phase flow, the time averaged basic equations for dispersed phase are obtained based on the assumption that the physical quantities within each dispersed phase are uniform and each dispersed phase is treated as a particle. In the obtained basic equations, the averaged pressure gradient term, the averaged viscous stress term, and averaged heat flux term due to molecular diffusion do not appear. The averaged physical quantities are shown to be given by the value at interface. These results are very reasonable in physical interpretation of each term and in relating each averaged term to the practical measurements in experiments.

The mathematical characteristic of obtained basic equations are examined and the result shows that the obtained basic equations are shown to have always real eigenvalue and well-posedness as an initial value problem whereas previous basic equations based on one pressure two-fluid model usually shows ill-posedness.

The present basic equations are expected to be more useful and reasonable in analyzing gas-liquid dispersed two-phase flow along with the appropriate constitutive equations for dispersed phase.

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Appendix: The derivation and validity of Eq.(10)

Mathematically, equation (10) is written in more rigorous form by

$$-\phi_d \ \text{grad} \ P_d = -P_{di} n_{di} a_i \tag{A-1}$$

The right hand side of Eq.(A-1) is given by

$$-\phi_d \ \text{grad} \ P_d = -\text{grad} \ \phi_d P_d + P_d \ \text{grad} \ \phi_d \tag{A-2}$$

The assumption of uniform pressure within dispersed phase gives

$$\text{grad} \ \phi_d P_d = 0 \tag{A-3}$$

Due the the mathematical feature of the gradient of characteristic function $\phi_d{}^2$, one obtains

$$P_d \ \text{grad} \ \phi_d = P_{di} n_{di} a_i \tag{A-4}$$

From Eqs. (A-2) through (A-4), we obtain Eq.(A-1) which is rigorous form of Eq.(10).

For the pressure field, the assumption of Eq.(A-3) may not be rigorously valid because pressure field must be continuous at the interface when the surface tension force is negligible. Physically, there is a boundary layer near interface where pressure gradient is very large. Here, we assume that the thickness of this boundary layer is very thin and pressure gradient is represented by Eq.(A-4) (delta function included in a_i). For small droplet and bubble, this assumption is considered to be approximately valid.