



THE EFFECT OF THERMAL CONDUCTANCE OF VERTICAL WALLS ON NATURAL CONVECTION
 IN A RECTANGULAR ENCLOSURE

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ABSTRACT

This paper deals with the experimental results of natural convective heat transfer in a rectangular water layer bounded by vertical walls of different thermal conductance. The vertical walls were made of copper or stainless steel. A minimum was observed in the horizontal distribution of temperature near the heating wall since a secondary reverse flow occurred outside the boundary layer. For copper case the experimental results of Nusselt number agreed well with calculations under an isothermal wall condition. For stainless steel case, however, the measured values were lower than the calculations since a three-dimensional effect appeared in convection due to non-uniformity in wall temperature.

INTRODUCTION

There are many plenums and cavities in the coolant system of nuclear reactors. Flow is stagnant in such enclosures and natural convection occurs as a result of gradients in density which are due to variations in temperature. The design of coolant systems therefore requires a knowledge of natural convection as well as forced convection. In particular, attention has been recently focused on this convective process owing to increasing emphasis on the safety of nuclear reactors. The accurate knowledge of natural convection is important for analyzing the decay heat removal after the shutdown of nuclear reactors.

A number of review papers have appeared during the past decade or so. Ostrach¹ and Catton² provide a comprehensive and interpretive review of natural convection in enclosures. Batchelor³ is one of the earliest investigators who analyzed the behavior for a channel made up of vertical walls at different uniform temperatures and insulated horizontal end plates. He showed that the flow pattern changes with increasing Rayleigh number Ra. Eckert and Carlson⁴, Mordchelles-Regnier and Kaplan⁵, and Edler^{6,7} confirmed the flow pattern predictions and showed that there are three regimes: (1)

conduction regime, (2) transition regime and (3) boundary layer regime. In the conduction regime a temperature gradient is observed in the center portion. In the boundary layer regime, however, a flat temperature distribution is dominant in the central core.

Elder^{6,7} carried out a very comprehensive work on natural convection in a vertical cavity with aspect ratio ranging from 1 to 60 and showed that a laminar flow condition is maintained up to $Ra=10^7$. MacGreger and Emery⁸ obtained an empirical correlation of heat transfer coefficient and predicted that a transition to turbulent motion occurs in the range of $3 \times 10^6 < Ra < 3 \times 10^7$. An analysis of the boundary layer regime by Raithby et al.⁹ presents that the flow pattern and heat transfer depend strongly on the aspect ratio as well as on the Rayleigh and Prandtl numbers. Takeuchi et al.¹⁰ studied natural convection in thermal stratification where a vertical gradient exists in the temperature of the fluid in the core of the vertical cavity.

These experimental results for natural convection are generally less accurate than for forced convection owing to difficulty in evaluating the heat fluxes through and along the nonisothermal walls. As a consequence, discrepancies between sets of data are observed,

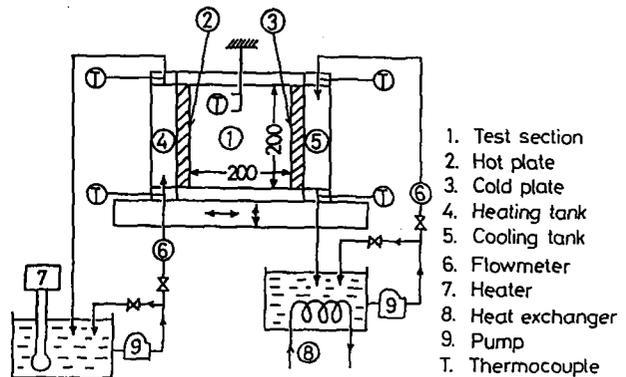


Fig.1 Experimental apparatus

especially for liquid coolant in which the thermal conduction of walls affects natural convection in the cavity. So far, there exist a few works¹¹⁻¹⁴ which treat combined conductive and convective heat transfer. This led to the present authors to carry out an experimental study of natural convection in a rectangular enclosure with vertical walls of different thermal conductance. The experimental results will be compared with some numerical calculations by a conventional model which treats natural convection under the boundary conditions of uniform temperature or uniform heat flux.

EXPERIMENTAL APPARATUS AND PROCEDURES

A schematic diagram of the experimental apparatus is shown in Fig. 1. The overall dimensions of the test section (fluid layer) are height, $H=200$ mm and length, $L=200$ mm (see Fig.2). The fluid layer of water was bounded by two vertical metallic (copper or stainless steel) plates of 200 mm high, 200 mm wide and 10 mm thick. Other side and horizontal walls are made of acrylic resin to keep in an adiabatic condition. The temperature of hot/cold wall was held constant with thermostatically controlled water circulating through a heating/cooling tank, which was attached to the back of the heat transfer wall. The heating/cooling rate was obtained from the enthalpy difference between the inlet and outlet of the heating/cooling tank.

Eighteen thermocouples were used to measure the temperatures of metallic plates. The thermocouple grooves in the plates were drilled to within 3 mm of the surface in contact with the test fluid. For temperature measurement of the fluid layer a small thermocouple (0.25 mm in diameter) was inserted into the layer through the upper boundary. The fluid velocity was measured using the laser-Doppler anemometer system.

The experiments were performed in the following manner. In order to maintain the side walls at prescribed temperatures, hot/cold water was circulated into the heating/cooling tank. The fluid layer was held nearly at room temperature in order to minimize heat losses. The output signals from thermocouples were recorded on a strip chart. Sufficient time (15 to 20 hours) was allowed for the convection development to reach a steady-state condition. Upon reaching a steady state all the temperature readings were taken by a multichannel digital voltmeter, which was controlled by a personal computer. Another set of temperature readings were taken 1 hour later, and when good agreement

* In order to investigate the effect of heating method on heat transfer characteristics, an electrical heater was also used in the present experiments.

was reached within 0.2 K for each set of temperature readings, data of temperature as well as velocity were recorded.

RESULTS AND DISCUSSION

A. Temperature and Velocity Distributions

Figure 3 shows vertical distributions of temperature in the fluid layer for two wall materials at nearly constant Rayleigh number. The abscissa is the temperature difference, $T-T_{bm}$ where T_{bm} is the mixed mean temperature. The ordinate is the dimensionless height, z/H .

It can be seen from the figure that for copper case an isothermal condition is nearly maintained through the heating and cooling surfaces which correspond to the vertical lines of $x/L=0$ and 1, respectively. The isothermal condition disappears in the core portion far from the wall and the temperature increases in the vertical direction. This temperature gradient becomes steeper for stainless steel case, especially in the lower portion of heating surface ($x/L=0$) and the upper portion of cooling surface ($x/L=1$). This is attributed to low thermal conductivity of stainless steel.

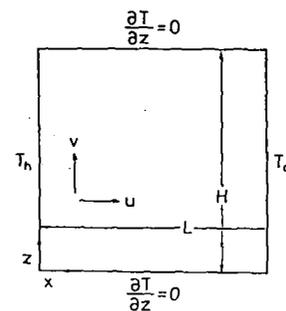


Fig.2 Coordinate system

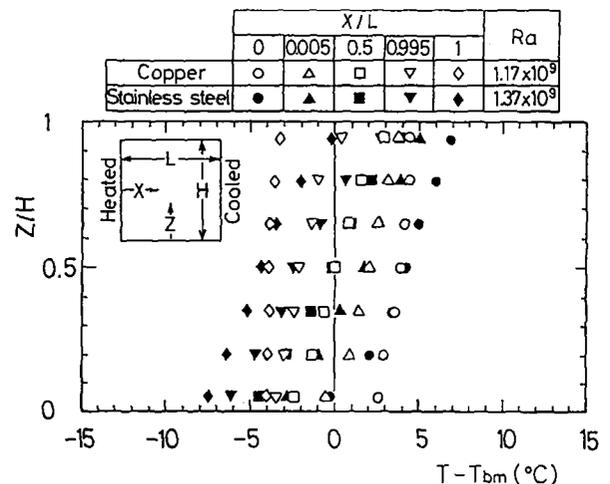


Fig.3 Vertical temperature distribution

Figure 4 shows vertical distributions of heat flux at the heating surface for various wall materials and Rayleigh numbers. The local heat flux was obtained from the relationship:

$$q_z = \lambda(dT/dx)_{x=0} \quad (1)$$

where the temperature gradient $(dT/dx)_{x=0}$ is assumed to be equal to the value measured near the wall at each height of the fluid layer. The uncertainty of heat flux is within 5%. For copper case the heat flux becomes extremely high in the lower region ($z/H < 0.2$). This tendency is more accentuated in higher Rayleigh number. For stainless steel case, however, a comparatively flat distribution of heat flux is maintained even in the lower z/H region.

The experimental results are compared with calculations by an analytical model, which treats two-dimensional natural convection in enclosures. A brief description of the analytical model is given in Appendix. Calculations were carried out for $Ra = 1.2 \times 10^8$ under isothermal wall condition. The calculated results agree fairly well with measured values for copper case. For stainless steel case, however, a discrepancy is observed between calculations and experimental data. Measured heat fluxes are extremely lower in the lower portion ($z/H < 0.1$) of the heating surface. This is attributed to non-uniformity in wall temperature.

Figure 5 shows detailed horizontal distributions of velocity and temperature which were measured near the heating surface for two wall materials: (a) copper and (b) stainless steel. For any cases the velocity has a peak at $x/L = 0.005$ in the boundary layer and decreases to be almost zero in the core region ($x/L > 0.05$) far from the surface. The peak velocity has the highest value at the horizontal line of $z/H = 0.5$. A secondary reverse flow is also observed at $x/L = 0.03$.

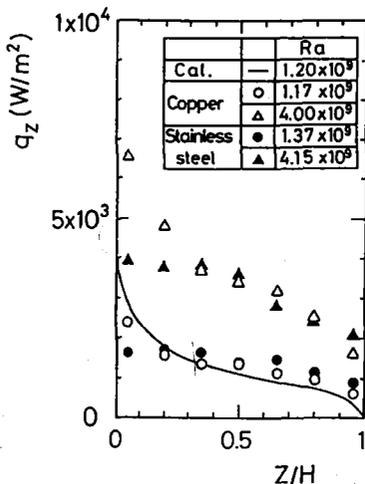


Fig.4 Vertical heat flux distribution

Similar distributions are observed in lower ($z/H = 0.125$) and higher ($z/H = 0.875$) regions for copper case. For stainless steel case, however, the velocity at $z/H = 0.125$ is lower than that at $z/H = 0.875$. This low velocity is consistent with low heat flux in the lower z/H region, as already indicated in Fig.4.

On the other hand, the temperature first tends to decrease sharply with increasing the horizontal distance (x/L), and after attaining a minimum near $x/L = 0.015$, increases somewhat to remain more or less constant in the core region. The minimum temperature is caused since the secondary hot fluid flows downward in the outer ($x/L = 0.03$) region although the main cold fluid flows upward near the wall, as already indicated in the figure of velocity distributions.

The experimental results for copper case are compared with analytical calculations. A fairly good agreement is observed in velocity distributions, but the calculated wall temperatures are different from the measured values, except at $z/H = 0.5$. This is attributed to the assumption of isothermal vertical walls in the calculation.

B. Heat Transfer

Figure 6 shows a relationship between local Nusselt number Nu_z and local Rayleigh number Ra_z at the heating z surface. The dimensionless numbers Nu_z and Ra_z are defined in terms of the local bulk temperature T_{bz} which was measured at each horizontal line in the core region. Although data are scattered slightly, the Nu_z increases with higher Ra_z for both copper and stainless steel cases. The measured values

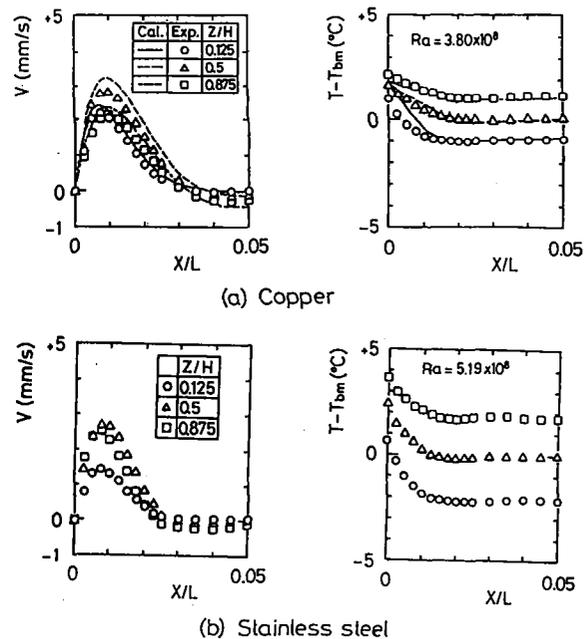


Fig.5 Velocity and temperature distributions

agrees fairly well with the correlation of Takeuchi et al.¹⁰ for natural convection from a vertical isothermal wall immersed in thermally stratified fluid. In the region of $Ra_z < 4 \times 10^6$ the Nu_z is lower for stainless steel than for copper since heat fluxes are lower for stainless steel in the lower portion ($z/H = 0.05$) of the heating surface, as already indicated in Fig. 3. In the region of $Ra_z > 10^6$ data scattering is enlarged due to the secondary reverse flow which affects strongly the local heat transfer.

Figure 7 shows a relationship between mean Nusselt number Nu_H and mean Rayleigh number Ra_H where the height H of the fluid layer is used as the reference length. In this figure are also shown data for electrical heating as well as for hot water heating. In the case of hot water heating there is a tendency of higher Nu_H for copper than for stainless steel. This discrepancy lessens in the case of electrical heating.

The experimental results are compared with analytical calculations as well as the correlation of Raithby et al.⁹, as

$$Nu_H = 0.354 Ra_H^{1/4} \quad (2)$$

in the range of $10^6 < Ra_H < 5 \times 10^9$ for isothermal walls. The present data, except for copper wall heated by hot water, are much lower than the calculated values as well as Raithby et al.'s correlation. An upgraded model is therefore needed for analyzing heat transfer on the wall of low thermal conductivity materials.

In conventional natural convection in the vertical fluid layer it has been customary that the Nusselt number is defined in terms of the length L , as

$$Nu = qL / \lambda \Delta T \quad (3)$$

The experimental results of Nu are brought together in Fig. 8. The abscissa is the Rayleigh number which is defined in terms of this same length, as

$$Ra = g\beta \Delta T L^3 / \alpha \nu \quad (4)$$

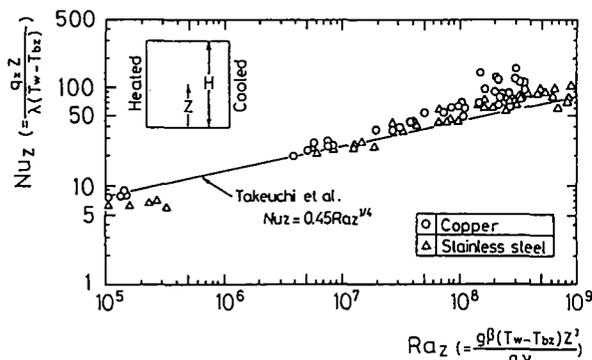


Fig.6 Relation between local Nusselt number and local Rayleigh number

The Nu is higher for copper than for stainless steel in the lower Ra region ($Ra < 5 \times 10^9$). This discrepancy tends to disappear in higher Ra region ($Ra > 10^{10}$).

The experimental results are also compared with analytical calculations. In the case of copper wall heated by hot water the measured values agree well with calculations under isothermal wall condition. In other cases, however, the experimental data are much lower than the calculated values and the discrepancy becomes larger for stainless steel since a three-dimensional effect appears in convection flow due to non-uniformity in wall temperature.

The solid line is the empirical correlation of MacGreger and Emery⁸, as

$$Nu = 0.046 Ra^{1/3} \quad (5)$$

They carried out an experimental study of natural convection in a fluid layer with the hot wall at constant heat flux and the cold wall at constant temperature. It can be seen that their correlation is lower than the copper data but higher than the stainless-steel data in the present experiments.

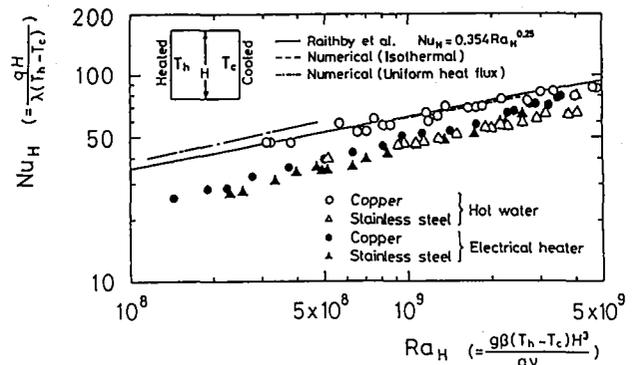


Fig.7 Relation between mean Nusselt number and Rayleigh number

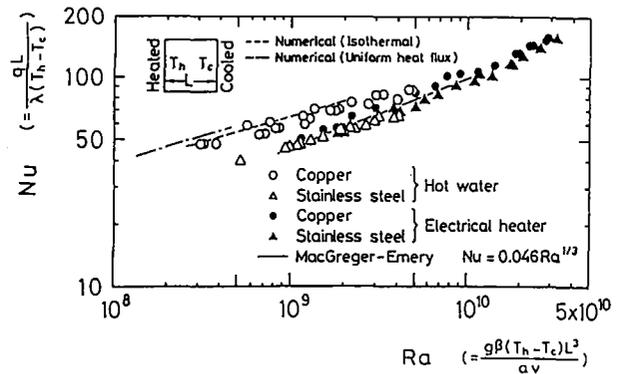


Fig.8 Relation between mean Nusselt number and Rayleigh number

CONCLUSIONS

An experimental study has been conducted to investigate the effect of thermal conductance of vertical walls on natural convection in a rectangular water layer. Copper and stainless steel were used as the wall material. Measurements were made of velocity and temperature distributions in the layer as well as the vertical surface temperatures. Comparison of the experimental results with analytical calculations yielded the following conclusions:

(1) A steep temperature gradient is observed in the boundary layer region near the wall. In the core region far from the wall, however, a flat distribution is dominant in horizontal direction.

(2) There is a minimum in the horizontal distribution of temperature in the boundary layer since the secondary hot inverse flow occurs outside of the main cold upflow.

(3) The experimental results of Nusselt number for copper case agree well with calculations under an isothermal wall condition. For stainless steel case, however, the measured values are lower than the calculations.

NOMENCLATURE

a : thermal diffusivity
c : specific heat
g : gravitational acceleration
H : height of fluid layer
h : enthalpy
L : distance between vertical walls
Nu : mean Nusselt number, $qL/\lambda\Delta T$
Nu_H : mean Nusselt number, $qH/\lambda\Delta T$
Nu_z : local Nusselt number, $\alpha_z z/\lambda$
p : pressure
q : mean heat flux
q_z : local heat flux
Ra_a : Rayleigh number, $g\beta L^3\Delta T/av$
Ra_H : Rayleigh number, $g\beta H^3\Delta T/av$
Ra_z : local Rayleigh number, $g\beta z^3(T_w - T_{bz})/av$
T : temperature
T_c : mean temperature of col surface
T_h : mean temperature of hot surface
T_h : local wall temperature
T_w : mixed mean temperature of fluid, $(T_h + T_c)/2$
T_{bm} : local bulk temperature of fluid
T_{bz} : surface-to-surface temperature difference, $T_h - T_c$
u : horizontal velocity
v : vertical velocity
x : horizontal coordinate, $0 < x < L$
z : vertical coordinate, $0 < z < H$

Greek symbols

α_z : local heat transfer coefficient, $q_z/(T_w - T_{bz})$
 β : isobaric coefficient of volumetric expansion
 λ : thermal conductivity
 μ : viscosity

ν : kinematic viscosity
 ρ : density

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APPENDIX. ANALITICAL MODEL

The geometry of the enclosure is shown in Fig.2. The enclosure has isothermal vertical walls, one hot (T_h) and one cold (T_c). The horizontal walls are insulated. Viscous dissipation and radiation effects are neglected. For steady, laminar 2-D flow the governing equations are:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial z}(\rho v) = 0 \quad (A1)$$

$$\begin{aligned} \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial z}(\rho v u) - \frac{\partial}{\partial x} \left(\mu \frac{\partial u}{\partial x} \right) - \frac{\partial}{\partial z} \left(\mu \frac{\partial u}{\partial z} \right) \\ = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left(\mu \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial z} \left(\mu \frac{\partial v}{\partial x} \right) \end{aligned} \quad (A2)$$

$$\begin{aligned} \frac{\partial}{\partial x}(\rho u v) + \frac{\partial}{\partial z}(\rho v^2) - \frac{\partial}{\partial x} \left(\mu \frac{\partial v}{\partial x} \right) - \frac{\partial}{\partial z} \left(\mu \frac{\partial v}{\partial z} \right) \\ = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x} \left(\mu \frac{\partial u}{\partial z} \right) + \frac{\partial}{\partial z} \left(\mu \frac{\partial v}{\partial z} \right) - \rho g \end{aligned} \quad (A3)$$

$$\begin{aligned} \frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial x}(\rho u h) + \frac{\partial}{\partial z}(\rho v h) \\ = \frac{\partial}{\partial x} \left(\frac{\lambda}{c} \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{\lambda}{c} \frac{\partial h}{\partial z} \right) \end{aligned} \quad (A4)$$

Here, the Boussinesq approximation has not been used. The boundary conditions are:

$$\begin{aligned} x = 0 : T = T_h, u = v = 0 \\ x = L : T = T_c, u = v = 0 \\ \left. \begin{aligned} z = 0 \\ z = H \end{aligned} \right\} \frac{\partial T}{\partial z} = 0, u = v = 0 \end{aligned} \quad (A5)$$

The governing equations were solved numerically using the SIMPLE method¹⁵. A 100×100 grid was used in computation. The grid was packed close to the wall so that the boundary layer could be well resolved. A staggered grid was used for the calculation of velocity.