EFFECTIVE FIELD THEORY FOR NN INTERACTIONS

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ABSTRACT: The effective field theory of NN interactions is formulated and the power counting appropriate to this case is reviewed. It is more subtle than in most effective field theories since in the limit that the S-wave NN scattering lengths go to infinity it is governed by nontrivial fixed point. The leading twobody terms in the effective field theory for nucleon selfinteractions are scale invariant and invariant under Wigner SU(4) spin-isospin symmetry in this limit. Higherbody terms with no derivatives (i.e. three: and fourbody terms) are automatically invariant under Wigner symmetry.

INTRODUCTION

Let us consider $N(\vec{p})N((\vec{p}) \rightarrow N(\vec{p}')N((-\vec{p}')$ scattering in the ${}^1S_0$ channel. Since the spin of the two nucleons are combined anti-symmetrically Fermi statistics, implies that this channel is I=1 where I is the isopin of the system (similarly the ${}^3S_1$ and ${}^3D_1$ channels are I=0).

The energy $E = \vec{p}^{\,2}/M = \vec{p}^{\,\prime 2}/M$ where $p' = |\vec{p}'|$ and the scattering matrix S is related to the scattering amplitude by $S = 1 + i\mu p/2\pi$. Since $S=e^{2i\delta}$ where δ is the phase shift.

$$A^{({}^1S_0)} = \frac{4\pi}{M}\,\frac{1}{p\cot g\delta^{({}^1S_0)} - ip} \qquad (1)$$

Where M is the nucleon mass. For $p < m_\pi/2$ the quantity $p$cotgδ can be expanded in a power series in p^2

$$p\mathrm{cotg}\,\delta^{({}^1S_0)} = -\frac{1}{a^{({}^1S_0)}} + \frac{1}{2}r_0^{({}^1S_0)}p^2 + \ldots \qquad (2)$$

Where a is called the scattering length and r_0 is called the effective range. The scattering length in the ${}^1S_0$ channel is very large [1] $a^{({}^1So)}$ = -23.7 fm or $1/a^{({}^1So)}$ = -8.3 MeV. On the other hand the nuclear potential is characterized by a momentum scale Λ ~200 MeV. The smallness of $|1/a^{({}^1So)}|$ compared with this scale is the result of an accidental cancellation which cause a state in the spectrum to be very near zero binding energy. ($a \rightarrow \infty$ as a scattering state approaches zero energy and $a \rightarrow \infty$ as a bound state approaches zero binding energy). Neglecting the small ${}^3S_1$-${}^3D_1$ mixing, formulas analogous to Eqs.(1) and (2) hold in the ${}^3S_1$-chanel. The scattering length is also large in that case [1] $a^{({}^3S_1)} \cong 5.4$ fm or $1/a^{({}^3S_1)} \cong 36$ Mev. The bound state in this channel that is near zero binding energy is the deuteron.

EXPANSIONS OF A

The simplest expansion of **A** is a momentum expansion. This is analogous to what is done in standard application of effective field theory. e.g. chiral perturbation theory for $\pi\pi$ scattering. For NN scattering in the $S = {}^1S_0$ or ${}^3S_1$ channel.

$$\mathbf{A}^{(S)} = \frac{4\pi}{M}\frac{1}{[-1/a^{(S)} + (1/2)r_0^{(S)}p^2 + ... - ip} \qquad (3)$$

$$= -\frac{4\pi}{M}a^{(S)}\{1 - ia^{(S)}p + (\frac{a^{(S)}r_0^{(S)}}{2} - a^{(S)^2})p^2 + \ldots\}$$

if $a^{(S)}$ was its natural size (i.e. $a^{(S)} \sim 1/\Lambda$) this would be the appropriate expansion to perform. However in nature the S-wave N-N scattering lengths are very large and the expansion above is only valid in the small region of momentum $p \leq |1/a^{(S)}| << \Lambda$. Since the underlying physics is set by m_π and Λ_{QCD} there sould be an expansion in p/Λ that is valid even when $p >> |1/a^{(S)}|$. It is not difficult to deduce what this expansion is. In Eq. (3) keep $-1/a^{(S)}$ –ip in the denominator and expand in the remaining terms. This yields

$$\mathbf{A}^{(S)} = -\frac{4\pi}{M}\frac{1}{1/a^{(S)} + ip}[1 + \frac{r_0^{(S)}p^2/2}{(1/a^{(S)} + ip)} + \ldots] \qquad (4)$$

Now $\mathbf{A}^{(S)} = \sum_{n=-1}^{\infty} \mathbf{A}^{(S)}_n$ where $\mathbf{A}^{(S)}_n \sim O(p^n)$. This is the appropriate expansion in the case where the scattering lengths are large. It has the unusual property that the leading term is order p^{-1}.

EFFECTIVE FIELD THEORY WITHOUT PIONS

The effective field theory without the pions integrated out contains only nucleon fields, $N = \begin{pmatrix} n \\ p \end{pmatrix}$ and we expect that the lowest dimension operators will be the most important ones. The Lagrangian density is written as $l = l_1 + l_2 + \ldots$ where l_n contains n-body operators. The one and two body terms are:

$$l_1 = N^+ [\, i\partial_t + \Delta/2M\,] N + \ldots. \qquad (5)$$

$$l_2 = -\sum_s C_0^{(S)} (N^T P_i^{(S)} N)^+ (N^T N^T P_i^{(S)} N) + \ldots. \qquad (6)$$

Here $S = {}^1S_0$ or ${}^3S_1$ the elipses denote higher dimension operators and $P^{(S)}$ are the spin-isospin projectors

$$P_i^{({}^1S_0)} = \left(\frac{\delta_2\tau_2\tau_i}{\sqrt{8}}\right),\quad P_i^{({}^3S_1)} = \left(\frac{\delta_2\delta_i\tau_2}{\sqrt{8}}\right) \qquad (7)$$

Where the Pauli matrices σ_i act in spin space and the Pauli matrices τ_i act in isospin space.

Neglecting higher dimension operators the scattering amplitudes in ${}^1S_0$ and ${}^3S_1$ channels come from the sum of bubble-type Feynman diagrams shown in fig.1

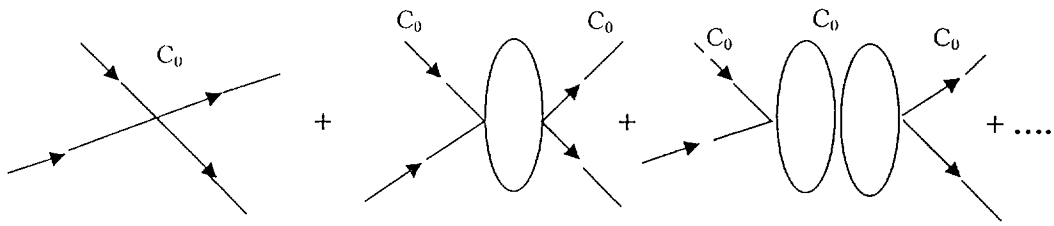

Fig.1. The leading contribution to NN scattering

Each bubble above is linearly divergent in the ultraviolet region so the coefficients $C^{(S)}_0$ depend on the regulator and substraction scheme adopted. We use dimensional regularization and start with minimal substraction (we will switch to a different substraction scheme momentarily). Since the divergences are linear the Feynman diagrams have pole at $D = 3$ but not at $D = 4$. In MS (minimal substraction) the coefficients of the operators explicitly displayed in Eq. (6) are substraction point independent and we denote them by $\bar{C}^{(S)}_0$. In this scheme sum of bubble-type Feynman diagrams gives.

$$A^{(S)} = -\frac{\bar{C}_0^{(S)}}{1 + iMp\bar{C}_0^{(S)}/4\pi} \tag{8}$$

Comparing Eq. (8) with Eqs (1) and (2) it is evident that this corresponds to keeping only the scattering length term in the expansion of $p\mathrm{cotg}\delta^{(S)}$ (i.e. the first term of Eq. (4) and that

$$\bar{C}_0^{(S)} = 4\pi a^{(S)}/M \tag{9}$$

So in this substraction scheme the coefficients $\bar{C}^{(S)}_0$ are very large and also very different in the two channels. However as $a^{(S)} \to \infty$, $A^{(S)} \to 4\pi i/Mp$ which is the same in both channels. This form for the scattering amplitudes is consistent with Wigner spin-isospin SU(4) symmetry and also with scale invariance.

In MS when $p > 1/a^{(S)}$ the terms in the perturbative series for the scattering amplitude get larger. We would like to use a substraction scheme where the various Feynman diagrams in Fig.1 are the same size as their sum and where the symmetries that arise as $a^{(S)} \to \infty$ are manifest at the level of the Lagrangian.

Examples of such substraction scheme are [2] PDS where poles at D=3 are also substracted and the OS momentum space substraction scheme [3,4]. In these schemes the coefficients are substraction point dependent $C^{(S)}_0 = C^{(S)}_0(\mu)$ and the sum of bubble diagrams gives

$$A^{(S)} = -\frac{C_0^{(S)}(\mu)}{1 + M(\mu + ip)C_0^{(S)}(\mu)/4\pi} \tag{10}$$

This still corresponds to keeping just the scattering length and is the leading term in Eq. (4). But now

$$C_0^{(S)}(\mu) = -(4\pi/M)\,(1/\mu - 1/a^{(S)}) \tag{11}$$

which as $a^{(S)} \to \infty$ becomes $C_0^{(S)}(\mu) = -4\pi/M\mu$. In this limit the coefficients are the same in both channels and with $\mu \sim p$ each term in sum of bubble-type diagrams in Fig.1 is the same size as the sum itself.

The operators with coefficients $C^{(S)}_0$ are nonrennormalizable dimension six operators. Naively they are irrelevant operators and at low momentum can be treated in perturbative theory. However as $a^{(S)} \to \infty$ the coefficients $C^{(S)}_0(\mu)$ flow to a nontrivial fixed point [2,3] where $\mu d(\mu C^{(S)}_0(\mu))^0/d\mu = 0$. For larger $a^{(S)}$ the power counting is controlled by this fixed point and the leading contribution to the N-N scattering amplitude comes from treating $C^{(S)}_0$ non-perturbavely. It is straightforward to show that in PDS or OS the coefficients of S-wave operators with $2n$ spatial derivatives scale as [2]

$$C_{2n}^{(S)} \sim 4\pi/M\Lambda^n\mu^{n+1} \tag{12}$$

For $\mu >> | 1/a^{(S)} |$, with $\mu \sim p$, $C^{(S)}_{2n}(\mu)p^{2n} \sim p^{n+1}$ and the two body operators with derivatives can be treated perturbatively. In a nonrelativistic theory a loop integration $\int d^4q = \int dq^0 d^3q \sim O(p^5)$ (since the dq^0 integration is of order p^2 and the d^3q is order p^3) and the nucleon propagator $i/(p^0-p^2/2M+i\varepsilon) \sim O(p^2)$. Consequently each loop gives a factor p plus whatever factors of p are associated with the vertices. The power counting [2,5] is now evident. The leading order (LO) contribution $A_{-1}^{(S)}$ comes from $C^{(S)}_0$ treated nonperturbativery, the next leading order (NLO) contribution $A_0^{(S)}$ comes from $C^{(S)}_0$ treated nonperturbativery and $C^{(S)}_2$ inserted once, the next-to-next to leading order (N^2LO) contribution comes from $C^{(S)}_0$ treated nonperturbativery and $C^{(S)}_2$ inserted twice or $C^{(S)}_4$ inserted once, etc.

With the piont integrated out the effective field theory expansion applied to N-N scattering reproduces Eq. (4) and has no more content than the momentum expansion of $p\cot g\delta^{(S)}$. However even with the pions integrated out one can couple photons or W and Z gauge bosons to the nucleons.

The relative importance of operators containing these fields depends on their renormarization group scaling near the fixed point.

In the two nucleons sector predictions based on the effective field theory with not pions are similar to those made by effective range theory [6]. However the effective field theory approach has a number of advantages. Predictions based on effective range theory are only valid to a given order in the p/Λ expansion. In the effective field theory new twobody operators containing the gauge fields arise which spoil the predictions of effective range theory.

As $a^{(S)} \to \infty$, $L_2 \to -(2\pi/M\mu)(N^+N)^2 + \ldots$ where the elipses denote two body operators with derivatives. In this limit the leading one and twobody terms are invariant under the following symmetries:

(i.) Wigner symmetry

Under infinitesimal Wigner symmetry SU(4) transformations

$$\delta N = i\alpha_{\mu\nu}\sigma^\mu \tau^\nu N \tag{13}$$

where $\sigma^\mu = (1,\sigma)$ and $\tau^\nu = (1, \tau)$ with $\mu = 0,1,2,3$ and repeated indices summed. The symmetry group corresponding to Eq. (13) is actually SU(4) x U(1) with α_{00} the group parameter for the additional baryon number U(1). Associated with this symmetry are the conseved charges

$$Q^{\mu\nu} = \int d^3\vec{x}\ N^{+} 6^{\mu} \tau^{\nu} N \qquad (14)$$

The two body terms with derivatives are not invariant under Wigner symmetry even if $a^{(S)} \to \infty$. Hence in the two body sector the violations of Wigner symmetry go as ($1/ [a^{(^1S_0)}p]$ - $1/ [a^{(^3S_1)}p]$) and p/Λ. Wigner symmetry will not be a good approximation the mommentum p is too low or if it is too large.

Wigner symmetry is relevant for nuclei with many nucleons [7]. It is not difficult to see that the higher body terms with no derivatives are automatically invariant under Wigner symmetry. Since these contact terms are antisymmetric in the nucleon fields N and in the hermitian conjugates N^{+}, contact terms without derivatives cannot occur for the five body operators and higher. The nucleons are in the 4 of SU(4) and the $N^{+}s$ are in the $\bar{4}$. Four nucleons combined antisymmetrycally are an SU(4) singlet and so the fourbody terms are invariant under SU(4). The threebody terms transform as $\bar{4}\otimes 4 = 1\oplus 15$. However the operators in the 15 are not invariant under the total spin or isospin SU(2) subgroup of SU(4). Hence the allowed threebody terms are also invariant under SU(4) Wigner symmetry.

A complete extention of the general fixed point power counting to the higherbody terms has not been made. However there has been considerable recent progress [5]. This work indicates that the threebody terms with no derivatives is leading order (i.e. as important as effects coming from $C^{(S)}{}_{0}$)

(ii.) Scale invariamce

The leading one and two terms are invariant under the scale transformation $N(t,\vec{x}) \to N'(t, \vec{x})$ and $\mu \to \mu'$ where

$$N'(t, \vec{x}) = \lambda^{-3/2} N(t/\lambda^2, \vec{x}/\lambda) \qquad (15)$$

$$\mu = \mu'/\lambda \qquad (16)$$

Note that Eq. (15) corresponds to $N'(t, \vec{x}') = \lambda^{-3/2} N(t, \vec{x})$

With $\vec{x}' = \lambda\vec{x}$ and $t' = \lambda^2 t$. The different scaling of space and time coordinates is dictates by invariance of leading onebody terms in the Lagrangiandensity.

INCLUDING PIONS

With pion included the power counting is taken to be in power of Q/λ_{NN} where $Q \sim p \sim m_\pi$. A subscript N-N has been put on λ as a reminder that the expansion should work better if the pions are included as explicit fields, i.e. we expect that $\lambda_{NN} > \lambda$. Potential pion exchange arises the term

$$\mathbf{L}_{int} = -\frac{9A}{\sqrt{2}f_\pi}\nabla^i \pi^j N^{+} 6^i \tau^j N \qquad (17)$$

where $g_A = 1.25$ is the axial coupling and $f_\pi \cong 131$ MeV is the pion decay constant. Exchange of a potential pion between nucleons is order Q^0 (the two factors of Q from the vertices cancel the $1/Q^2$ from the pion propagator) this is the same size as the two body contact terms with two derivatives and consequently pion exchange can be treated perturbatively. Including pion exchange without the two derivatives two body contact terms is not a systematic improvement and is not better (from a power counting perspective) than just including the effects of $C^{(S)}_0$. Note that this power counting is very different from the one originally proposed by Weinberg [3,9] where the leading contribution came from treating both potential pion exchange and $C^{(S)}_0$ nonperturbatively. The effects of twobody terms with derivatives and insertions of the light quarks mass matrix were considered subdominant.

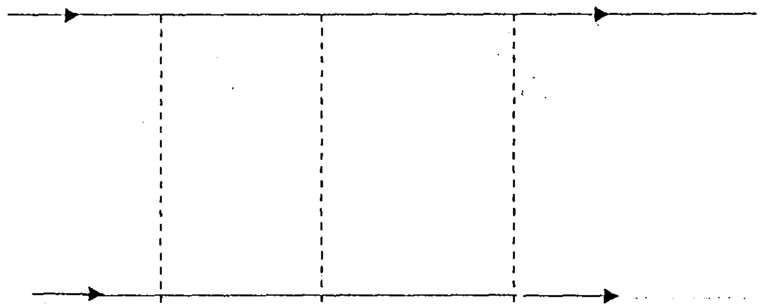

Fig.2 Contribution to $^3S_1$ scattering from three potential pion exchange the dashed denote potential pion exchange

In the $^3S_1$ channel the Feynman diagram shown in Fig.1 with three potential pion exchange is logarithmically divergent. Neglecting the pion mass it gives a contribution to $A^{(^3S_1)}$ of order

$$(4\pi/M)(Mg^2_A/8\pi f^2_\pi)^3 p^2 \,[\, \ln\mu^2 + K \,] \qquad (18)$$

where K is constant, the μ – dependence above is cancelled by the μ – dependence of $C^{(^3S_1)}_2$.

There is point to including this Feynman diagram without including the effects of the twobody $^3S_1$ operator with 2-derivatives. Eq. (18) is an N^{3}LO contribution . With the pions included a single insertion of $C^{(S)}_2$ is not just NLO it contributes at higher levels at the Q expansion as well. For that reason $C^{(S)}_2$ and other contact term coefficients are sometimes written as a sum $C^{(S)}_2 = \sum_{n=1}^{\infty} C^{(S)}_{2,n}$ where $C^{(S)}_{2,1}$ gives the NLO contribution, etc. When this is done predictions for physical quantities are exactly μ independent at each order in the Q expansion. If $C^{(2)}$ is not expanded is this way then predictions at a given order on the Q expansion have some substraction point dependence, which is higher order in the Q expansion

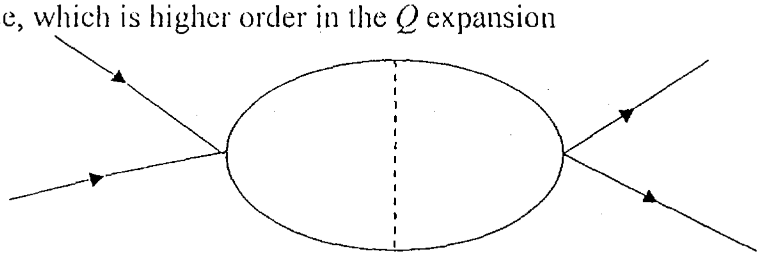

Fig.3. Contribution to N-N scattering that renormalized $D^{(S)}_2$

There are twobody S-wave contact terms with no derivatives but with one insertion of the light quark matrix

$$m_q = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \qquad (19)$$

since $m^2_\pi \alpha (m_u + m_d)$ and insertion of m_q counts as two powers of Q and coefficients of these operators $D^{(S)}{}_2$ scale with μ in the same way as the coefficients $C^{(S)}{}_2$. At NLO they must also be included. The Feynman diagram in Fig. 3 is logarithmically divergent and it gives a contribution to the ${}^1S_0$ and ${}^3S_1$ scattering amplitudes of order

$$(4\pi/M)(g^2{}_A M/8\pi f^2{}_\pi)\ (C_0^{(S)}\ M/4\pi)^2\ m_\pi^2\ [\ ln\mu^2 + K\ ] \qquad (20)$$

where K is constant. The μ dependence here is cancelled by that of the coefficients $D_2^{(S)}$. Including one pion exchange without the effects of the two body terms with oneinsertion of the light quart mass matrix does not systematically improve the theoretical prediction the N-N scattering amplitude.

If a momentum cutoff regulator is used instead of dimentional regularization then including pion exchange without the twobody contact operators that have an insertion of the quark mass matrix results in a cutoff dependent amplitude $\mathbf{A}^{(S)}$. It is possible in the ${}^1S_0$ channel to sum to all orders potential pion exchange and when this is done the cutoff dependence does not become subdominant [10] (compared with the finite cutoff independent parts of pion exchange). The effects of local four nucleon (i.e. twobody) operators with an insertion of the quark mass matrix cannot be viewed as less important than the effects of pion exchange.

AN APPLICATION OF WIGNER SYMMETRY

Potential pions have $k^0 \sim \vec{k}^2/M$ while radiation pions have $k^0 \sim \sqrt{\vec{k}^2 + m_\pi^2}$. The coupling of the radiation pions to the nucleons is done by performing a multipole expansion on Eq. (17). At leading order this amounts to evaluating the pion field in Eq. [(1&) at the space time point $(t, \vec{x}) = (t,0)$. Hence for radiation pions the term in the action corresponding to Eq. (17) is

$$S_{int} = -\frac{9A}{\sqrt{2}f_\pi} \int dt(\nabla^i \pi^j)\big|\ \vec{x} = 0 \cdot Q^{ij} \qquad (21)$$

Where Q^{ij} are the charges of Wigner symmetry in Eq. (14). In the limit $a^{(S)} \to \infty$ these charges are conserved and the Q^{ij} are time independent. Hence as $a^{(S)} \to \infty$ only the $k^0 \approx 0$ mode of pion couples in Eq. (21). This is incompatible with the radiation pions is suppressed by $(1/a^{({}^1S_0)} - 1/a^{({}^3S_1)})$.

CONCLUSION

Effective field theory methods are viable model independent approach to the physics of two nucleon sector. The power counting is slightly unusual due to the large S-wave N-N scattering lengths. This approach is useful up to a center of mass momentum around 200 MeV, however the expansion parameter at such a momentum is probably not much smaller than 1/2. It seems likely that for many quantities calculations at N^2LO with reach the same precision as conventional potential model approaches, however with such a large expansion parameter there are likely to be some failures.

Extension of the effective field theory approach to the three nucleon sector is underway. Several theoretical issues remain to be resolved before there is a complete power counting, but recent progress in this area is very encouraging.

The holy grail of this field is the application of the effective field theory methods to nuclear matter. We are still a long way from having the theoretical tools to tackle this problem and even with these tools the Fermi momentum associated with nuclear density may be too large for a Q expansion to be useful. However given the importance of understanding the properties of nuclear matter continuing to develop the effective field theory approach is very worth while.

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