THE 4TH INTERNATIONAL TOPICAL MEETING ON
NUCLEAR THERMAL HYDRAULICS, OPERATIONS AND SAFETY
April 6-8, 1994, Taipei, Taiwan

CHF PREDICTOR DERIVED FROM A 3D THERMAL-HYDRAULIC CODE AND AN ADVANCED STATISTICAL METHOD

D.BANNER, S. AUBRY
Electricité de France, Direction des Etudes et Recherches
6, quai Watier, 78401 Chatou, FRANCE
Tel: (1)-30.87.77.51 ; Fax: (1)-30.87.79.49
E-Mail: Didier.Banner@der.edf.fr

ABSTRACT

A rod bundle CHF predictor has been determined by using a 3D code (THYC) to compute local thermal-hydraulic conditions at the boiling crisis location. These local parameters have been correlated to the critical heat flux by using an advanced statistical method based on spline functions. The main characteristics of the predictor are presented in conjunction with a detailed analysis of predictions (P/M ratio) in order to prove that the usual safety methodology can be applied with such a predictor. A thermal-hydraulic design criterion is obtained (1.13) and the predictor is compared with the WRB-1 correlation.

INTRODUCTION

Critical heat flux (CHF) is one of the most important limits on nuclear power plant operation. Safety analysis requires that no onset of CHF occurs under operating conditions. Since no accurate theory of CHF exists, prediction of the phenomenon is obtained by correlations compiled from experimental data.

These CHF predictors are determined by a two-step approach for fuel assemblies. In a first step, local thermal-hydraulic conditions (e.g. quality, mass flow rate ..) are computed by subchannel analysis codes (e.g. COBRA, THINC) at the CHF location. Then, these local parameters are correlated with the critical heat flux by statistical regressions, so that a local predictor of the occurrence of CHF is obtained.

In this paper, the thermal-hydraulic approach and the statistical approach have both been revised.

On the one hand, the standard subchannel analysis of thermal-hydraulics has been substituted by a 3D porous body analysis. The code used here is THYC (Thermal HYdraulics for Cores). On the other hand, an advanced statistical technique has been used to derive a CHF predictor that is not based on a parametric least-square method. This has been achieved by the pseudo-cubic spline method (PCSM), a statistical technique based on spline functions.

The predictor has been built from CHF experimental data taken from the EPRI compilation. Eight data banks, that include 570 data points, have been investigated, so that four geometrical effects are accounted for : the heated length, the grid spacing, the axial heat flux profile and the presence of tube thimbles.

The paper is presented as follows. First, the thermal-hydraulic code (THYC) and the statistical method (PCSM) are briefly described. Special attention is then focused on the characteristics of the predictor obtained. The ability of the predictor to determine the CHF is investigated in terms of a P/M (predicted to measured) distribution analysis. Moreover, it is shown that the predictor meets safety analysis requirements in terms of trends and bias with independent variables. A design criterion of 1.13 is then established for this CHF predictor.

I THE THERMAL-HYDRAULIC *THYC* CODE

THYC is a thermal-hydraulic code developed by 'Électricité de France' (EDF) [1]. Its application scope is single or two-phase flow in rod bundles. It is especially devoted to heat and mass transfer in the following nuclear components: reactor cores, steam generators and condensers.

The code differs from subchannel analyses which assume a prevailing axial component of velocity and uniform pressure at each elevation. Here, a fully three-dimensional representation of the flow is proposed in conjunction with a porous-body approach.

The liquid-vapour flow is described by three equations that respect strict conservation of mass, momentum and energy. An additional equation can be implemented into the model in order to allow thermodynamic non-equilibrium between the vapour and the liquid phase (subcooled boiling or super-heated steam).

In this paper, conservation equations have only been solved for the gas-liquid mixture.

$$\frac{\partial \varepsilon \rho}{\partial t} + div\,(\varepsilon Q) = 0 \qquad \text{(Mass Eq.)} \qquad (1)$$

$$\frac{\partial \varepsilon Q}{\partial t} + div\,(\varepsilon\, u \otimes Q) + div\,(\varepsilon\, \rho\, C\,(1-C)\, V_r \otimes V_r)$$
$$= -\varepsilon\, grad\, P + div\,(\varepsilon\, T_v) + \varepsilon\, \rho\, g - \varepsilon\, K\, Q$$
$$\text{(Momentum Eq.)} \qquad (2)$$

$$\frac{\partial \varepsilon \rho h}{\partial t} + div\,(\varepsilon h Q) + div\,(\varepsilon \rho £ C\,(1-C)\, V_r)$$
$$= \varepsilon \frac{\partial P}{\partial t} + \varepsilon\, u\, grad\, P - div\,(\varepsilon \varphi) + \varepsilon \phi$$
$$\text{(Energy Eq.)} \qquad (3)$$

with :

ε	porosity
ρ	mass density (kg/m^3)
Q	mass flow rate ($kg/s/m^2$)
u	average velocity (m/s)
C	vapor quality (kg/kg)
V_r	relative velocity (m/s)
P	pressure (Pa)
T_v	turbulent stress tensor (N/m^2)
g	gravity (m/s^2)
K	pressure tensor (s^{-1})
h	enthalpy (J/kg)
£	latent heat (J/kg)
φ	thermal heat flux (W/m^2)
ϕ	heat source (W/m^3)

Equation (2) is solved for the three components of velocity. No simplified equation is used for transverse flows even if the axial component is prevalent, so that the model is really three-dimensional. In addition to boundary conditions, closure relationships are needed to determine primary variables (P : Pressure, G : mass flow rate, H : enthalpy).

In the case of two-phase flow in CHF test sections (5 x 5 rod experiments), the main closure laws are listed in table 1 and references are in [1] .

A key-parameter for the determination of mass flow rates in rod bundles is the turbulent viscosity ν_t. In particular, mixing grids enhance turbulence in the bundle and have a strong impact on ν_t.. The THYC software can represent this effect by three options [2] . First, it is assumed that mixing grids increase the rate of turbulence in a uniform manner throughout the flow, so that ν_t can be expressed as a function of turbulent viscosity in a bare bundle $\nu_{t,bare}$ by

Table 1 Closure relationships

Closure Laws	*Correlation*
Drift velocity	Lellouche, Zolotar
Two phase flow friction	EPRI
Axial pressure drop	Blasius
Turbulent viscosity	See next section
Thermal diffusivity	$Pr_t = 1$

$\nu_t = gf \,.\, \nu_{t,bare}$ where gf (grid factor) accounts for the grids. A second possibility is to represent generation and decay of turbulence by a k-L model. In addition to that model, oriented cross flows generated by mixing vanes can be modelled by non-diagonal pressure drop tensors.

In this paper, the first turbulent viscosity model is used, so that grids are represented by a grid factor.

II THE PSEUDO-CUBIC SPLINE METHOD (PCSM)

The PCSM is a smoothing technique, based on spline functions, developed by the French Atomic Energy Commission (CEA) [3]. It is designed to analyse experimental data points and, in particular, thermal-hydraulic data. Detailed and theoretical information can be found in [3,4]. Attention is focused on the main features of the method. It is also compared with usual correlation techniques.

II.1. Main assumptions

Let us consider $\vec{t}$ a vector containing all independent variables (e.g. $\vec{t}$ (P,G,X) with P pressure, G mass flow rate and X quality). It is assumed that y the physical phenomenon (here CHF) is smooth enough to be expressed as follows :

$$y = f(\vec{t}) + \varepsilon(\vec{t}) \qquad (4)$$

where $f(\vec{t})$ is the most likely value at point $\vec{t}$ and $\varepsilon(\vec{t})$ is a random function of mean and standard deviation equal to 0 and σ_r, respectively. σ_r is termed the residual error. Moreover, it will be assumed that $\varepsilon(\vec{t})$ is characterized by a normal (gaussian) distribution in order to apply statistical tests (Student, Fischer..).

II.2. Scope of the PCSM

The method provides:

- $\hat{f}$ an estimate of the f function, $\hat{f}$ is called a predictor or a 'thin-plate' [3],
- $\hat{\sigma}_r$ an estimate of the residual error σ_r,
- σ_p the uncertainty on the predictor ($\hat{f}$),
- N_F the number of used degrees of freedom where $N_F < N$ (N being the number of data points)

The $\hat{f}$ function is determined by using multidimensional cubic spline functions and minimizing a certain energy E [4]. A smoothing parameter ρ of considerable importance is used, so that $\hat{f}$ can continuously vary from a multilinear regression ($N_F \approx 4$) to a spline interpolation function through all data points ($N_F \approx N$). An ideal compromise is reached by the so-called generalized cross validation technique [4].

II.3. Comparison with existing methods

CHF correlations are usually determined by defining beforehand a mathematical function with a certain number of parameters that are optimized by least-square considerations. For example, it is often assumed that the CHF (ϕ) varies with quality (X) as follows: $\phi = A - BX$. This kind of assumption is not required by the PCSM.

In addition to that asset, the PCSM yields significantly lower residual errors (σ_r), so that more accurate predictions are obtained by the method. It is a very adaptive technique, especially dedicated to non-linear physical phenomena that vary with independent variables (e.g. from burn-out to dry-out). Nevertheless, the PCSM requires more computational power than usual correlations but this is not a strong limitation with modern computers. For CHF calculations, the computational time devoted to the DNBR prediction with the PCSM is significantly smaller than the time required to estimate local thermal-hydraulic conditions.

III THE CHF PREDICTOR

Experimental data analyzed with THYC and the PCSM are presented. The main characteristics of the predictor are given. This is followed by detailed analysis of P/M ratio in order to check that the usual safety margin methodology can still be applied with such a predictor. Finally, a thermal-hydraulic design criterion is determined and the predictor is compared with the WRB-1 correlation.

III.1. Data and geometrical effects

Eight CHF data banks have been analyzed by the THYC software in order to derive local thermal-hydraulic parameters. These data [5] are listed in table 2.

Table 2 : CHF data banks

Database	*156*	*157*	*158*	*160*	*161*	*162*	*163*	*164*
Number of data points	90	79	68	67	71	74	41	80
Heated length (ft)	14	8	8	8	14	14	8	14
Grid spacing (in)	26	26	26	22	22	22	22	22
Axial profile	Unif	Unif	Unif	Unif	Unif	Cos	Unif	Cos
Tube thimble	Yes	No	Yes	No	No	Yes	No	No

These CHF data are 5 x 5 rod bundle experiments with rod diameter and pitch equal to 9.5 mm and 12.6 m, respectively. As far as 9.5 mm rods are concerned, these data banks are identical to those analyzed to develop the WRB-1 correlation [6], except that E163 has been added to derive our own predictor.

Four geometrical effects can be investigated with the data bases studied here :

- heated length (14 to 8 ft)
- grid spacing (22 or 26 in)
- presence or absence of tube thimble
- axial profile (uniform or cosine)

These eight data bases include 570 data points, so that a CHF predictor with a wide validity range (Table 3) can be obtained. Correlations do not only predict CHF from thermal-hydraulic conditions (P,G,X) but also include geometrical parameters (e.g. grid spacing).

In table 2, geometry-related characteristics can only take two values, so that they will be represented by binary variables if relevant for thermal-hydraulic analysis. An effect is said to be significant if the residual error σ_r is significantly increased when not accounted for. Non-uniform axial profiles are treated by introducing the Tong factor F, so that F.ϕ (with ϕ being the local critical heat flux) is correlated to independent variables [7])

Table 3: thermal-hydraulic range

Variable	*Minimum*	*Maximum*
Pressure (10^5 Pa)	103	168
Mass flow rate ($kg/m^2/s$)	1300	4850
Quality	-0.12	0.33

Table 4: Geometrical effects

Effect	*Ratio of residual errors*	*Value*
A (heated length)	$\sigma_{BCD} / \sigma_{ABCD}$	1.39
B (grid spacing)	$\sigma_{ACD} / \sigma_{ABCD}$	1.40
C (axial profile)	$\sigma_{ABD} / \sigma_{ABCD}$	1.04
D (tube thimble)	$\sigma_{ABC} / \sigma_{ABCD}$	1.055

Nevertheless, one can test if predictions are improved by using a binary variable that represents the axial profile. In table 4, ratios of residual errors σ are listed for the four geometrical effects; subscripts denote binary effects.

III.2. Characteristics of the predictor

An excessive number of binary variables should not be included in the predictor. Consequently, only the heated length (A) and the grid spacing (B) effects are worth being accounted for (table 5).

It can be observed that N_F is lower than one sixth of N, so that the complexity of the predictor is reasonable. A low $\sigma_{M/P}$ value is given by the PCSM (6,5%), an indication of the good accuracy of the method to predict CHF over a wide thermal-hydraulic range. Information given in table 5 is not sufficient to determine whether the predictor can be applied to safety analysis. Further information is required.

III.3. Statistical analysis

DNB margins are determined by defining tolerance intervals (95%) and confidence levels (95%), so that CHF is not observed. To do so, Owen's one sided tolerance interval criterion is used [8]. At least, the two following requirements have to be met :

- a normal distribution of the P/M ratio,
- bias and trends with independent variables should be avoided, so that a unique safety criterion can be applied for all thermal-hydraulic regions.

Table 5: CHF predictor characteristics

Number of points N	*Nb. of used degrees of freedom* N_F	*Residual error* σ MW/m²	$\overline{\left(\frac{M}{P}\right)}$	$\sigma_{P/M}$ (%)	$\sigma_{M/P}$ (%)
570	94	0.116	1	**6,77**	**6,50**

In figure 1, the P/M plot is presented. It appears that this distribution fits a normal (bell-shaped) distribution but visual estimations are not sufficient. A Chi-square statistical

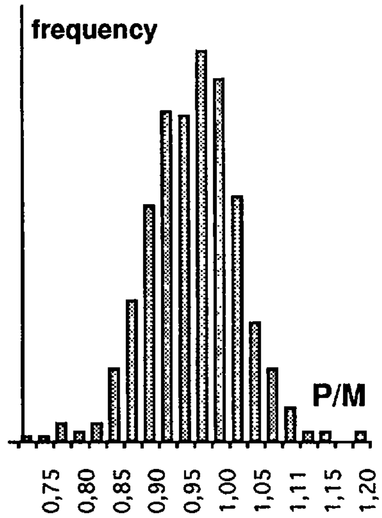


figure 1: P/M distribution (predicted to measured CHF)

test has been applied in order to determine whether a normality assumption can be accepted. The test proceeds as follows. If the P/M population is split into N_{tot} divisions with N_k being the population of the k-th division and $\tilde{N}_k$ the theoretical number of data points in division k for a normal distribution, then $D^2 = \sum_{k=1}^{N_{tot}} \frac{(N_k - \tilde{N}_k)^2}{\tilde{N}_k}$ follows a $\chi^2_{N_k-\ell-1}$ distribution if normality can be assumed (with ℓ being the number of estimated parameters, here $\ell = 2$ since the mean and the standard deviation have been estimated)

For $N_{tot} = 12$, it turns out that the probability of a false assertion by not accepting the normality assumption is greater than 15%. Therefore, a normal distribution assumption appears to be reasonable.

In figures (2-4), P/M ratios are plotted as a function of the three main thermal-hydraulic variables (pressure, mass flow rate and quality). It can be observed for the three plots that the mean of the P/M distribution does not vary from one thermal-hydraulic range to another. Therefore, the predictor does not exhibit bias with independent variables. Moreover, dispersion around the mean value $\left(\frac{M}{P} = 1\right)$ does not fluctuate. It follows that the two prerequisites for the application of Owen's criterion are verified.

III.4. Design criterion. Comparison with WRB-1

Since the predictor does not exhibit bias and a non-uniform dispersion, a design criterion C can be computed for safety analysis. In order to ensure that critical conditions are not attained with a probability of 95% with a 95% confidence level, C is determined by :

$$C = \frac{1}{\left(\frac{M}{P}\right) - k\,\sigma_{M/P}}$$

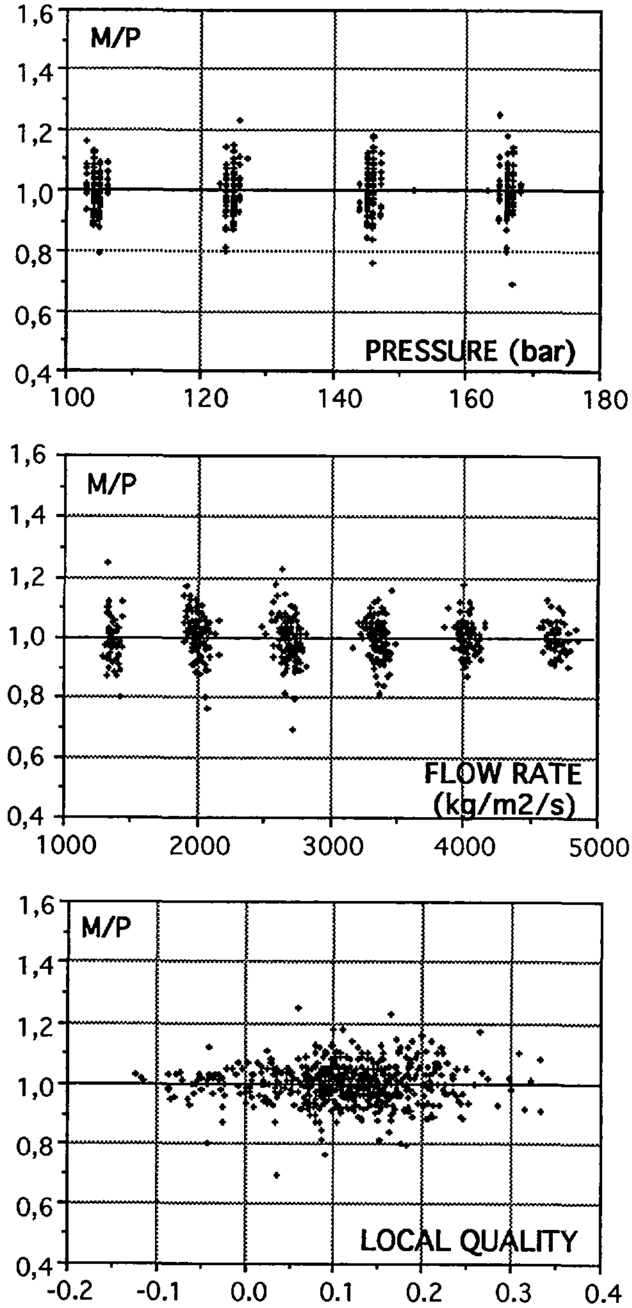
with k being the Owen factor equal to 1.755 for 570 data points and the 95/95 criterion [8]. This yields C = 1.13. This value is significantly lower than the WRB-1 design criterion (1.17) [6]. In figure 5, predicted (P) and measured (M) values are plotted as well as the solid line given by equation P = C.M. Operating conditions have to be represented on the lower part of the plot in order to verify the 95/95 criterion.

In table 6, comparison between the WRB-1 correlation and our predictor is presented. Data given for the WRB-1 are taken from [6] and thermal-hydraulic analysis has been performed by the THINC code.

Lower standard deviations are obtained for all data banks, so that the overall predictor yields more accurate predictions for the four geometrical effects investigated (heated length, grid spacing..).

Table 6: Comparison between our predictor and WRB-1

	Mean		Standard dev. $\sigma_{P/M}$ (%)	
Data base	THYC + PCSM	THINC + WRB-1	THYC + PCSM	THINC + WRB-1
E157	1.00	1.01	**4.84**	**8.48**
E158	0.997	1.030	**7.73**	**10.48**
E160	1.00	1.050	**5.11**	**10.20**
E161	0.973	0.996	**5.20**	**6.55**
E162	0.975	1.00	**6.21**	**7.96**
E164	1.05	1.002	**6.40**	**8.52**



Figures 2-4: M/P (measured to predicted) versus thermal-hydraulic parameters

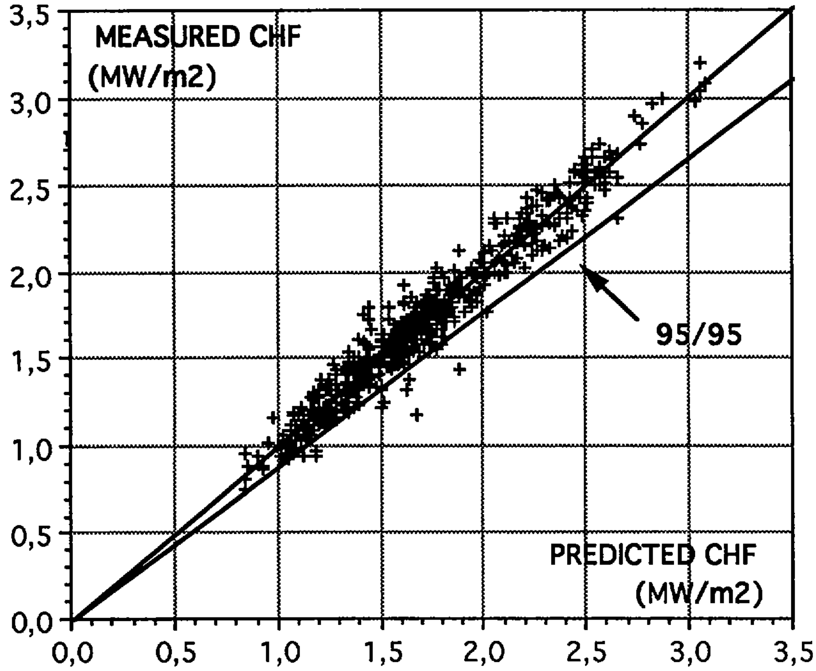


figure 5: Measured versus predicted CHF

CONCLUDING REMARKS

A CHF predictor determined over a wide thermal-hydraulic range has been computed. The design criterion is equal to 1.13, a value significantly lower than for most rod bundle CHF correlations.

This predictor is compatible with the THYC code which allows a 3D representation of the flow. The statistical method (PCSM) has turned out to be a flexible tool to derive CHF predictors.

In the future, on the one hand, more sophisticated thermal-hydraulic models will be investigated to derive more accurate results. On the other hand, the predictor will be applied to core design calculations and compared with the existing methodology.

REFERENCES

[1] C. CAREMOLI, P. RASCLE, S. AUBRY and J. OLIVE, "THYC, a Thermal HYdraulic Code for 3D Two-Phase Flows in Tube Bundles", Second International Seminar on subchannel Analysis, Electric Power Research Institute, 19th Nov 1993.

[2] D. BANNER and S. AUBRY ; Impact on DNB Predictions of Mixing Models Implemented into the Three-Dimensional Thermal-Hydraulic Code THYC, NURETH-6 Proceedings, Grenoble, France, Oct 5-8, (1993)

[3] F. de CRECY; Pseudo-Cubic Thin-Plate Type Spline Method for Analyzing Experimental Data; NURETH-6 Proceedings, Grenoble, France, Oct 5-8, 1993

[4] E. J. WEGMAN and I. WRIGHT; Spline in Statistics; Journal of the American Statistical Association, Volume 78, Number 382, June 1983

[5] C. F. FIGHETTI, D.G. REDDY; Parametric Study of CHF data, Columbia University, NP2609, Research Project, Final Report (September 1989)

[6] F. E. MOTLEY; New Westinghouse Correlation WRB-1 for Predicting Critical Heat Flux in Rod Bundles with Mixing Vane Grids, WCAP-8763 (Class 3), July 1976

[7] L. S. TONG; Boiling Crisis and Critical Heat Flux, TID-25887, 1972

[8] D. B. OWEN; Factors for One-Sided Tolerance Limits and For Variables Sampling Plans, SCR-607, March, 1963