w-MODE PERTURBATION THEORY AND REACTOR KINETICS FOR ANALYZING ACCELERATOR-DRIVEN SUBCRITICAL SYSTEMS

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Abstract

An ω-mode first-order perturbation theory is developed for analyzing the time- and space-dependent neutron behavior in Accelerator-Driven Subcritical Systems (ADSS). The generalized point-kinetics equations are systematically derived using the ω-mode first-order perturbation theory and Fredholm Alternative Theorem. Seven sets of the ω-mode eigenvalues exist with using six groups of delayed neutrons and all ω eigenvalues are negative in ADSS. Seven ω-mode adjoint and forward eigenfunctions are employed to form the point-kinetic parameters. The neutron flux is expressed as a linear combination of the products of seven ω-eigenvalue-mode shape functions and their corresponding time functions up to the first order terms, and the lowest negative ω-eigenvalue mode is the dominant mode.

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Introduction

A critical reactor is prompt-subcritical and uses delayed neutrons to make it critical. However, a mixed-oxide-fuel core for nuclear waste transmutation contains lot of long-lived actinides (Pu239, Pu240, Pu241, Am241, etc.) with very small delayed neutron fractions and is very difficult to be operated safely. To overcome this difficulty, conceptual designs of ADSS [1,2] have been developed to use an external neutron source produced by accelerator-generated high speed proton or electron bombardment as a substitute for part of delayed neutrons.

Cacuci [3] pointed out that the point-kinetics equations obtained by using the traditional Wigner-type λ-mode adjoint shape function developed originally for a critical reactor is inadequate to model the time- and space-dependent neutron behavior in ADSS for the following reasons:

1) Physically, a critical reactor becomes steady by maintaining a self-sustained chain reaction, whereas an ADSS becomes steady by maintaining a source-driven chain reaction.
2) Mathematically, a critical reactor is governed by a homogeneous λ-mode eigenvalue equation with a non-trivial homogeneous flux solution, whereas an ADSS is governed by an inhomogeneous λ-mode equation with trivial homogeneous flux solution but a non-trivial source-driven particular flux solution.

Although the λ-mode eigenvalues do not exist in ADSS, physically the ω-mode eigenvalues do exist in this source-driven subcritical system [4,5]. Consequently, an ω-mode first-order perturbation theory is developed below to model the time- and space-dependent neutron behavior in ADSS.

Theory

To model the time- and space-dependent neutron behavior in a near steady state of ADSS, let us consider that 1) an ADSS is a subcritical system perturbed by an accelerator-driven external neutron source, 2) the subcritical system itself is an unperturbed system with an effective neutron multiplication factor very close to 1, e.g., 0.995, 3) an initial neutron flux exists in the subcritical system, and 4) there are six groups of delayed neutrons.

The transport kinetics equations for the unperturbed neutron flux $\varphi(\mathbf{r},E,\boldsymbol{\Omega},t)$ and the delayed neutron precursors $C_i(\mathbf{r},t)$ in the subcritical system are:

$$\frac{1}{v}\frac{\partial}{\partial t}\varphi(\mathbf{r},E,\boldsymbol{\Omega},t) = -\,\boldsymbol{\Omega}\bullet\nabla\varphi - \Sigma_t(\mathbf{r},E)\varphi + \mathbf{S}\varphi + (1-\beta)\chi_p\mathbf{F}\varphi + \sum_{i=1}^{6} \chi_i\lambda_i C_i(\mathbf{r},t) \qquad (1)$$

$$\frac{\partial}{\partial t}C_i(\mathbf{r},t) = \beta_i\mathbf{F}\varphi - \lambda_i C_i(\mathbf{r},t) \;,\quad i=1,\ldots,6 \qquad (2)$$

where β_i, λ_i, χ, Σ_t, Σ_s and Σ_f are delayed neutron fraction, delayed neutron decay constant, fission spectrum, total cross section, scattering cross section, and fission cross section, respectively;

$$\mathbf{S}\varphi \equiv \iint \Sigma_s(\mathbf{r},E'\rightarrow E,\Omega'\rightarrow\Omega)\,\varphi(\mathbf{r},E',\Omega',t)\,dE'\,d\Omega' \equiv \langle\Sigma_s\varphi\rangle$$

$$\mathbf{F}\varphi \equiv \frac{1}{4p}\iint \nu(E')\,\Sigma_f(\mathbf{r},E')\,\varphi(\mathbf{r},E',\Omega',t)\,dE'\,d\Omega' \equiv \frac{1}{4p}<\nu\Sigma_f\varphi>.$$

In the ω-mode development [4], the unperturbed neutron flux is decomposed into a time function P(t) and a shape function $\psi(\mathbf{r},E,\Omega)$, the unperturbed delayed neutron precursors are decomposed into time functions $B_i(t)$, and shape functions $\zeta_i(\mathbf{r})$, and all time functions are proportional to $e^{\omega t}$:

$$\varphi(\mathbf{r},E,\Omega,t) = P(t)\,\psi(\mathbf{r},E,\Omega) = P_o\,e^{\omega t}\,\psi(\mathbf{r},E,\Omega) \qquad (3)$$

$$C_i(\mathbf{r},t) = B_i(t)\,\zeta_i(\mathbf{r}) = B_{io}\,e^{\omega t}\,\zeta_i(\mathbf{r}) \;,\; i=1,\ldots,6\,. \qquad (4)$$

Substituting Eqs. (3) and (4) into Eqs. (1) and (2) leads to the ω-mode unperturbed transport equation with an ω-mode transport operator **L**:

$$\mathbf{L}\psi(\mathbf{r},E,\Omega) \equiv -\,\Omega\bullet\nabla\psi - (\Sigma_t + \frac{w}{n})\psi + \mathbf{S}\psi + (1-\beta)\chi_p\mathbf{F}\psi + \sum_{i=1}^{6}\chi_i\frac{libi}{w+li}\mathbf{F}\psi = 0\,. \qquad (5)$$

Eq. (5) is an implicit eigenvalue equation with an initial neutron flux and seven sets of ω eigenvalues, all ω eigenvalues are negative for a subcritical system, the initial neutron flux decays with time as a linear combination of the products of seven ω-eigenvalue-mode eigenfunctions and their corresponding time functions, and the lowest negative ω-eigenvalue mode is the dominant mode.

The transport kinetics equations for the perturbed neutron flux $\varphi'(\mathbf{r},E,\Omega,t)$ and the precursors $C_i'(\mathbf{r},t)$ in ADSS are:

$$\frac{1}{v}\frac{\partial}{\partial t}\varphi'(\mathbf{r},E,\Omega,t) = -\,\Omega\bullet\nabla\varphi' - \Sigma_t\varphi' + \mathbf{S}\varphi' + (1-\beta)\chi_p\mathbf{F}\varphi' + \sum_{i=1}^{6}\chi_i\lambda_i C_i'(\mathbf{r},t) + Q(\mathbf{r},E,\Omega,t) \qquad (6)$$

$$\frac{\partial}{\partial t}C_i'(\mathbf{r},t) = \beta_i\mathbf{F}\varphi' - \lambda_i C_i'(\mathbf{r},t) \;,\; i=1,\ldots,6 \qquad (7)$$

where $Q(\mathbf{r},E,\Omega,t)$ is the accelerator-driven external source. Similarly, the perturbed neutron flux is decomposed into a time function P'(t) and a shape function $\psi'(\mathbf{r},E,\Omega)$, the perturbed delayed neutron precursors are decomposed into time functions $B_i'(t)$, and shape functions $\zeta_i'(\mathbf{r})$ as

$$\varphi'(\mathbf{r},E,\Omega,t) = P'(t)\,\psi'(\mathbf{r},E,\Omega) \qquad (8)$$

$$C_i'(\mathbf{r},t) = B_i'(t)\,\zeta_i'(\mathbf{r}) \;,\; i=1,\ldots,6 \qquad (9)$$

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Since the ADSS is considered as a perturbed system to the subcritical system, the perturbed portions of the flux and the precursors are expressed in terms of $P\Delta\psi+\psi\Delta P$ and $B_i\Delta\zeta_i+\zeta_i\Delta B_i$ based on the first-order perturbation theory (the second-order terms $\Delta P\Delta\psi$ and $\Delta B_i\Delta\zeta_i$ are neglected) such that:

$$\varphi' = P'\psi' = (P+\Delta P)(\psi+\Delta\psi) = P\psi+P\Delta\psi+\psi\Delta P+ \Delta P\Delta\psi \cong \varphi + P_o e^{\omega t}\Delta\psi+\psi\Delta P \qquad (10)$$

$$C_i' = B_i'\zeta_i' = (B_i+\Delta B_i)(\zeta_i+\Delta\zeta_i) = B_i\zeta_i+B_i\Delta\zeta_i+\zeta_i\Delta B_i+\Delta B_i\Delta\zeta_i \cong C_i + B_{io}e^{\omega t}\Delta\zeta_i +\zeta_i\Delta B_i \, . \qquad (11)$$

Substituting Eqs. (10) and (11) into Eqs. (6) and (7) with use of Eqs. (1) and (2) leads to:

$$\mathbf{L}\Delta\psi(\mathbf{r},E,\Omega) = \frac{1}{Po} e^{-\omega t} \{ \Omega\bullet\nabla\psi\Delta P + \psi\frac{1}{v}\frac{d}{dt}\Delta P + \Sigma_t\psi\Delta P - \mathbf{S}\psi\Delta P - (1-\beta)\chi_p\mathbf{F}\psi\Delta P - \sum_{i=1}^{6}\chi_i\frac{li}{w+li}[\beta_i\mathbf{F}\psi\Delta P - \frac{d}{dt}\Delta B_i\zeta_i + \omega\zeta_i\Delta B_i] - Q(\mathbf{r},E,\Omega,t)\} \, . \qquad (12)$$

Eq. (12) is an inhomogeneous equation to the ω-mode eigenvalue equation Eq. (5). Based on the Fredholm Alternative Theorem, Eq. (12) has a particular solution if and only if the generalized source terms of the right hand side are orthogonal to the adjoint eigenfuctions, ψ_j^+, of Eq. (5). This solubility condition leads to:

$$\frac{d}{dt}\Delta P_j(t) - \frac{rj - bj,eff}{\Lambda j}\Delta P_j(t) - \sum_{i=1}^{6}\frac{li}{w+li}[\beta_{ij,eff}\Delta P_j(t) - \frac{d}{dt}\Delta B_{ij}(t) +\omega\Delta B_{ij}(t)] - Q_j(t) = 0, \quad j=1,\ldots,7 \qquad (13)$$

where

$$\Lambda_j \equiv \frac{1}{Fj}<\psi_j^+\frac{1}{n}\psi_j>, \quad \rho_j \equiv \frac{1}{Fj}<\psi_j^+(-\Sigma_t\psi_j + \mathbf{S}\psi_j + \chi_p\mathbf{F}\psi_j)>$$

$$\beta_{ij,eff} \equiv \frac{1}{Fj}<\psi_j^+\chi_i\beta_i\mathbf{F}\psi_j> , \quad \beta_{j,eff} \equiv \sum_{i=1}^{6}\beta_{ij,eff} , \quad F_j \equiv <\psi_j^+\chi\mathbf{F}\psi_j>$$

$$\Delta B_{ij}(t) \equiv \frac{1}{\Lambda jFj}<\psi_j^+\chi_i\zeta_i>\Delta B_i , \quad Q_j(t) \equiv \frac{1}{\Lambda jFj}<\psi_j^+ Q(\mathbf{r},E,\Omega,t)> \, .$$

The balance equations for the precursor first-order perturbed time functions can be recovered by defining:

$$\Delta B_{ij}(t) \equiv \frac{1}{w+li}[\beta_{ij,eff}\Delta P_j(t) - \frac{d}{dt}\Delta B_{ij}(t) +\omega\Delta B_{ij}(t)] \, . \qquad (14)$$

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Substituting Eq. (14) into Eq. (13) and rearranging both equations lead to the point kinetics equations:

$$\frac{d}{dt}\Delta P_j(t) = \frac{rj - bj,eff}{\Lambda j}\Delta P_j(t) + \sum_{i=1}^{6} \lambda_i \Delta B_{ij}(t) + Q_j(t), \qquad (15)$$

$$\frac{d}{dt}\Delta B_{ij}(t) = \beta_{ij,eff}\Delta P_j(t) - \lambda_i \Delta B_{ij}(t), \qquad i=1,\ldots,6;\ j=1,\ldots,7\ . \qquad (16)$$

Then, the particular solution of Eq. (12), $\Delta\psi(\mathbf{r},E,\Omega)$, can be effectively solved from

$$\mathbf{L}\Delta\psi(\mathbf{r},E,\Omega) = \Omega\bullet\nabla\psi \frac{1}{Po} e^{-\omega t}\Delta P\ . \qquad (17)$$

Consequently, the solution for the neutron flux in the ADSS is:

$$\varphi'(\mathbf{r},E,\Omega,t) \cong \sum_{j=1}^{7} \{P_{oj}e^{\omega j\, t}\psi_j(\mathbf{r},E,\Omega) + P_{1j}e^{\omega j\, t}\Delta\psi_j(\mathbf{r},E,\Omega) + P_{2j}\psi_j(\mathbf{r},E,\Omega)\Delta P_j(t)\}, \qquad (18)$$

where P_{oj}, P_{1j} and P_{2j} are coefficients. The lowest negative ω-eigenvalue mode in Eq. (18) is the dominant mode.

For illustration, it should be simpler to analyze the time- and space-dependent neutron behavior in an ADSS with an effective one group of delayed neutrons, i.e., i=1 and j=1,2 for Eqs. (15) and (16), as a starting point.

Conclusion

An ω-mode first-order perturbation theory has been developed for analyzing the time- and space-dependent neutron behavior in ADSS. The generalized point-kinetics equations have been systematically derived using the ω-mode first-order perturbation theory and Fredholm Alternative Theorem. Seven sets of the ω-mode eigenvalues exist with using six groups of delayed neutrons and all ω eigenvalues are negative in ADSS. Seven ω-mode adjoint and forward eigenfunctions have been employed to form the point-kinetic parameters. The neutron flux is expressed as a linear combination of the products of seven ω-eigenvalue-mode shape functions and their corresponding time functions up to the first order terms, and the lowest negative ω-eigenvalue mode is the dominant mode.

References

1. H. Safa et al., "A Super-conducting Proton Linear Accelerator For Waste Transmutation", 3rd International Conference on Accelerator Driven Transmutation Technologies and Applications (ADTT'99), Praha, Czech Republic, 1999.
2. D. Ridikas, H. Safa, and B. Bernardin, "A Beta Compensated Reactor (BCR) Driven by Electron Accelerator'', Session 4B, International Conference on the New Frontiers of Nuclear Technology:

Reactor Physics, Safety and High-Performance Computing, PHYSOR 2002, Seoul, Korea, October 7-10, 2002.

3. D.G. Cacuci, "On Perturbation Theory and Reactor Kinetics: From Wigner's Pile Period to Accelerator Driven Systems," Plenary Session, International Conference on the New Frontiers of Nuclear Technology: Reactor Physics, Safety and High-Performance Computing, PHYSOR 2002, Seoul, Korea, October 7-10, 2002.
4. Akcasu, G. S. Lellouche and L. M. Shokin, "Mathematical Methods in Nuclear Reactor Dynamics," Academic Press, New York, 1971.
5. R-T. Chiang and J. J. Dorning, "A Spatially Asymptotic Solution to the Energy-Dependent ω-mode Transport Equation," Trans. Am. Nucl. Soc. 34, 280 (1980).