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WATER HAMMER IN ELASTIC PIPES

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ABSTRACT

One dimensional two-fluid six-equation model of two-phase flow, that can be found in computer codes like RELAP5, TRAC, and CATHARE, was upgraded with additional terms, which enable modelling of the pressure waves in elastic pipes. It is known that pipe elasticity reduces the propagation velocity of the shock and other pressure waves in the piping systems. Equations that include the pipe elasticty terms are used in WAHA code, which is being developed within the WAHALoads project of 5th EU research program.

1 INTRODUCTION

Safety analyses of nuclear reactors require computations of complex two-phase flows. Several computer codes like RELAP5, CATHARE, TRAC were developed for this purpose. All these codes are based on one-dimensional two-fluid six-equation models of two-phase flow. Despite the fact that these codes are not verified for water hammer fast transients, they are often the only available tools for two-phase water hammer simulations. It is the fact that the RELAP5 code successfully covers the area of the transients with characteristic time scale determined by the fluid velocity, but it has to be used with extreme caution for transients that include acoustic waves. One of the features not covered by upper codes is elasticity of the pipe wall, which affects the propagation of the pressure waves in the pipe. This paper presents an extension of the RELAP5's two-fluid model and computer code based on characteristic upwind numerical scheme described in [2,3] that takes into account the elasticity of the pipe.

The current version of the RELAP5 code describes two-phase flow with a six equation two-fluid model. The system of the six first-order partial differential equations is derived from the cross-section averaged Navier-Stokes equations. Diffusion terms with second-order derivatives are replaced with empirical correlations, which are flow regime dependent. The type of the flow regime is determined by the flow parameters and the geometry. The basic RELAP5 equations are ill posed, i.e. non-hyperbolic with complex eigenvalues. The ill-posedness of the discretised equations is avoided by the first-order accurate numerical scheme used in RELAP5. The scheme is based on a semi-implicit finite difference scheme with staggered grid and donor-cell discretization of the convective terms. The method is first-order accurate in time and space. The main advantages of such a scheme are robustness and efficiency. The weak side of the scheme is its numerical dissipation, which tends to smear discontinuities on coarse grids [1,2,3].

WAHA code development within the WAHALoads project, which is part of 5th research program of European Union, aims to improve the existing two-fluid model and to use new, second-order accurate method for the simulation of fast transients in 1D two-phase flow. Simulations with second-order scheme of WAHA code described in the present paper require a hyperbolic set of equations. Equations of two-fluid model were made hyperbolic with improved form of the virtual mass term. The second-order accurate numerical scheme applied in WAHALoads code development is based on the Godunov method. Its description and behaviour with the equations of the two-fluid model was analysed in [1].

Next chapter is reserved for the presentation on how the pipe elasticity is taken into account in the basic equations. Chapter 3 discusses the implementation of new code on basic benchmark example of a water-hammer transient, where the advantage of equations with elasticity terms is obvious. Experimental results presented and used for validation of developed code are from two-phase water-hammer transient initiated with rapid valve closure performed by A. R. Simpson [5].

2 PIPE ELASTICITY IN BASIC EQUATIONS

2.1 Influence of pipe elasticity

Elasticity modulus has a remarkable influence on a propagation of pressure waves in 1D pipes. The equation from Streeter, Wylie [6] gives the speed of the small pressure wave c in 1D elastic pipe filled with single-phase fluid:

$$\frac{1}{c^2} = \frac{1}{c_0^2} + K \cdot \rho \tag{1}$$

With speed of sound in pure fluid: $c_0^2 = \left(1 - \left(\frac{\partial \rho}{\partial u}\right)_p \frac{p}{\rho^2}\right) \Big/ \left(\frac{\partial \rho}{\partial p}\right)_u$, and contribution of the elasticity: $K = D/(E \cdot d)$, where D is pipe diameter, d is pipe wall thickness and E modulus of elasticity. Fig. (1) shows the speed of the small pressure wave as a function of the elasticity modulus. Constant values used in Eq. (1) to plot the curve in Fig. (1) were: c_0 = 1490 m/s, D = 1.905 cm, d = 1.588 mm (pipe from Simpson's [1] experiment), and variable elasticity modulus E. The small pressure waves propagate along the pipe with the speed of sound modified by the pipe elasticity. If the elasticity modulus is infinite (stiff pipe) then the propagation velocity of the small pressure wave is equal to the speed of sound. Figure (1) shows the need to introduce the pipe elasticity into the basic equations for the given pipe. If the speed of sound in the stiff pipe of given diameter and wall thickness is approximately 1490 m/s, we can see in Fig. (1) that reduced speed (E = 1.25 N/m^2 copper) is about 1360 m/s. The difference is significant and it is approximately 9 %.

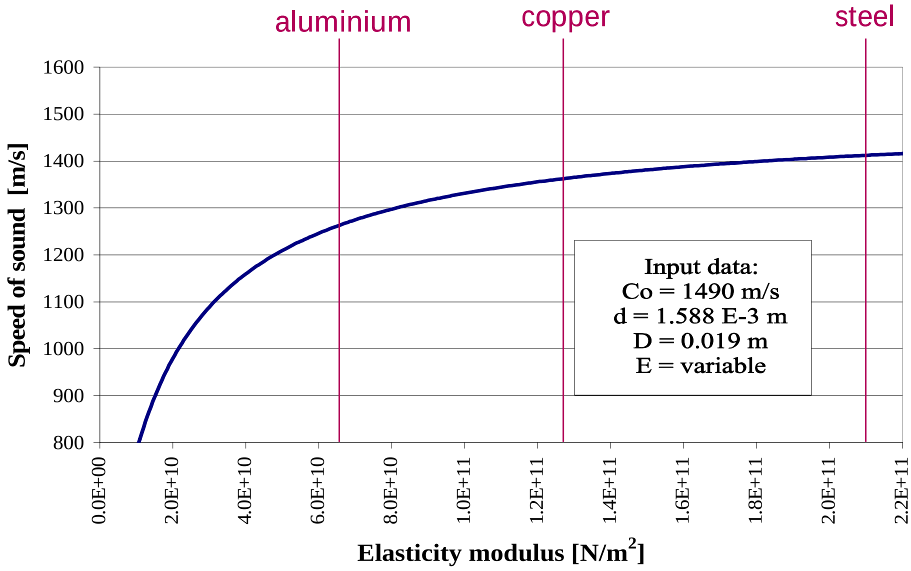

Figure 1: *Speed of sound - dependency on Elasticity modulus*

The reduction of the small pressure wave speed c has another important impact. Simple equation of Joukowsky [6] shows the influence of the speed c on the pressure jump Δp when the fluid flow with density ρ and initial velocity v_0 is abruptly stopped by a sudden valve closure:

$$\Delta p = \rho \cdot v_0 \cdot c \tag{2}$$

Thus, reduction of the speed c means also reduction of the pressure jump. This is one of the basic reasons why the existing computer codes cannot describe the water hammer phenomena in elastic pipes.

2.2 Basic facts about WAHA and RELAP5 code

RELAP5 6-equation two-fluid model was used as the basis of the WAHA code. Some of the main differences between the WAHA and RELAP5 equations are:

- terms with spatial derivatives in virtual mass correlation, which are neglected in RELAP5, are included in WAHA code and virtual mass coefficient is slightly modified to ensure the hyperbolicity of the equations,
- inter-phase mass, momentum and heat transfer between two phases are assumed to be infinitely fast (only in the current version of the WAHA code - more realistic models are being developed),
- simplified flow regime map is applied in WAHA code (recognizes only dispersed and separated flow),
- terms describing pipe elasticity are present in WAHA code,
- different numerical scheme (first-order accurate in RELAP5, second-order in WAHA).

Homogeneous-equilibrium assumption is judged to be acceptable for the calculation of fast transients like the Simpson's experiment [5], where only a small amount of vapor appears in the pipe. This vapor is very close to thermal and mechanical equilibrium with liquid phase. Thus, the present simulation is actually simulation of homogeneous-equilibrium flow with six-equation two-fluid model. Of course, the two-fluid model is used in WAHA code, because it can also describe thermal and mechanical non-equilibrium when necessary.

2.3 Equations of the two-fluid model

The following modification of the two-fluid equations is required to introduce the pipe elasticity: pipe cross-section is assumed to be varying due to the variable initial pipe's geometry and due to the pressure changes along the pipe:

$$A(x,t) = A(x) + A_e(p(x,t)) \tag{3}$$

Second term with A_e is much smaller than the first one. The pressure pulse changes the pipe diameter in accordance with linear relation (Streeter, Wylie [6]):

$$\frac{dA_e}{A(x,t)} = \frac{D}{d} \cdot \frac{dp}{E} = K \cdot dp \tag{4}$$

With both relations taken into account the six-equation two-fluid model (continuity, momentum and internal energy equations for fluid and gas) can be rearranged into the following form.

Continuity equation for liquid phase:

$$\begin{aligned} &\frac{\partial (1-\alpha)\rho_f}{\partial t} + (1-\alpha)\rho_f K \frac{\partial p}{\partial t} + \frac{\partial (1-\alpha)\rho_f v_f}{\partial x} + (1-\alpha)\rho_f v_f K \frac{\partial p}{\partial x} = \\ &-\Gamma_g - (1-\alpha)\rho_f v_f \frac{1}{A(x)} \frac{dA(x)}{dx} \end{aligned} \tag{5}$$

and for vapor (gas) phase:

$$\begin{aligned} &\frac{\partial \alpha \rho_g}{\partial t} + \alpha \rho_g K \frac{\partial p}{\partial t} + \frac{\partial \alpha \rho_g v_g}{\partial x} + \alpha \rho_g v_g K \frac{\partial p}{\partial x} = \\ &\Gamma_g - \alpha \rho_g v_g \frac{1}{A(x)} \frac{dA(x)}{dx} \end{aligned} \tag{6}$$

where the temporal changes of cross-section *A(x,t)* are neglected in the denominators of the last term of Equations (5) and (6).

Internal energy equation for liquid phase with terms for pipe elasticity is:

$$\begin{aligned} &(1-\alpha)\rho_f \frac{\partial u_f}{\partial t} + (1-\alpha)\rho_f v_f \frac{\partial u_f}{\partial x} - p\frac{\partial \alpha}{\partial t} + p(1-\alpha)K\frac{\partial p}{\partial t} + p\frac{\partial (1-\alpha)v_f}{\partial x} + p(1-\alpha)v_f K \frac{\partial p}{\partial x} = \\ &Q_{if} - \Gamma_g (u_f^* - u_f) + v_f F_{f,wall} - (1-\alpha)v_f p \frac{1}{A(x)} \frac{dA(x)}{dx} \end{aligned} \tag{7}$$

and for vapor phase:

$$\alpha\rho_g\frac{\partial u_g}{\partial t}+\alpha\rho_g v_g\frac{\partial u_g}{\partial x}+p\frac{\partial\alpha}{\partial t}+p\alpha K\frac{\partial p}{\partial t}+p\frac{\partial\alpha v_g}{\partial x}+p\alpha v_g K\frac{\partial p}{\partial x}= \\ Q_{ig}+\Gamma_g(u_g^*-u_g)+v_g F_{g,wall}-\alpha v_g p\frac{1}{A(x)}\frac{dA(x)}{dx} \tag{8}$$

Again, the temporal changes of cross-section $A(x,t)$ are neglected in the denominators of the last terms of Equations (7) and (8),

Momentum equations are equal because they do not contain cross-section term. Momentum equation for liquid phase is:

$$(1-\alpha)\rho_f\frac{\partial v_f}{\partial t}+(1-\alpha)\rho_f(v_f-w)\frac{\partial v_f}{\partial x}+(1-\alpha)\frac{\partial p}{\partial x}-CVM-p_i\frac{\partial\alpha}{\partial x}= \\ C_i\,|v_r|\,v_r-\Gamma_g(v_i-v_f)+(1-\alpha)\rho_f g\cos\theta-F_{f,wall} \tag{9}$$

and for vapor phase:

$$\alpha\rho_g\frac{\partial v_g}{\partial t}+\alpha\rho_g(v_g-w)\frac{\partial v_g}{\partial x}+\alpha\frac{\partial p}{\partial x}+CVM+p_i\frac{\partial\alpha}{\partial x}= \\ -C_i\,|v_r|\,v_r+\Gamma_g(v_i-v_g)+\alpha\rho_g g\cos\theta-F_{g,wall} \tag{10}$$

Additional closure equations are:

- Two equations of state for phase k (fluid or gas)

$$d\rho_k=\left(\frac{\partial\rho_k}{\partial p}\right)_{u_k}dp+\left(\frac{\partial\rho_k}{\partial u_k}\right)_p du_k \tag{11}$$

Derivatives on the right hand side of the equation (11) are determined by the water property subroutines.

- The virtual mass term CVM in momentum equations (9) and (10)

$$CVM=C_{VM}\left(\frac{\partial v_g}{\partial t}+v_f\frac{\partial v_g}{\partial x}-\frac{\partial v_f}{\partial t}-v_g\frac{\partial v_f}{\partial x}\right) \tag{12}$$

The main purpose of the virtual mass term is to ensure hyperbolicity of equations.

Values of the inter-phase friction coefficient C_i in Eqs. (9) and (10) and heat transfer coefficients (hidden in Γ_g and $Q_{if,ig}$) in Eqs. (5)-(8) are assumed to be infinite in the present version of the code. Realistic correlations were applied for the wall friction terms $F_{f,g,wall}$ in Eqs. (9) and (10).

2.4 Vectorial form of equations

With the relations (9) the derivatives of densities are expressed with derivatives of pressure and internal energies and together with (10) the equations of the six-equations model can be written in vectorial form as:

$$\mathbf{A}\frac{\partial\vec{\psi}}{\partial t} + \mathbf{B}\frac{\partial\vec{\psi}}{\partial x} = \vec{S} \tag{12}$$

where $\vec{\psi}$ represents the vector of the independent variables $\vec{\psi} = (p, \alpha, v_f, v_g, u_f, u_g)$. **A** and **B** are matrices of the system and $\vec{S}$ is a vector with non-differential terms in the equations.

3 APPLICATION OF WAHA CODE

One of the benchmarks for application of WAHA code is Simpson's pipe experiment [5]. Scheme for Simpson's experiment with initial conditions is shown in Fig. (2). Results in Fig. (3) are given for the same geometry and initial conditions as results in Fig. (4); the only difference between both figures is initial velocity, which was 0.2 m/s in Fig. (3) and 0.4 m/s in Fig. (4). Simpson's experiment [5] is one of basic benchmarks for two-phase flow code due to its simple geometry and wide range of phenomena that it covers. In WAHA code it is modeled as a single pipe divided into 100 volumes and connected to the tank (constant pressure boundary condition) in one side, while the other side represents the boundary, i.e., instantaneously closed valve. Pipe is filled with liquid water and steady state with initial velocity of water in the pipe is present.

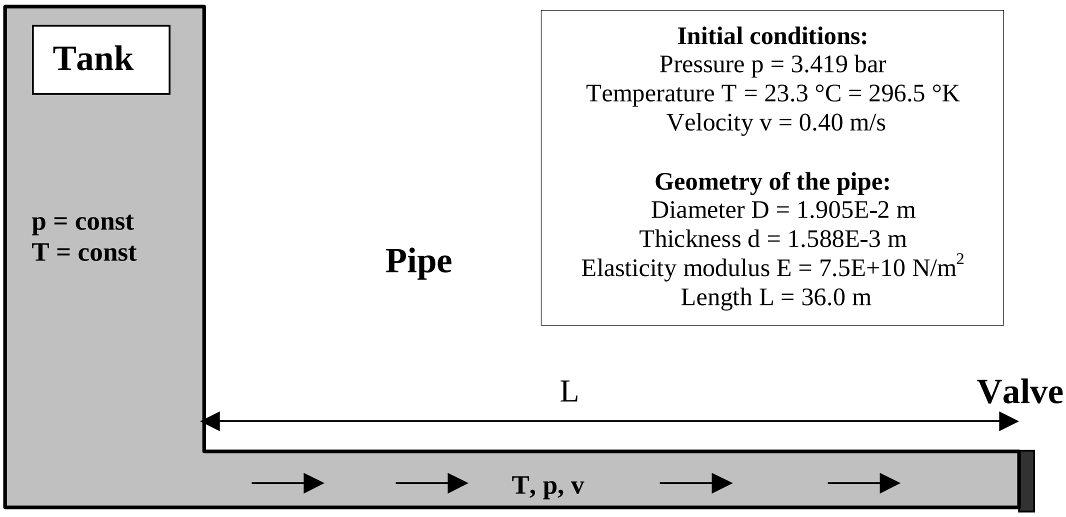

Figure 2: *Geometry and initial conditions of Simpson's experiment*

At the moment of valve closure the pressure at the valve would increase by an amount $\Delta p \approx \rho_f c_f v$ (see Eq. (2)). The pressure is maintained until the expansion wave reflected from the free end and arrives back after time $2L/c_f \approx 0.05s$. Then the pressure should drop for the same value $\Delta p \approx \rho_f c_f v$ beyond the initial pressure. Two cases are possible:

- *Lower pressure limit is greater or equal than saturation level* - no flashing appears in that case and flow in the pipe remains single-phase (Fig. 3). Single pressure wave propagates up and down along the pipe and pressure history near the valve is periodic.

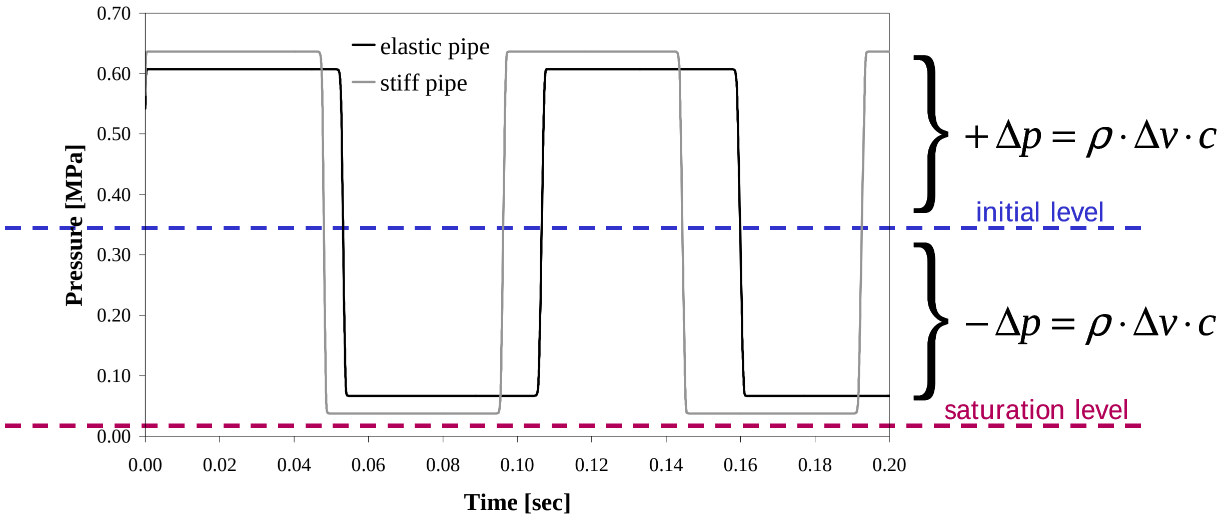

Figure 3: *Pressure history near the valve – no flashing (v_0 = 0.20 m/s)*

- *Lower pressure limit is less than saturation level* – as pressure drops bellow the saturation level the flashing appears near the valve and the created vapor maintains the pressure at the local saturation conditions (Fig. 4).

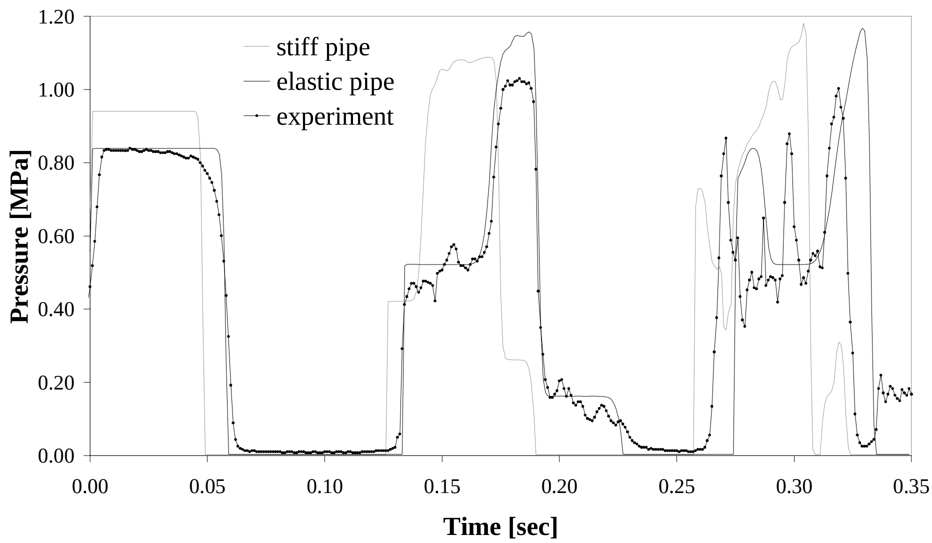

Figure 4: *Pressure history near the valve – flashing (v_0 = 0.40 m/s)*

Flashing thus appears in the second benchmark case of Simpson's experiment. When the second expansion wave reflected from the tank reaches the valve two reflections appear: one from the vapor (Fig. (4), t = 0.13 sec) and the other from the valve after condensation collapse (Fig. (4), t = 0.17 sec). Combination of both waves creates peak pressure (Fig. (4), t = 0.18 sec), which is higher than the initial pressure rise. Each appearance and disappearance of the vapor pockets in the system creates a new pressure wave and adds to the complexity of the transient. It is important to stress that the two-phase water hammer transient results in more then one pressure wave in the system. When these pressure waves interact they produce pressure peaks higher than the initial pressure rise near the valve, which is equal to the pressure rise in single-phase system. It is also important to underline that elasticity of the pipe in the second Simpson's experiment results in pressure peak that is higher that would be the same pressure peak in the stiff pipe. Again, such result can appear only in two-phase systems.

4 CONCLUSIONS

It is clear from the results obtained with the stiff and elastic pipe (Fig. (3) and (4)), that elasticity of the pipe cannot be neglected in the Simpson's pipe experiment. However, it is important to add that the elasticity modulus was modified as suggested by Simpson [5] on the basis of the measured sonic velocity: value $E = 0.75 \times 10^{11}$ N/m^2 was used in our calculation instead of 1.2×10^{11} N/m^2 (copper). Exact reason for that discrepancy is not given in [5].

Pressure waves in single-phase systems are weaker if pipe elasticity is taken into account. However, in the two-phase systems, where peak pressures are result of the interaction of several pressure waves, pressures in the elastic pipes can be even larger than in the stiff pipes as shown in the second Simpson's pipe experiment.

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