# Progress in the theory of magnetic reconnection phenomena

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**Abstract.** Recent theoretical work on magnetic reconnection in hot plasma confinement devices is reviewed. The presentation highlights the common aspects of reconnection phenomena, and current research trends are emphasised. Progress in understanding the dynamics of slowly evolving modes of the tearing family, based on advanced analytic techniques and numerical simulation, as well as of faster modes that lead to internal disruptions, is reported.

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## 1. Introduction

Magnetic reconnection is a common phenomenon in laboratory and space plasmas. In this work, a review of recent advances and trends is given.

To illustrate the concept, in a way that will be convenient for the rest of this article, consider first a simple two-dimensional (2D) system where, with respect to a set of Cartesian coordinates $(x, y, z)$, all the physical quantities of interest are constant in a given direction $z$. In this system, the magnetic field $\mathbf{B}$ can be written as

$$\mathbf{B} = \mathbf{B_z}\hat{\mathbf{z}} + \nabla \times \psi\hat{\mathbf{z}}, \qquad (1)$$

where $\psi$ is the vector potential component along $z$, also called magnetic flux function, $\hat{z}$ is the versor in the $z$ direction and $B_z$, often called the guide field, can be in general a function of $(x, y)$.

In an ideal plasma, characterized by the ideal Ohm's law $\mathbf{E} + (\mathbf{v}/\mathbf{c}) \times \mathbf{B} = \mathbf{0}$, magnetic field lines are "frozen in the flow", which means that two material points that belong to a field line at a given time, will belong to the same field line at a later time.

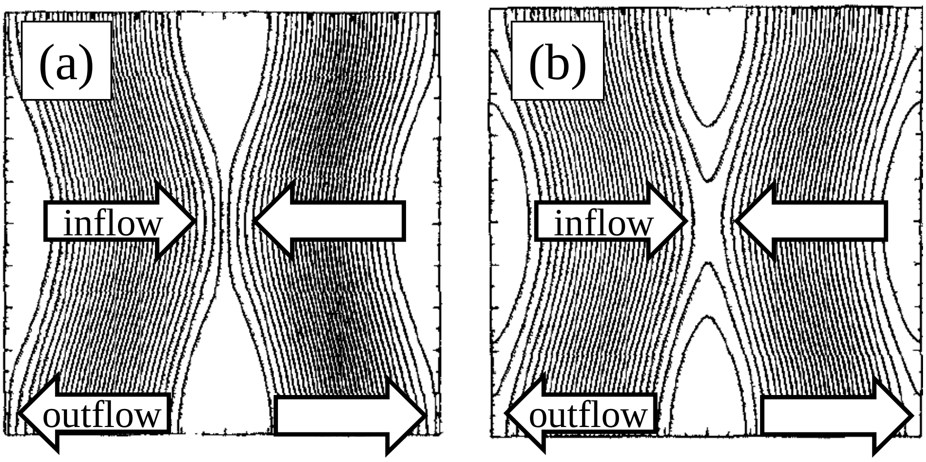


**Figure 1.** Deformation of the magnetic field lines in 2D under a model flow. a) ideal plasma and b) non-ideal plasma with reconnection

For systems possessing an ignorable coordinate, like the one described in Eq. 1, ideal Ohm's low implies the conservation of the magnetic flux function as it can be seen by taking the z component of Ohm's law:

$$\partial\psi/\partial t + \mathbf{v}\cdot\nabla\psi = \mathbf{0} \qquad (2)$$

where $\mathbf{v}$ and $\nabla$ operate in the $(x, y)$ plane. Thus $\psi$ is conservatively advected by the flow. Note that this occurs regardless of what causes the flow, or whether a guide field is present and irrespectively of its size or spatial dependence.

Consider now a magnetic configuration where initially the magnetic flux function depends on a spatial variable $(x)$ only, $\psi = \psi_0(x)$. Assume that $\psi_0(x)$ has an extremum at given location, $x = 0$. This defines a line, called the neutral line, where the magnetic field perpendicular to $\hat{z}$ is null. At points on the neutral line where the flow is convergent, $\partial v_x/\partial x < 0$, the local curvature of the flux function increases, which corresponds to an increase of the magnetic field gradient and of the local current density in the $z$ direction, $J$, since, from Ampere's law, $4\pi J/c = -\nabla^2\psi$. At the same time, the value of $\psi$ stays constant on the neutral line. This situation is depicted in Fig. 1a for a model flow. While the flow can in principle change the shape of the neutral line, material points initially situated on different sides of the neutral line will stay so, since connections by field lines are maintained and the initial topology of the system is preserved.

For slightly non-ideal plasmas, connections by field lines are not preserved. However, this has little consequence on the overall topology almost everywhere, except near the neutral line where the non-conserved flux function develops local saddle points (X-points) and local extrema (O-points). This situation is depicted in Fig. 1b, where the formation of a magnetic island that topologically connects regions that where initially unconnected is apparent. Physical phenomena where reconnection plays an important role abound in space as well as in laboratory plasmas. Known examples are the region in the Earth's magnetopause where the Earth's magnetic field is connected with the interplanetary magnetic field; magnetic substorms in the Earth's magnetotail, and solar flares. In plasma confinement machines, current driven instabilities of the tearing mode family are also common examples. In the following, we refer in particular to the simple cylindrical approximation of a tokamak with a constant axial (toroidal) field $B_\phi$ along $z$. Considering helical states, where all the physical quantities depend on the radial coordinate $r$ and on the angular coordinates $\theta$ (poloidal) and $\phi$ (longitudinal or toroidal) only through the combination $m\theta - n\phi$, one finds that the helical flux $\psi_{*mn} = \psi + nr^2 B_T/2mR$ is conserved by the ideal MHD dynamics. Thus, as far as reconnection is concerned, the flux $\psi_{*mn}$ plays the role previously played by $\psi$ in the ignorable $z$ case. Specifically, if one assumes initial conditions, here denoted by the suffix 0, where all the quantities depend only on $r$, one finds that $\psi_{*mn}$ has a neutral line where $0 = d\psi_{*mn0}/dr = d\psi_0/dr + nrB_\phi/mR$. Upon introduction of the poloidal magnetic field $B_{\theta 0} = -d\psi_0/dr$ and of the safety factor $q = rB_\phi/RB_{\theta 0}$, one finds that $\psi_{*mn0}$ has an extremum on the "rational" surface $q(r) = m/n$. This extremum is a minimum for monotonic, increasing $q$ profiles, which is the most common situation in tokamaks. Around rational surfaces, non-conservation of $\psi_{*mn}$ by non-ideal plasma effects leads to a chain of $m$ islands in the $\psi_{*mn}$ isocontours as it can be seen in the poloidal plane. For single helicity states, $\psi_{*mn}$ isocontours identify the magnetic surfaces, which are helical, with their proper magnetic axis, in a neighbourhood of the rational surfaces. When $q$ is non-monotonic, multiple rational surfaces with the same helicity exist. If the conditions for a tearing instability are satisfied, double (or possibly multiple) chains of islands occur. An important example is discussed in Sec. 4. Finally, island chains of different helicity can also develop. In this case, chaotic field lines occupy part of the volume, where magnetic surfaces do not exist. This situation will be briefly considered in Sec. 5 in conjunction with ongoing work.

## 2. Tearing mode energetics and classification

In this section, we concentrate more specifically on the case where magnetic reconnection occurs as a consequence of a tearing mode instability. The stability properties of tearing modes depend on the parameter[1]

$$\Delta' \equiv \lim_{x \to 0_+} \frac{dlog(\psi_{\rm L})}{dx} - \lim_{x \to 0_-} \frac{dlog(\psi_{\rm L})}{dx} > 0\,, \tag{3}$$

where $\psi_{\rm L}$ is the ideal MHD magnetic flux eigenfunction and $x$ denotes the distance from the reconnection layer. Taken literally, if one could use linear ideal MHD in the whole system, $\Delta'$ would represent a singular, modulated current density at $x = 0$, proportional to the magnetic field discontinuity. Since this singularity is resolved by non-ideal plasma effects, for thin reconnecting layers $\Delta'$ is in reality proportional to the total modulated current in the layer. Also, one finds that this current is destabilising[2], in the sense that it acts in the direction to increase an initial perturbation, when $\Delta' > 0$. For a simple MHD model, this is a necessary and sufficient condition for instability in the limit of small resistivity, although the actual $\Delta'$ tearing mode threshold can be substantially

| Δ' regime | linear | nonlinear |
|---|---|---|
| $\Delta' L << 1$ | $\gamma \sim \eta^{3/5} \Delta'^{4/5} \gamma_A$ | Rutherford growth $w \sim \eta \Delta' t$ and saturation |
| $\Delta' L >> 1$ & $\Delta' d_L << 1$ | | $w \sim \eta \Delta' t$, then $w \sim \eta t^2$ (Sweet-Parker) |
| $\Delta' d_L >> 1$ | $\gamma \sim \eta^{1/3} \gamma_A$ | $w \sim \eta t^2$ complete reconnection ? |

**Table 1.** Regimes of linear and nonlinear behaviour of the resistive tearing mode

different when additional effects are included. As expected, one also finds that the modes become progressively more energetic as $\Delta'$ increases. The various regimes of linear[3] and nonlinear growth of the resistive tearing are summarised in Table 1. As one can see, the smallest $\Delta'$ regime is defined qualitatively by the condition $\Delta' L < 1$, where $L$ is the spatial scale of the system. In this situation the mode grows linearly with growth rate $\gamma \sim \eta^{3/5} \Delta'^{4/5}$[1], nonlinearly *à la* Rutherford[2], with an island size $w$ behaving like $w \sim \eta \Delta' t$ and eventually saturates on the resistive timescale $\tau_R \sim L^2/\eta$ with a width of order $w \sim \Delta' L^2$. The precise saturation condition has been calculated only recently[4, 5] as discussed in the next section. One notes that throughout its entire growth the island width satisfies $w\Delta' < 1$. Technically, this means that the flux function $\psi$ can be treated as approximately constant in the island region (constant-$\psi$ approximation), a fact that can be exploited in the calculations. Linearly, the $\eta^{3/5}$ growth regime occurs as long as the constant-$\psi$ condition is satisfied, which implies, for linear modes, $\Delta' d_L < 1$ where $d_L \sim \eta^{2/5} \Delta'^{1/5}$ is the width of the singular layer. As $\Delta'$ is increased, this condition is eventually violated as $\Delta' \sim \eta^{-1/3}$. At larger values, the growth rate becomes $\Delta'$-independent, $\gamma \sim \eta^{-1/3}$. This is what happens in particular for the resistive $m = 1$ kink. As its width exceeds $d_L$, the island grows nonlinearly as $w \sim \eta t^2$, with a characteristic timescale $\tau_{SP} \sim \eta^{-1/2}$, obtained by extending the Sweet-Parker scenario to time-dependent situations[6]. As $\Delta'$ approaches infinity the system becomes ideally unstable and it eventually grows on the Alfven timescale. In the interesting intermediate regime of Tab. 1, when $\Delta' L >> 1$, but $\Delta' d_L << 1$, the nonlinear growth occurs initially *à la* Rutherford, but then, as $w\Delta' \sim 1$, the constant-$\psi$ is broken and the faster $w \sim \eta t^2$ sets in[7].

## 3. Growth and saturation of magnetic islands in the small $\Delta'$ regime

Determining the size of magnetic islands is of primary importance in magnetic fusion research. In the regions occupied by the islands the reconnected field lines effectively short-circuit the plasma in the radial direction allowing increased particle and energy losses through parallel transport, reduction of overall confinement, and possibly disruptions. This problem has been tackled with a variety of theoretical approaches and models of increasing complexity. Key progress in the analytic treatment of tearing modes within the reduced MHD approximation, and recent numerical studies of drift-tearing modes and of neoclassical tearing modes with diamagnetic effects are reviewed in this section.

The question of tearing mode saturation in a symmetric plasma sheet has long remained open since the work of Rutherford on the nonlinear growth of the tearing mode[2]. While the correct scaling of the saturated island width $w_s \sim \Delta'$ had long been known, only recently, a quantitatively accurate calculation became available[4, 5]. These works exploit, directly or indirectly, the constant-$\psi$ approximation to solve the nonlinear MHD equations in the island region and to match them asymptotically to the linear, ideal MHD solutions in the outer region, far from the reconnection layer. The width of this layer, $\delta$, is essentially proportional to the island width, and it replaces the thinner linear layer when the island growth becomes nonlinear. As a consequence of the smallness of $\Delta'$ and of the constant-$\psi$ approximation, the flux function has the form[4]

$$\psi(x, y, t) = \psi_0(x) + \delta^2 \cos ky + \delta^4 \tilde{\psi}(\frac{x}{\delta}, ky, \delta, t), \tag{4}$$

where, referring to a magnetic field of the form (1), we have assumed a one dimensional equilibrium $\psi_0(x)$ with tearing perturbations of wavenumber $k$ along $y$. One can see that the leading order, $O(\delta^2)$ modification of the initial equilibrium is independent of $x$, which is valid as long as $x << 1$, which includes the matching region. The magnetic field lines are approximately given by the condition that $\psi_0(x) + \delta^2 \cos ky = \text{const}$, or, developing around $x = 0$, and choosing suitable normalisations, $(x/\delta)^2 - 2cos(ky) = \text{const}$. As a consequence, the shape of the island is unchanged throughout its growth, a fact that can be exploited in the calculations. The final evolution equation takes the form[4]

$$\frac{dw}{dt} \approx 1.22\, \eta\, [\Delta' - \alpha' w], \tag{5}$$

with $\alpha' \approx 0.41 J_0''(0)/J_0(0)$, in terms of the equilibrium current $J_0(x)$. The saturated width $w_s$ computed from Eq. 5 for the Harris pinch is shown in Fig. 2 compared with the numerical points given in Ref. [6] and with the curves given by previous, approximate, work[8, 9].

In the more general case, when the equilibrium is not symmetric, contributions coming from the first derivative of the equilibrium current, $J_0'(0)$ must be taken into account. A calculation by Thyagaraja[10] gave $\Delta' \approx 0.41[J_0'(0)/J_0(0)]^2 w \ln(w^{-1})$. Although this is formally the leading order contribution for very small islands, because of the $ln(w)$ factor, in practice the $O(w)$ terms can be equally or more important. Extension of the theory to include all the relevant terms is under way[11, 12].

In magnetic confinement machines, additional effects, as those coming from the presence of pressure and pressure gradients must be taken into account. Moreover the island dynamics depends in general on the collisionality regime. Inclusion of the pressure (or density) gradient to reduced MHD brings in the effect of the diamagnetic frequency. Then the mode becomes mainly oscillatory, with a substantial reduction of growth rate with respect to ordinary tearing modes (drift-tearing modes[13]). Depending on the

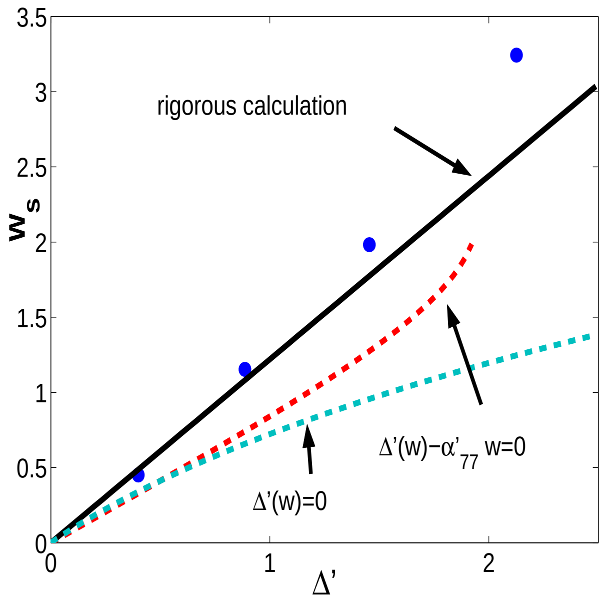


**Figure 2.** Comparison of the numerical solution of the island width of the sheet pinch (dots), with the asymptotic solution given here (solid line) and two former predictions.

collisionality regime, full stabilisation at significant positive $\Delta'$ can be achieved through coupling with finite beta, ion sound dynamics (see Ref. [14]). Finally, the addition of the bootstrap current[15]can lead to a nonlinear destabilisation even at negative $\Delta'$ as a consequence of the bootstrap current modification that follows the evolution of the density profile when the island develops to a sufficiently large size (neo-classical tearing modes[16, 17, 18]).

All this physics is included in the two-dimensional $(x, y)$, modified version of the four field model[19] equations

$$dU/dt = [J, \psi] + \mu\nabla^2 U \tag{6}$$

$$d\psi/dt + v_*\partial\psi/\partial y = [n, \psi] - \eta(J - J_{\rm eq} - J_b) \tag{7}$$

$$dn/dt + v_*\partial\varphi/\partial y = \rho_*^2[J, \psi] + D\nabla^2 n - \beta[v, \psi] \tag{8}$$

$$dv/dt - v_*\partial\psi/\partial y = -[n, \psi] + \chi\nabla^2 v \tag{9}$$

where $\varphi$ is the electrostatic potential (the stream function of the perpendicular plasma flow), $\psi$ is the magnetic flux function, $n$ is the perturbed electron density, $v$ is the parallel ion flow, $U = \nabla^2\varphi$ is the vorticity, $J = -\nabla^2\psi$ the current density along the $z$-direction, $J_{\rm eq}$ the equilibrium current density, which can be assumed to include the equilibrium bootstrap current, and $J_b = C_b\partial_x n$ is the bootstrap current perturbation, proportional to the perturbed gradient and dependent on the plasma parameters through the constant $C_b$. The first two equations are the usual reduced MHD equations (with a generalised Ohm's law), the third is the continuity equation and the fourth is the parallel plasma momentum equation. The Poisson bracket is defined as $[A, B] = (\partial_x A)(\partial_y B) - (\partial_y A)(\partial_x B)$. The total time derivative includes

$\vec{E} \times \vec{B}$ velocity advection, $d/dt \equiv \partial_t + [\varphi, \cdot]$. For a given equilibrium, the model is controlled by nine parameters, which are $\Delta'$, the perpendicular viscosity, $\mu$, the electrical resistivity, $\eta$, the particle diffusion coefficient, $D$, the electron diamagnetic drift velocity, $v_* = -(cT/eB_0 L_s)(dn_{eq}/dx)$, where $B_0$ is the dominant magnetic field component along the $z$-direction and the equilibrium (ambient) density gradient $dn_{eq}/dx$ is assumed constant, the normalized ion sound Larmor radius, $\rho_*$, the electron $\beta$, and the parallel viscosity coefficient $\chi$. The bootstrap current parameter $C_b$ scales like $\sim \beta/\rho_*^{1/2}$, but it is treated independently in the studies reported here.

The case $C_b = 0$ has been considered in Ref. [20]. A systematic study of the four field model was carried out by exploring the $(\omega_*, \beta)$ plane at moderate $\Delta'$, in the semi-collisional regime satisfying the condition $\rho_* > \eta^{1/3}$. Several dynamical regimes were found. In some of these, the occurrence of multiple stable solutions, accessible from different initial conditions, was observed. At small enough $(\omega_*, \beta)$, the only stable state is an ordinary tearing-mode magnetic island, which can be obtained by allowing a perturbation to grow from its drift-tearing linear phase. In this process, the island frequency $\omega$, initially close to the diamagnetic frequency, gradually decreases as the island grows, eventually settling to a low value $\omega << \omega_*$. In the saturated state, the initial density gradient is compensated, in the island region, by its perturbation caused by the enhanced radial transport caused by field line reconnection. One also observes the formation of a self-generated zonal flow that contributes to the final island frequency. By increasing $\beta$ at constant $\omega_*$, one crosses the boundary above which drift-tearing modes are linearly stable. This is given approximately by the value $\beta_{\mathrm{cr}} = \rho_*^2 \Delta'^{4/3}/3^{4/3}\omega_*^{2/3}$, in the semi-collisional regime, as discussed in Ref. [14]. One observes that when this boundary is slowly exceeded, starting from an initially saturated island, the island survives, provided that $\omega_*$ is not too large. This identifies a region where two stable solutions of the model equations exist, one corresponding to a saturated, tearing-like island and the other to the stable initial equilibrium. When $\omega_*$ is increased at constant, not-too-large $\beta$, the initial equilibrium remains always unstable. However, at large enough diamagnetic frequency, a new solution coexists, in a certain domain of the $(\omega_*, \beta)$ plane, with the usual tearing island. The new solution is accessed by starting with a small perturbation, close to the unstable equilibrium, and allowing the mode to grow. One observes that the mode saturates at small amplitude, with a frequency close to $\omega_*$, and with delocalised $\mathbf{E} \times \mathbf{B}$ flow. This state appears as an electromagnetic drift-wave, which is a property of the four-field model. Finally, at large enough $(\omega_*, \beta)$, full stabilisation occurs. More details on the four-field model at $C_b = 0$ can be found in Ref. [20].

Theoretical studies of neoclassical tearing modes are usually carried out in the framework of simple MHD, modified with the neoclassical Ohm's law that includes the bootstrap current. Additional effects previously described in the context of the drift-tearing modes are not included. The outcome is a generalized Rutherford equation

(GRE) for the island evolution that, in its simplest form, can be written as

$$\frac{dw}{dt} \approx 1.22\,\eta\,[\Delta' - \alpha' w + C_{\rm BS}/w + ...], \tag{10}$$

where $C_{\rm BS}$ is a constant proportional to the previous $C_b$, and the dots indicate additional terms that may be included in the refined theory. The bootstrap current contribution $C_{\rm BS}/w$ is obtained by assuming complete flattening of the pressure gradient in the island, which would occur if transport along the island field lines is fast enough. In this simple form, Eq. 10 has a solution $w \approx -C_{\rm BS}/\Delta'$ for all negative values of $\Delta'$. However, parallel transport is less efficient when the island is smaller. Eventually, at small enough island size, the gradient-restoring effect of perpendicular transport prevails and below some critical island size $w_c$ no flattening occurs and the bootstrap current effect disappears. The value of $w_c$ was estimated in Ref. [21] to scale like $w_c \sim (\chi_\perp/\chi_\parallel)^{1/4}$. This effect can be taken into account with a modification of the bootstrap current term in Eq. 10 to $C_{\rm BS} w/(w^2 + w_c^2)$. The important consequence is the appearance of a minimum value of $\Delta'_{\rm NTM} \approx -C_{\rm BS}/w_c$ below which full stabilisation occurs. Since actual tokamak operating scenarios often refer to a given family of current profiles, which means that $\Delta'$ can be considered as given, the previous condition can be seen as a condition on $C_{\rm BS}/w_c$, hence on $\beta$, above which NTMs are possible.

The problem with the GRE is that it is too simple to describe island rotation. However, certain terms in the GRE, and in particular the polarisation current term associated with the vorticity, depend on the frequency, which must then be treated as a free parameter. Moreover the possibly stabilising effects of the diamagnetic frequency and of the parallel ion dynamics are not included.

To explore the occurrence of NTMs with all the drift-tearing effects, a systematic investigation of the full model (6-9) has been undertaken. Starting from a saturated island solution obtained at fixed $(\omega_*, \beta)$, positive $\Delta'$ and $C_b = 0$, the bootstrap current term is initially switched on and a new finite island solution is found. Then, a sequence of neoclassical drift-tearing solutions is obtained by reducing $\Delta'$ in steps through zero. Substantial island size is measured for negative $\Delta'$ until, below a $C_b$-dependent critical value, the island suddenly disappears. This effect is consistent with the existence of $w_c$ as discussed before. One has to note that parallel and perpendicular competing density transport effects are automatically built in the model. However, besides collisional transport, $\mathbf{E} \times \mathbf{B}$ convection and time-dependent effects associated with the island rotation can also play a role, so at present it is not clear how the observations must be related to Ref. [21]. Another important observation is the reduction of the island width with respect to the value given by the GRE. Whether or how this is related to the additional physics present in the four-field model is under investigation. If confirmed by a wider parameter scan, this would be an important result, since impact of magnetic islands on transport must be minimised in the future fusion reactor.

## 4. Double tearing modes in fully non inductive discharges in Tore Supra

Fully non-inductive current discharges in tokamaks often develop non-monotonic $q$ profiles. This occurs rather naturally, and it is a desirable feature since slightly reversed magnetic shear profiles are also often characterised by reduced transport. However, non-monotonic $q$ profiles allow double tearing modes (DTMs) to develop[22]. The resistivity scaling of the growth rate of DTMs is essentially tearing-like ($\sim \eta^{3/5}$) except when the two rational surfaces are very close, in which case the internal kink dependence ($\sim \eta^{1/3}$) is found. Nonlinearly, an intermediate regime where Rutherford's growth is followed by a faster dynamics was found[23]. This phenomenology is indeed consistent with the observations in the Tore Supra tokamak, where a transition between a regime of good confinement with low MHD activity and a regime of worse confinement occurs during the slow evolution of this type of discharges. An example of this behaviour is shown in Fig. 3a. A tearing mode stability calculation that employs the $q$ profile obtained from the CRONOS code[24] has shown a change in the nature of the $(2, 1)$ tearing modes occurring around the $q = 2$ surfaces. In the low MHD phase, the modes are effectively localised around their respective surface, as for ordinary low-$\Delta'$ tearing modes. As the transition to the strong MHD phase is approached, the circulation pattern in between the two rational surfaces becomes more delocalised as the minimum value of $q$ and $\Delta'$ correspondingly increases until a crash associated with complete reconnection between the two $q = 2$ sets the beginning of the strong MHD phase. The $q$ profile and the numerically computed linear displacement in the low MHD regime and shortly before the onset of the strong MHD regime are shown in Fig. 3b

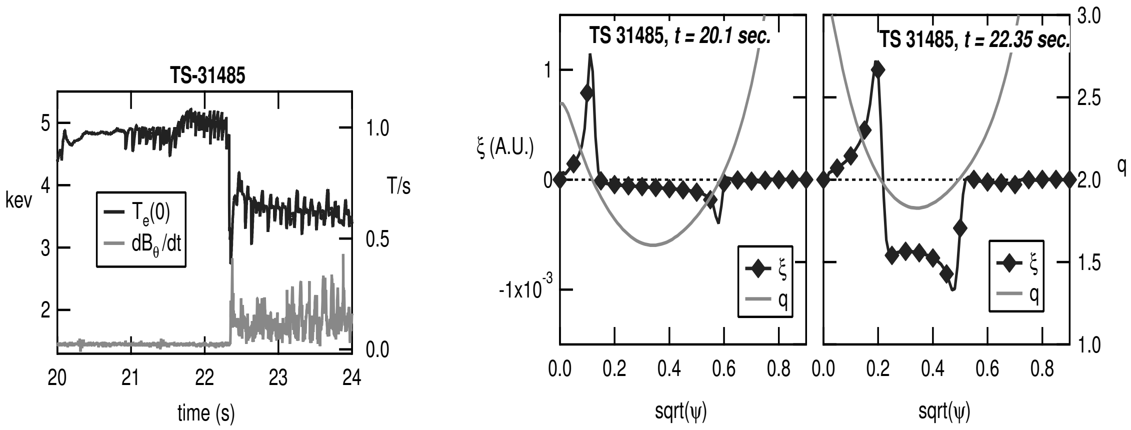


**Figure 3.** Transition between MHD regimes and development of double-tearing modes in Tore Supra. a) central electron temperature and MHD signal; b) $q$ profile and linear displacements in the two double-tearing modes regimes

## 5. Fast reconnection

The sawtooth crash is one of the fastest phenomena occurring in tokamaks. Since its first observation[25], it was identified as mainly a combination of an $m = 1$,$n = 1$ (internal kink) mode coupled with an $m = 0$,$n = 0$ relaxation. In an early work, Kadomtsev[26] proposed a full reconnection model in cylindrical geometry, which, consistently with an extension to time-dependent reconnection of the Sweet-Parker scenario[6], predicted that the crash time would occur on the nonlinear resistive timescale $\tau_{\rm crash} \sim \eta^{-1/2}$.

Later observation in larger, less collisional machines, and in particular in JET[27] showed that crash times can be shorter than the collisional timescale. From the theoretical viewpoint this implies that a form of generalised Ohm's law with additional non-collisional effects must be employed. By including the electron inertia[28], and the plasma pressure[29], which bring in the electron skin depth $d_e = c/\omega_{pe}$ and the ion sound Larmor radius $\rho_s = (T_e/m_i)^{1/2}/\Omega_{ci}$ as characteristic scales, it was shown that the reconnection time scales like $\tau_{\rm rec} \sim 1/d_e$, for cold enough electron plasmas ($\rho_s < d_e$), and like $\tau_{\rm rec} \sim 1/(\rho_s^{2/3} d_e^{1/3})$, for a warm electron plasmas ($\rho_s > d_e$). This would give a timescale of the order of $40\mu$s, for JET parameters, in agreement with the observations. In the nonlinear fluid simulations of Refs. [30, 31], this result was found to hold well into the nonlinear phase where a super-exponential growth is also observed. Moreover, saturation of the island width can occur for certain systems due to the phase mixing of the relevant Lagrangian invariants of the problem[32]. For more details one should refer to a recent review[33].

These results, obtained in simple planar geometry, may serve as a paradigm not only for the sawtooth crash problem, the original motivation, but also for more general space plasma applications where reconnection timescales based on resistive MHD are found too long to explain the observations. Whether the non-collisional rates given before are short enough to agree with the observation of a given phenomenon, or whether even shorter, truly Alvenic rates are required, is a matter of debate, and it may depend on the specific situation.

In the last couple of years, work on fast reconnection has continued mainly with numerical tools. New interesting results are becoming available, and while lack of space prevents a complete review, some trends in current studies, aiming at different directions, are briefly recalled in the rest of this work. The question of the stability of the current sheet that develops in the nonlinear phase has been addressed in Ref. [34]. By using the model of Ref. [31], it was found that, at some stage of the nonlinear evolution, a secondary, Kelvin-Helmholtz instability of the current sheet can occur. This instability breaks the original tearing-mode parity of the system, which is essentially preserved in the linear and in the early nonlinear phase. Interestingly, no definite change in the overall reconnection rate was reported. The condition for the existence of this instability is that the plasma is cold enough ($\rho_s < d_e$). In the opposite case, the system can grow to saturation as discussed in Ref. [32].

The problem of the growth of magnetic islands of different helicity in three dimensions, and of the related generation of magnetic stochasticity and possibly enhanced transport is an important one, in particular in the context of the energy losses during the sawtooth crash when toroidicity effects are taken into account. A numerical study presented at this Conference[35] considers this problem in the collisionless regime. Three phases of the dynamics were observed, a linear phase when the islands grow independently, an early nonlinear phase when stochastic layers develop, which is characterised by an acceleration of the reconnection rate, and a second nonlinear phase when global stochasticity develops. The consequence of these dynamics on particle transport across the reconnected (stochastic or regular) region is currently under investigation.

The validity of fluid equations to describe collisionless reconnection is sometimes questioned. Whereas the importance of kinetic effects has been shown in applications with weak or zero guide field, when the plasma is weakly magnetised near the reconnection layer, comparatively less attention has been given to the possible role of kinetic effects in strong guide field situations typical of tokamaks. Since the reconnection rate in these machines would still satisfy the low frequency conditions $\omega << \Omega_{ci}$, the proper tools for kinetic studies in these regimes would be the gyrokinetic equation for ions (to treat sub-$\rho_i$ scales) and the drift-kinetic equation for the electrons. First results in two dimensions with cold ions and drift kinetic electrons are now available[36, 37]. These studies employ an electron drift kinetic equation of the form

$$\partial f_e/\partial t + [H_{GCe}, f_e] = 0 \tag{11}$$

Where $H_{GCe} = \phi - (P_z + \psi)^2/2$ is the guiding centre Hamiltonian associated with the conserved canonical momentum $P_z$, on which the electron guiding centre distribution function $f_e = f_e(x, y, P_z)$ depends parametrically. The system is closed by Ampere's law, to determine $\psi$, and by an equation for the ion dynamics that evolves $\phi$. The first studies, carried out in the warm plasma regime, have found fast reconnection rates in the early nonlinear phase, comparable to those seen in fluid simulations. The system also exhibits a breaking of tearing-mode parity in the nonlinear phase, consistent with the absence of a specific symmetry in Eq. 11. Regimes where strong particle acceleration can occur as the plasma flow, associated with reconnection, increases, are being explored. Indeed, one can estimate that the conservation of the canonical momentum implies that a radial displacement of the order of a poloidal electron Larmor radius is enough to double the electron velocity. This effect competes with the fact that electrons move along the field lines and therefore would explore regions where the $\mathbf{E}\times\mathbf{B}$ drift is alternatively inward or outward. In colder plasmas regime this effect is reduced and particle acceleration is more likely. Generation of fast electrons is a fairly common phenomenon that accompanies the sawtooth crash in tokamaks.

## 6. Conclusions

The theoretical investigation of various kinds of magnetic reconnection phenomena, based primarily on detailed numerical study of relatively simple model systems, has continued to yield interesting information. In the last couple of years, one has also observed a revival of the use of analytic tools, particularly for the more slowly evolving modes, with improved techniques and perhaps a slight change of perspective. This approach is clearly necessary to interpret the increasing wealth of available numerical data. There is also a clear need of improved analytic modelling to interpret the experimental data of tearing modes and NTMs, that take into account all the key physics in the weakly collisional parameter regimes of present devices. Studies of fast reconnection with fluid codes in simple geometry have confirmed the important role of electron inertia and plasma pressure in allowing a rapid evolution of a reconnecting instability well into the nonlinear regime. New lines of investigation open up with non-symmetric or 3D systems being explored. First studies of collisionless magnetic reconnection with drift-kinetic codes that deal with the large scale-separation between the reconnection layer and the system size, that characterises fusion research devices, have become available. This approach would allow in principle to assess how good the fluid approach can be, and to determine under which conditions particle acceleration and significantly non-Maxwellian distributions can occur. Gyrofluid codes, originally developed for turbulence studies, which allow treating non-isotropic plasmas quite naturally, are a yet unexplored option worth considering.

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