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## ON FREE FALL OF A RELATIVISTIC PARTICLE

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In the present work the free fall of a relativistic particle is considered: the well-known fact of the light velocity constancy is taken into account in the Galilean problem about the movement of a particle free from nongravitational forces and its fall onto the ground. The velocity hodograph and the world line of the particle are found.

В настоящей работе рассмотрено свободное падение релятивистской частицы: известный факт постоянства скорости света учтен в задаче Галилея о свободном от негравитационных сил полете частицы и ее падении на землю. Найдены годограф скорости и мировая линия этой частицы.

The question about taking into account the light velocity constancy in the Galilean problem has been raised by Einstein in his first attempt [1] to construct a relativistic gravity theory.

In the Galilean problem the earth surface looks like a fixed Euclidean plane, the gravitation field intensity above the earth surface is characterized by a positive constant, called the free fall acceleration, usually denoted by  $g$ . It is the simplest gravitational field.

As in each problem, the consideration of the light velocity constancy in the Galilean problem is reduced to the replacement of the Euclidean geometry in the velocity space with the Lobachevsky geometry, since the light velocity  $c$ , entering the Lorentz transformations, turns out to be the Lobachevsky parameter in the geometry of the velocity space.

The notion of velocity space for the case of Euclidean geometry in this space was introduced by Hamilton in [2].

The notion of velocity space for the case of Lobachevsky geometry in this space was introduced by Kotelnikov in [3].

Poincare found in [4] that the Lobachevsky geometry is realized on a hyperboloid. From the last result it follows that in the velocity space the Lobachevsky geometry is realized.

Indeed, the end of 4-vector  $(U^1, U^2, U^3, U^4)$  of a particle velocity lies upon the upper half of the hyperboloid

$$cU^4 = \sqrt{c^2 + U^1U^1 + U^2U^2 + U^3U^3}. \quad (1)$$

The interior geometry of the hypersurface (1) in the Minkowski space is given by the metric

$$dS^2 = dU^1 dU^1 + dU^2 dU^2 + dU^3 dU^3 - c^2 dU^4 dU^4, \quad (2)$$

where

$$c^2 dU^4 dU^4 = \frac{(U^1 dU^1 + U^2 dU^2 + U^3 dU^3)^2}{c^2 + U^1U^1 + U^2U^2 + U^3U^3}. \quad (3)$$

It is the known metric in the Lobachevsky space. Equation (3) follows from Eq. (1).

The equation  $U^1 = 0$  determines on the upper half of the hyperboloid (1) the Lobachevsky plane, and the couple of equations  $U^1 = 0, U^2 = 0$  determines the Lobachevsky straight line. The transition from the Galilean to the Lorentz transformations leads to the transition from Euclidean to Lobachevsky geometry in the velocity space. For more information on this subject, see [5].

In the considered problem the space of events  $M$  in regard to the background connection  $\check{\Gamma}_{mn}^a$  is the 4-dimensional affine space with map  $(x^1, x^2, x^3, x^4)$ , in which  $\check{\Gamma}_{mn}^a$  is equal to zero.

The notion of a fixed plane (let us denote it by  $E_2$ ) demands the introduction in the space of events of the structure of a direct multiplication  $M = P \times T$  of the 3-dimensional affine space  $P$  with the affine time line  $T$ . The numbers  $x^1 = x, x^2 = y, x^3 = z$  are the affine coordinates in  $P$ , the number  $x^4 = t$  is the affine coordinate in  $T$ .

The history of the fixed plane  $E_2$  is represented in  $M$  by the hyperplane  $z = 0$  with the following metric:

$$c^2 d\check{\tau}^2 = c^2 dt^2 - dx^2 - dy^2. \quad (4)$$

The gravitational field is considered in the domain  $z > 0$ , where the field metric equals

$$c^2 d\tau^2 = c^2 \left(1 + \frac{gz}{c^2}\right)^2 dt^2 - dx^2 - dy^2 - dz^2. \quad (5)$$

The field connection  $\Gamma_{mn}^a$  is the Christoffell connection for this metric. The particle's equations of motion are the geodesic equations for this connection. In the considered case they can be transformed to the following form:

$$\frac{d}{d\tau} \left( \frac{dx}{d\tau} \right) = 0, \quad \frac{d}{d\tau} \left( \frac{dy}{d\tau} \right) = 0, \quad (6)$$

$$\frac{d}{d\tau} \left( \frac{dz}{d\tau} \right) + g \left(1 + \frac{gz}{c^2}\right) \left( \frac{dt}{d\tau} \right) \frac{dt}{d\tau} = 0, \quad (7)$$

$$\frac{d}{d\tau} \left[ \left(1 + \frac{gz}{c^2}\right) \frac{dt}{d\tau} \right] + \frac{g}{c^2} \left( \frac{dz}{d\tau} \right) \frac{dt}{d\tau} = 0. \quad (8)$$

On the particle's world line the equation

$$c \left(1 + \frac{gz}{c^2}\right) \left( \frac{dt}{d\tau} \right) = \sqrt{c^2 + \left( \frac{dx}{d\tau} \right)^2 + \left( \frac{dy}{d\tau} \right)^2 + \left( \frac{dz}{d\tau} \right)^2} \quad (9)$$

is fulfilled. As a consequence of (8), the energy

$$E = \left[ \left(1 + \frac{gz}{c^2}\right)^2 \frac{dt}{d\tau} - 1 \right] c^2 \quad (10)$$

of the particle, divided by its rest mass, is conserved.

The solution of Eqs. (6)–(8) was obtained in [6] in the following way. The system of these equations splits up into the following couple of systems:

$$\frac{du^1}{dt} = 0, \quad \frac{du^2}{dt} = 0, \quad \frac{du^3}{dt} = -gu^4, \quad \frac{du^4}{dt} = -\frac{g}{c^2}u^3, \quad (11)$$

$$u^1 = \left( \frac{dx}{d\tau} \right), \quad u^2 = \left( \frac{dy}{d\tau} \right), \quad u^3 = \left( \frac{dz}{d\tau} \right), \quad u^4 = \left(1 + \frac{gz}{c^2}\right) \left( \frac{dt}{d\tau} \right). \quad (12)$$

From (11) follows the Lorentz transformation

$$\begin{aligned} u^1 &= U^1, & u^2 &= U^2, \\ u^3 &= U^3 \cosh \frac{s}{c} - cU^4 \sinh \frac{s}{c}, \\ cu^4 &= cU^4 \cosh \frac{s}{c} - U^3 \sinh \frac{s}{c}, \end{aligned} \quad (13)$$

where  $U^a$  is the value of  $u^a$  at  $t = 0$ , and  $s = gt$  is rapidity of the particle. It is important that the rapidity  $s$  is proportional to the time  $t$  with a coefficient of proportionality equal to  $g$ . Values  $U^1, U^2, U^3$  may be chosen arbitrarily. As to  $U^4$ , from (9) and (12) it follows that equality (1) is correct.

From (12) and (13) it follows that

$$x = X + U^1\tau, \quad y = Y + U^2\tau, \quad (14)$$

$$\left(\frac{c^2}{g} + z\right) \cosh \frac{s}{c} = \frac{c^2}{g} + Z + U^3\tau, \quad \left(\frac{c^2}{g} + z\right) \sinh \frac{s}{c} = cU^4\tau. \quad (15)$$

Here we assume that at  $\tau = 0$  the coordinate  $t$  of the particle takes zero value, and also the coordinates  $x, y, z$  take values  $X, Y, Z$ . Values  $X, Y$  may be chosen arbitrarily, but  $Z > 0$ .

According to (10), the energy  $c^2 + E$  of the particle is equal to

$$c^2 + E = (c^2 + gZ)U^4. \quad (16)$$

From (15) it follows that

$$\tanh \frac{gt}{c} = \frac{cU^4g\tau}{c^2 + gZ + U^3g\tau}, \quad (17)$$

$$c^2 + gz = \sqrt{(c^2 + gZ + g\tau U^3)^2 - (g\tau cU^4)^2}. \quad (18)$$

From the condition  $z \geq 0$  it follows that the time  $\tau$  of the particle's flight above the ground is restricted by the roots of the quadratic equation

$$g(c^2 + U^1U^1 + U^2U^2)\tau^2 = 2U^3(c^2 + gZ)\tau + 2c^2Z + gZ^2. \quad (19)$$

At any energy  $E$  and from any height  $Z$  the particle falls to the ground, flying from the point  $(X, Y, Z)$  over a distance less than  $\sqrt{2}Z$  in a time  $t$  less than  $(-U^4/U^3)Z$  if

$$U^3 < 0 \quad \text{and} \quad U^1U^1 + U^2U^2 < U^3U^3. \quad (20)$$

Under these conditions the world line of the falling particle lies in the domain

$$0 < z < Z - \sqrt{(x - X)^2 + (y - Y)^2}, \quad 0 < t/Z < -U^4/U^3. \quad (21)$$

The gravitational field of the Earth may be thought to be constant under these conditions if the height  $Z$  is small enough.

Therefore, the results obtained can be applied to neutral decay products falling from a small height and neutral particles being constituents of cosmic rays which are registered at the

same height and continuing their fall after registration. For instance,  $gZ/c^2 = 6.95 \cdot 10^{-14}$ , if  $Z = 10^{-4}R = 638$  m, where  $R$  is the radius of the Earth. This height is small enough.

Now consider the movement of this particle in the uniformly accelerated reference frame

$$\begin{aligned}\hat{x} &= x, & \hat{y} &= y, \\ \hat{z} &= \left(\frac{c^2}{g} + z\right) \cosh \frac{gt}{c} - \frac{c^2}{g}, \\ c\hat{t} &= \left(\frac{c^2}{g} + z\right) \sinh \frac{gt}{c}.\end{aligned}\tag{22}$$

For such a frame, see [6] and [7].

According to (14) and (15) we have

$$\hat{x} = X + U^1\tau, \quad \hat{y} = Y + U^2\tau, \quad \hat{z} = Z + U^3\tau, \quad \hat{t} = U^4\tau.\tag{23}$$

So in the reference frame (22) the particle is moving on straight line without acceleration with a velocity equal to

$$\hat{u}^1 = \frac{d\hat{x}}{d\tau} = U^1, \quad \hat{u}^2 = \frac{d\hat{y}}{d\tau} = U^2, \quad \hat{u}^3 = \frac{d\hat{z}}{d\tau} = U^3, \quad \hat{u}^4 = \frac{d\hat{t}}{d\tau} = U^4.\tag{24}$$

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