Eddy Current Analysis by BEM Utilizing Loop Electric and Surface Magnetic Currents as Unknowns

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Abstract. The surface integral equations whose unknowns are the surface electric and magnetic currents are widely used in eddy current analysis. However, when the skin depth is thick, computational error is increased especially in obtaining electromagnetic fields near the edge of the conductor. In order to obtain the electromagnetic field accurately, we propose an approach to solve surface integral equations utilizing loop electric and surface magnetic currents as unknowns.

1. Introduction

Applying Green's theorem, we can obtain integral representations of electromagnetic fields from Maxwell's equation [1]. Obtaining the electromagnetic fields on the surface of the conductor and enforcing the continuity conditions of the tangential and normal components of the electromagnetic fields, we get surface integral equations whose unknowns are the surface electric and magnetic currents, $\boldsymbol{J}_s$ and $\boldsymbol{K}_s$ [2].

At the sharp edge, the normal and tangential directions can not be defined and the continuity conditions become void. Therefore, the conventional BEM is employed for solving eddy current problems when the skin depth is small and the surface of the conductor is flat [3]. In order to get accurate solutions even when the skin depth is large, an approach to analyze the eddy current has been proposed by employing surface integral equations whose unknowns are loop electric and surface magnetic currents, I_ℓ and $\boldsymbol{K}_s$ [4]. By introducing I_ℓ, the essential electromagnetic condition that the divergence of $\boldsymbol{J}_s$ is zero is satisfied automatically. By employing a constant surface element for $\boldsymbol{K}_s$, the condition that the sum of the magnetic charge is zero is satisfied automatically.

As the electromagnetic fields at the edge can not be obtained by the conventional BEM, an edge boundary condition has been utilized to obtain the electromagnetic fields at the edge [5]. At the edge of the conductor, the electric current which flows on the surface is zero because the current can not get into the air. On the basis of this condition, the surface integral equation method is improved in order to get accurate computed results near the edge.

2. Formulation of Eddy Current

2.1 Integral Representations of Electromagnetic Fields

The integral representations of electromagnetic fields are derived from Maxwell's equation by applying Green's theorem [1]. The magnetic and electric fields, $\boldsymbol{H}_{op}$ and

E_{op}, in space with a permeability and permittivity, μ_o and ε_0, and the magnetic and electric field, H_{ip} and E_{ip}, in a conductor with a permeability and conductivity μ_i and σ, are given as follows.

$$H_{op} = H_{ep} + \int_S [J_s \times \nabla G_o + H_n \nabla G_o] dS, \tag{1}$$

$$E_{op} = E_{ep} - \int_S [j\omega\mu_o J_s G_o + K_s \times \nabla G_o + E_n \nabla G_o] dS, \tag{2}$$

$$H_{ip} = -\int_S [\sigma K_s G_i + J_s \times \nabla G_i + H_n \nabla G_i] dS, \tag{3}$$

$$E_{ip} = \int_S [j\omega\mu_i J_s G_i + K_s \times \nabla G_i] dS \tag{4}$$

where ω is the angular frequency, H_{ep} and E_{ep} are the exciting magnetic and electric fields at a calculating point P_o produced by electromagnetic sources, respectively, S is the surface of the conductor, the subscript p denotes the value at P_o and $j = \sqrt{-1}$. Also, we define

$J_s = n \times H_s$, $H_n = \mathbf{n} \quad \mathbf{H}_s$, $K_s = -n \times E_s$, $E_n = \mathbf{n} \quad \mathbf{E}_s$

with the electric and magnetic fields on the surface of the conductor E_s, H_s and the unit normal n directed from the inside to the outside of the surface,

$$G_o = \frac{\exp(-k_o r)}{4\pi r}, \qquad G_i = \frac{\exp(-k_i r)}{4\pi r}$$

with the distance r from a point on the surface to P_o and

$$k_o = j\omega\sqrt{\mu_o \varepsilon_o}, \qquad k_i = \sqrt{j\omega\mu_i \sigma}.$$

The normal components of E_s and H_s are given also as

$$E_n = -\nabla \mathbf{J}_s / (j\omega\varepsilon), \quad H_n = -\nabla \mathbf{K}_s / (j\omega\mu)$$

with the permittivety ε and the permeability μ of the medium.

2.2 Surface Integral Equations for Eddy Current Analysis

The surface electric and magnetic currents have to satisfy conditions that the divergence of J_s is zero and the sum of the surface magnetic charge is zero. In order to satisfy the former condition automatically, we use a loop current I_ℓ circulating along the edge of the surface element. In order to satisfy the latter condition automatically, we use a constant surface element for K_s.

Next, we derive the integral equations suitable for I_ℓ and K_s to be determined.

Obtaining the normal component of the magnetic field on the surface of the conductor from (1) and (3) and considering the boundary condition of the continuity of the magnetic flux density B, that is $\mu_o(n \cdot H_{op}) = \mu_i(n \cdot H_{ip})$, we get

$$n_p \cdot \int_S \left[\sigma K_s \nabla G_i - J_s \times \nabla \left(G_o + \frac{\mu_i}{\mu_o} G \right) + H_n \nabla (G_o + G_i) \right] dS = -n_p \cdot H_{ep} \tag{5}$$

Choosing P_o on the surface of the conductor and taking the vector product on (4) with $\boldsymbol{n}$, we obtain

$$\frac{\boldsymbol{K}_{sp}}{2} - \boldsymbol{n}_p \times \int_S \left[j\omega\mu_i \boldsymbol{J}_s G_i + \boldsymbol{K}_s \times \nabla G_i \right] dS = 0 \cdot \tag{6}$$

The unknowns of (5) and (6) are the loop electric and surface magnetic currents, I_ℓ and $\boldsymbol{K}_s$, because J_s and H_n are given as functions of I_ℓ and $\boldsymbol{K}_s$. By dividing the surface of the conductor to be analyzed into n small elements and introducing I_ℓ and $\boldsymbol{K}_s$, we obtain from (1)

$$\boldsymbol{J}_s = 2\boldsymbol{n}_p \times \boldsymbol{H}_{ep} + 2\boldsymbol{n}_p \times \sum_{i=1}^{n} \left[\oint_C I_\ell \boldsymbol{u} \times \nabla G_o d\ell + \int_{\Delta S} \frac{\nabla_s \cdot \boldsymbol{K}_s}{j\omega\mu_o} \nabla G_o dS \right], \tag{7}$$

$$H_n = 2\boldsymbol{n}_p ¥ \boldsymbol{H}_{ep} + 2\boldsymbol{n}_p ¥ \sum_{i=1}^{n} \left[\oint_C I_\ell \boldsymbol{u} \times \nabla G_o d\ell + \int_{\Delta S} \frac{\nabla_s \cdot \boldsymbol{K}_s}{j\omega\mu_o} \nabla G_o dS \right] \tag{8}$$

where $\boldsymbol{u}$ is the direction of the loop current, C is the line integral route along the loop current and ΔS is area of the small element.

2.3 *Surface elements for solving Integral Equations*

The electric current is zero at the edge of the conductor because the current can not get into the air. Therefore, the parallel component of $\boldsymbol{K}_s$ to the edge of the conductor is zero. In solving the surface integral equations, we adopt a surface element for $\boldsymbol{K}_s$ shown in Fig.1. The x and y components of $\boldsymbol{K}_s$, K_{sx} and K_{sy}, are given as

$$K_{sx} = a_x y + b_x, \tag{9}$$

$$K_{sy} = a_y x + b_y \cdot \tag{10}$$

As given in (9) and (10), K_{sx} and K_{sy} are constant along x and y axes, respectively, and b_x and b_y are zero on the element adjacent to the edge.

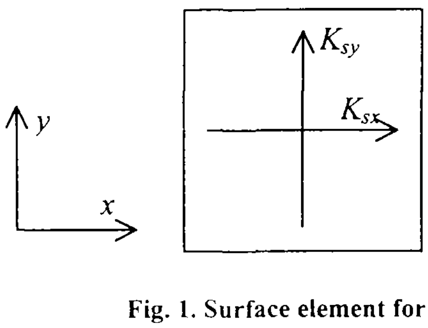

Fig. 1. Surface element for K_s

In solving surface integral equations, the surface of the conductor is divided into surface elements shown in Fig. 2. The surface magnetic current $\boldsymbol{K}_s$ has two components shown by arrows. The loop electric current I_ℓ circulates the edge of the element shown by the solid line. The calculating points for solving the surface integral equations are positioned at the center of each surface element.

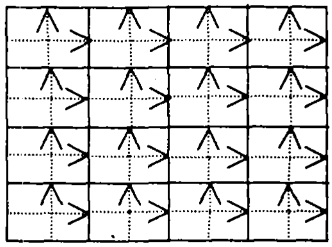

Fig. 2. Surface elements for solving the surface integral equations

3. Eddy Current Analysis

3.1 Eddy Current Analysis Model

In order to check the effectiveness and adequacy of the proposed BEM, we analyze eddy currents of a thick conductor plate (8x8x2 [cm]) shown in Fig.3. The plate is placed in a uniform magnetic field $\boldsymbol{H}_e$ directed perpendicular to the plate. The strength of the magnetic field is 1 A/cm and the frequency is chosen so that the skin depth δ_s becomes 1 cm. The conductivity σ and relative permeability μ_r of the non ferrous conductor are 352600 S/cm and 1. The frequency f is 71.8 Hz. Those of the ferrous conductor are as follows.σ=72500 S/cm, μ_r=200 and f=1.75 Hz.

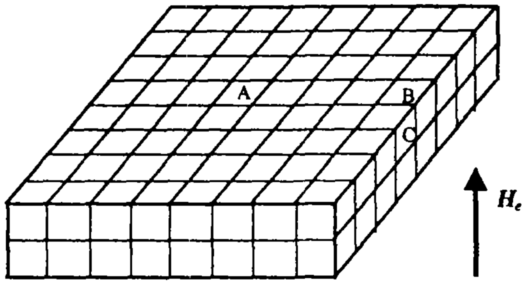

Fig. 3. A thick conductor plate to be analyzed

In solving (5) and (6), these equations are discretized to be simultaneous linear equations, $[A][x]=[B]$, by dividing the surface of the plate into small surface elements and choosing P_o at the center of the element. The upper and lower surfaces of the conductor are divided equally into 64 (8x8) surface elements, respectively, and each side surface is divided equally into 16 (8x2) surface elements as shown in Fig.3.

The number of the unknowns of I_ℓ is the same as the number of surface division n but one of the unknowns can be set free. The number of the unknowns of $\boldsymbol{K}_s$ is $2n$. Then, the total number of unknowns of the simultaneous equations is $3n$. On the other hand, the total number of unknowns in the conventional BEM is $4n$.

Adding one equation to fix one of the unknowns of I_ℓ, we solve (5) and (6) with the help of the least square method.

3.2 Check of Computed Electromagnetic Fields

Once the surface integral equations (5) and (6) are solved, the magnetic field outside and inside the conductor are obtained by using (1) and (3), respectively with the help of (7) and (8). The computed values of J_s and H_n of the nonferrous conductor are shown in Fig. 4. Those of the ferrous conductor are shown in Fig. 5. In these figures, A, B and C denote positions at the center of the upper surface, the edge and the center of the side surface as shown in Fig. 3. The symbols O, ◊ denote real and imaginary parts of the computed values of H_n. The symbols Δ, ∇ denote those of J_s. The solid and dotted lines denote those computed by FEM with fine meshes. In the case of the ferrous conductor, the imaginary parts of the magnetic field are almost zero.

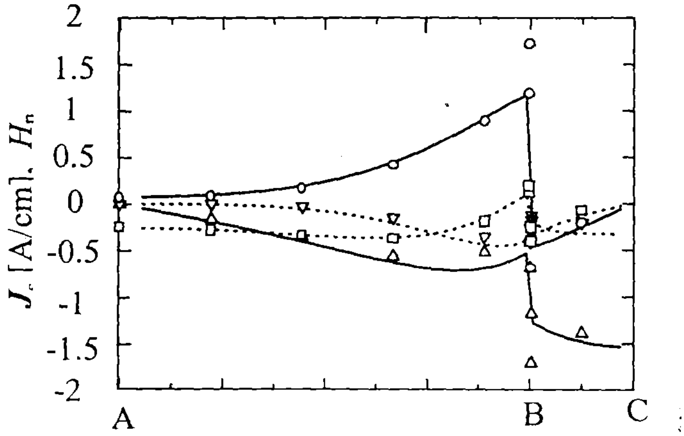

Fig. 4. Computed results of J_s and H_n (Nonferrous conductor)
Conductivity: σ=352600 S/cm, Relative Permeability: μ_r =1
Skin depth: δ_s =1 cm, f=71.8 Hz
O, ◊: Real and Imaginary Parts of Computed values of H_n
Δ, ∇: Real and Imaginary Parts of Computed values of J_s
Solid and Dotted lines: Computed values by FEM

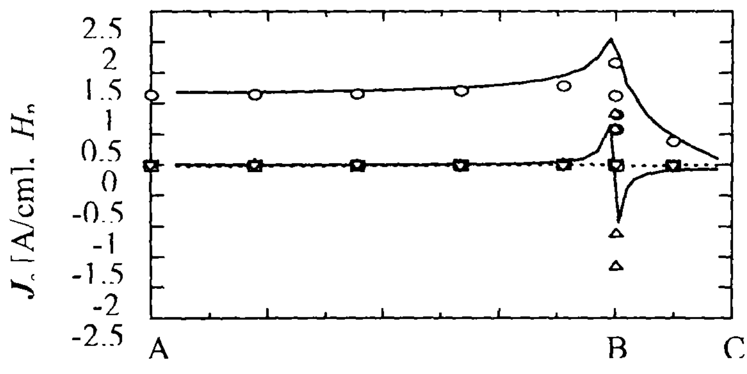

Fig. 5. Computed results of J_s and H_n (Ferrous conductor)
Conductivity:σ=72500 S/cm, μ_r=200
Skin depth: δ_s =1 cm, f=1.75 Hz
O, ◊: Real and Imaginary Parts of Computed values of H_n
Δ, ∇:: Real and Imaginary Parts of Computed values of J_s
Solid and Dotted lines: Computed values by FEM

In this eddy current formulation, the continuity of the normal component of $\boldsymbol{B}$, that is H_n, is satisfied positively as shown in (5). However, the continuity of the tangential component of $\boldsymbol{H}$, that is $\boldsymbol{J}_s$, is not satisfied positively. On the other hand, in the case of the conventional BEM, the continuities of both $\boldsymbol{J}_s$ and H_n are not satisfied positively.

It is checked how the computed values of $\boldsymbol{J}_s$ satisfy the continuity condition by

$$R_t = J_{so} / J_{si} ,$$

where J_s is the absolute value of $\boldsymbol{J}_s$ and the subscripts o and i denote the value outside and inside the conductor. If $\boldsymbol{J}_s$ computed satisfies the continuity condition completely, R_t=1. In Table 1 and 2, the results of R_t are given. In these tables, P_1-P_4 are the calculating points between A and B and P_5 is that between B and C shown in Fig. 3. From these results, we noticed that the magnetic fields computed by proposed method are adequate because these values satisfy both Maxwell's equation and boundary conditions.

Table 1. Check of the continuous condition of tangential component of the magnetic field
(Nonferrous conductor)

	P_1	P_2	P_3	P_4	P_5
R_t	0.989	0.994	1.01	0.826	0.982

Table 2. Check of the continuous condition of tangential component of the magnetic field
(Ferrous conductor)

	P_1	P_2	P_3	P_4	P_5
R_t	1.00	1.01	1.04	1.01	0.996

4. Conclusions

We have examined an approach for eddy current analysis by surface integral equations whose unknowns are loop electric and surface magnetic currents. These unknowns are very simple elements but these satisfy automatically the essential electromagnetic conditions that the divergence of the surface electric current is zero and the sum of the surface magnetic charge is zero. We compute the magnetic fields on the surface of the conductor and compare these with the values computed by FEM. It is found that the magnetic fields can be obtained correctly near the edge.

References

[1] J. A. Stratton, Electromagnetic Theory, McGRAW-HILL, pp.464-466, 1941

[2] Johnson J.H. Wang, Generalized Moment Method in Electromagnetics, John Wiley & Sons, 1991

[3] K. Ishibashi, "Eddy Current Analysis by Boundary Element Method utilizing Impedance Boundary Condition," IEEE Trans. Magn., Vol.31, No.3, pp.1500-1503, 1995

[4] K. Ishibashi, "Eddy Current Analysis by Integral Equation Method Utilizing Loop Electric and Surface Magnetic Currents as Unknowns," IEEE Trans. Magn., Vol.34, No.5, pp.2585-2588, 1998.

[5] K. Ishibashi, "Eddy Current Analysis by BEM Formulated by Loop Electric and Surface Magnetic Currents Utilizing Edge Boundary Condition," IEEE Trans. Magn., Vol.35, No.3, pp.1111-1114, 1999.

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