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# An Analysis of the Electromagnetic Field in Multi-Polar Linear Induction System

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**Abstract.** In this paper a new method for determination of the electromagnetic field vectors in a multi-polar linear induction system (LIS) is described. The analysis of the electromagnetic field has been done by four dimensional electromagnetic potentials in conjunction with theory of the magnetic loops . The electromagnetic field vectors are determined in the Minkovski's space as elements of the Maxwell's tensor . The results obtained are compared with those got from the analysis made by the finite elements method (FEM).

## 1. INTRODUCTION

The determination of the electromagnetic field in a multi-polar linear induction system (LIS) with moving metal sheet is considered .The system is supplied with a three-phase sine-wave power generator . Fig.1 represents the model of the LIS . Due to the presence of two-induction magnetic yoke the magnetic system is symmetric. The metal sheet put in the air gap is moving with a constant velocity $v$=*const* along the $z$ - axis . The driving Laplass' force is a result of the interaction between the excited magnetic field and the induced electric field in the aluminum sheet .

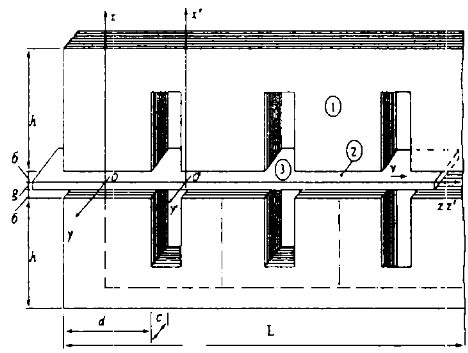

**Figure 1: An idealized model of multi-polar LIS ,where :**
*1-magnetic yoke , 2 –air gap , 3 –metal sheet*
*h , L are height and length of the inductor, respectively ; d , c are length and width of the pole, respectively; δ is air gap , g is thickness of the conductive sheet .*

The inductor with "*m*" number magnetic pole is connected with the fixed XOYZ while the metal sheet – with the mobile X'O'Y'Z' coordinate system.

A model of variable magnetic charge defined by sufficiently thin semi-endless solenoid with variable magnetic flux $\Phi(t)$ is used [1,3].

The cross-section of the magnetic pole is consisted of "*n*"-number magnetic charges (in this case n=5).

## 2. RESULTS

### 2.1. A four –dimensional electromagnetic potential

The loop with variable magnetic flux induces electromagnetic field with electric vector-potential $\vec{A}_\varepsilon$ [3].

$$\vec{A}_\varepsilon = \frac{\varepsilon}{20\pi}\left\{\sum_{i=1}^{n}\sum_{k=1}^{m}\iiint_{(v)}\frac{\partial B/\partial t}{r}dv\right\} = \varepsilon\frac{e(t)}{20\pi}\left\{\sum_{i=1}^{n}\sum_{k=1}^{m}\int_{-l}^{-\delta}\frac{\vec{dl}}{r} + \sum_{i=1}^{n}\sum_{k=1}^{m}\int_{\delta}^{l}\frac{\vec{dl}}{r'}dr\right\}\vec{x}^{o} \quad (1)$$

In an arbitrary point of the coordinate system XOYZ the scalar magnetic potential can be written as:

$$V_\mu = \sum_{i=1}^{n}\sum_{k=1}^{m}\int_{-\delta}^{\delta}\vec{H}\vec{dl} = \sum_{i=1}^{5}\sum_{k=1}^{m}\frac{\Phi(t)}{20\pi\mu}\int_{-\delta}^{\delta}\frac{1}{r}dr. \quad (2)$$

A four-dimensional electromagnetic potential is introduced as:

$$\bar{\Psi}_{(\varepsilon)} = \left\{A_{\varepsilon x}, A_{\varepsilon y} A_{\varepsilon z}, \frac{j}{c}V_\mu\right\}, \quad (3)$$

where $A_{\varepsilon x}, A_{\varepsilon y}, A_{\varepsilon z}$ are components of the electric vector-potential $\vec{A}_\varepsilon$ along *x*- , *y*- , *z*- axes, respectively and $V_\mu$ is scalar magnetic potential.

The Lorentz's transformations for the electromagnetic potentials [1,2 ] in the Minkovski's space $\vec{X} = \{x, y, z, jct\}$ are used:

$$A_{\varepsilon x} = A'_{\varepsilon x'}; A'_{\varepsilon y} = A'_{\varepsilon y'}; A_{\varepsilon z} = \alpha\left(A'_{\varepsilon z'} + \frac{v}{c^2}V'_\mu\right); V_\mu = \alpha\left(V'_\mu + vA'_{\varepsilon z'}\right), \quad (4)$$

were : $\alpha = \frac{1}{\sqrt{1 - v^2/c^2}}$ is relative factor ; $c$ is the speed of the light .

For the four-dimensional electromagnetic potential $\bar{\Psi}_{(\varepsilon)}$ we obtained:

$$\vec{\Psi}_\varepsilon = \left\{ \begin{array}{c} A_{\varepsilon x} = \varepsilon \frac{e(t)}{4\pi} \frac{1}{5} \sum_{i=1}^{5} \sum_{k=1}^{m} \left( Arsh \frac{x+\delta}{\sqrt{P_{ik}}} - Arsh \frac{x+h}{\sqrt{P_{ik}}} + Arsh \frac{x-h}{\sqrt{P_{ik}}} - Arsh \frac{x-\delta}{\sqrt{P_{ik}}} \right) \\ A_{\varepsilon y} = 0 \\ A_{\varepsilon z} = \frac{\alpha \varepsilon v}{20\pi \mu c^2} \sum_{i=1}^{5} \sum_{k=1}^{m} \left( \frac{\Phi_k(t)}{\sqrt{Q_{ik}}} - \frac{\Phi_k(t)}{\sqrt{M_{ik}}} \right) \\ V_\mu = \frac{j}{c} \frac{\alpha}{20\pi \mu} \sum_{i=1}^{5} \sum_{k=1}^{m} \left( \frac{\Phi_k(t)}{\sqrt{Q_{ik}}} - \frac{\Phi_k(t)}{\sqrt{M_{ik}}} \right) \end{array} \right\}, \tag{5}$$

where: $P_{ik} = (y + a_i)^2 + \alpha(z + d_k + b_i - vt)^2$;

$Q_{ik} = (x+\delta)^2 + (y + a_i)^2 + \alpha(z + d_k + b_i - vt)^2$;

$M_{ik} = (x-\delta)^2 + (y + a_i)^2 + \alpha(z + d_k + b_i - vt)^2$;

$C_{ik} = (x+h)^2 + (y + a_i)^2 + \alpha(z + d_k + b_i - vt)^2$;

$D_{ik} = (x-h)^2 + (y + a_i)^2 + \alpha(z + d_k + b_i - vt)^2$.

2.2. **Vectors of the electromagnetic field**

The components of the induced electric field are determined as elements of the dual Maxwell's tensor $F^{(\varepsilon)}$ [1, 3].

$$E_x = \frac{\alpha v}{4n\pi} \sum_{i=1}^{n} \sum_{k=1}^{m} \left[ \frac{\Phi_k(t)(y+a_i)}{Q_{ik}^{3/2}} - \frac{\Phi_k(t)(y+a_i)}{M_{ik}^{3/2}} \right] \tag{6}$$

$$E_y = -\frac{\alpha v}{4n\pi} \sum_{i=1}^{n} \sum_{k=1}^{m} \left[ \frac{\Phi_k(t)(x+\delta)}{Q_{ik}^{3/2}} - \frac{\Phi_k(t)(x-\delta)}{M_{ik}^{3/2}} \right] -$$

$$- \frac{\alpha}{4n\pi} \sum_{i=1}^{n} \sum_{k=1}^{m} \frac{d\Phi_k(t)}{dt} \frac{\alpha(z - vt + d_k + b_i)}{P_{ik}} \left[ \frac{x+\delta}{Q_{ik}^{3/2}} - \frac{x-\delta}{M_{ik}^{3/2}} + \frac{x-h}{D_{ik}^{3/2}} - \frac{x+h}{C_{ik}^{3/2}} \right] \tag{7}$$

$$E_z = \frac{1}{4n\pi} \sum_{i=1}^{n} \sum_{k=1}^{m} \frac{d\Phi_k(t)}{dt} \frac{(y+a_i)}{P_{ik}} \left[ \frac{(x+\delta)}{\sqrt{Q_{ik}}} - \frac{(x-h)}{\sqrt{C_{ik}}} + \frac{(x+h)}{\sqrt{P_{ik}}} - \frac{(x-\delta)}{\sqrt{M_{ik}}} \right] \tag{8}$$

For the magnetic field by analogy are obtained the following expressions:

$$H_x = \frac{\alpha}{4n\pi\mu}\sum_{i=1}^{n}\sum_{k=1}^{m}\left(\left[\frac{\Phi_k(t)(x+\delta)}{Q_{ik}^{3/2}} - \frac{\Phi_k(t)(x-\delta)}{M_{ik}^{3/2}}\right]\right) -$$

$$-\frac{\varepsilon}{4n\pi}\left\{\sum_{i=1}^{n}\sum_{k=1}^{m}\frac{d^2\Phi_k(t)}{dt^2}\left[Arsh\frac{x+h}{P_{ik}^{1/2}} - Arsh\frac{x+\delta}{P_{ik}^{1/2}} + \right.\right.$$

$$\left.\left. + Arsh\frac{x-\delta}{P_{ik}^{1/2}} - Arsh\frac{x-h}{P_{ik}^{1/2}}\right]\right\} -$$

$$-\frac{\varepsilon v\alpha}{4n\pi}\left\{\sum_{i=1}^{n}\sum_{k=1}^{m}\frac{d\Phi_k(t)}{dt}\frac{\alpha\left[z - vt + d_k + b_i\right]}{P_{ik}}\right\}\cdot$$

$$\cdot\left(\frac{x+\delta}{Q_{ik}^{1/2}} + \frac{x+h}{C_{ik}^{1/2}} - \frac{x-h}{D_{ik}^{1/2}} - \frac{x-\delta}{M_{ik}^{1/2}}\right)$$

(9)

$$H_y = \frac{\alpha}{4n\pi\mu}\sum_{i=1}^{n}\sum_{k=1}^{m}\left[\frac{\Phi_k(t)(y+a_i)}{Q_{ik}^{3/2}} - \frac{\Phi_k(t)(y+a_i)}{M_{ik}^{3/2}}\right]$$

(10)

$$H_z = \frac{\alpha^2}{4n\pi\mu}\sum_{i=1}^{n}\sum_{k=1}^{m}\left[\frac{\Phi_k(t)\alpha(z - vt + d_k + b_i)}{Q_{ik}^{3/2}} - \frac{\Phi_k(t)\alpha(z - vt + d_k + b_i)}{M_{ik}^{3/2}}\right] -$$

$$-\frac{\alpha\varepsilon v}{4n\pi}\sum_{i=1}^{n}\sum_{k=1}^{m}\frac{d\Phi_k(t)}{dt}\left[\frac{1}{Q_{ik}^{1/2}} - \frac{1}{M_{ik}^{1/2}}\right] -$$

$$-\frac{\alpha\varepsilon v^2}{4n\pi\mu}\left\{\sum_{i=1}^{n}\sum_{k=1}^{m}\Phi_k(t)\alpha(z - vt + d_k + b_i)\left(\frac{1}{Q_{ik}^{3/2}} - \frac{1}{M_{ik}^{3/2}}\right)\right\}$$

(11)

### 2.3. Numerical results

The calculations for determination of the electric and magnetic field vectors in the linear induction system are done using the following data: height of the magnetic yoke $h = 0.10\,m$, length of the inductor $L = 0.66\,m$, length of the magnetic pole $d = 0.04\,m$, width of the magnetic pole $c = 0.04\,m$, air gap $\delta = 0.01\,m$, thickness of the conductive aluminum sheet $g = 0.002\,m$, velocity of movement $v = 4\,m/s$. For this purpose a programme for computation of the electromagnetic

field vectors is assembled using FORTRAN 90.
The distribution of the magnetic flux lines of the linear induction system is presented in Fig.2.

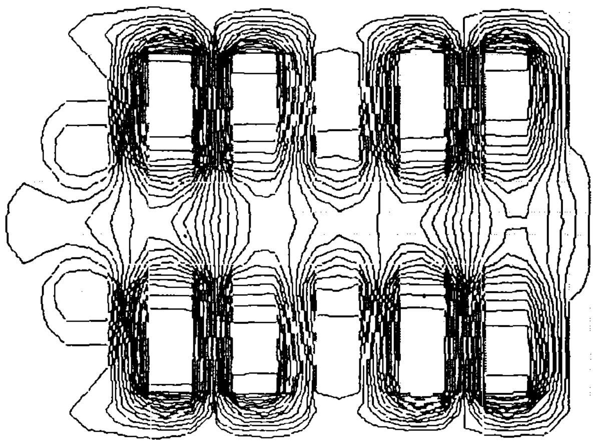

**Figure 2: 2D flux lines for multi polar linear induction system**

The distribution of the magnetic field strength along the $x$ - axis in the cross-section of the linear induction system is shown in Fig.3

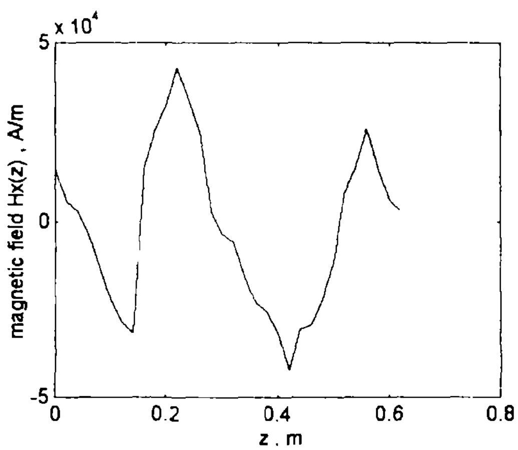


**Figure 3: Distribution of the magnetic field in the air gap of multi-polar LIS calculated by four-dimensional potential**

## 2.4 A comparative analysis with FEM

The outline of the linear induction system analyzed with finite elements method (using ANSYS software) is represented in Fig.4. The cross-section of the multi-polar linear induction system is considered. In this case the magnetic vector-potential is used for the analysis made by FEM.

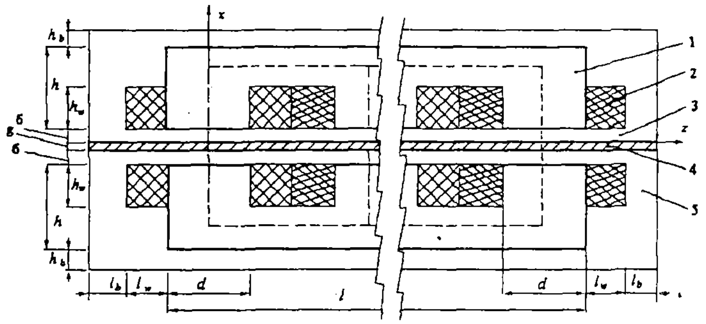

**Figure 4: Outline of multi-polar LIS for an analysis made by the FEM**
*Simulation: 1-magnetic yoke; 2- excitation windings; 3- air-gap; 4- metal sheet; 5- buffer band. Symbols: h-height of the magnetic yoke; l-length of the magnetic yoke;* $h_w$ *- height of the tooth; d-width of the tooth;* $h_b$ *-height of the buffer band;* $l_b$ *- length of the buffer band;* $\delta$ *-thickness of the conductive sheet.*

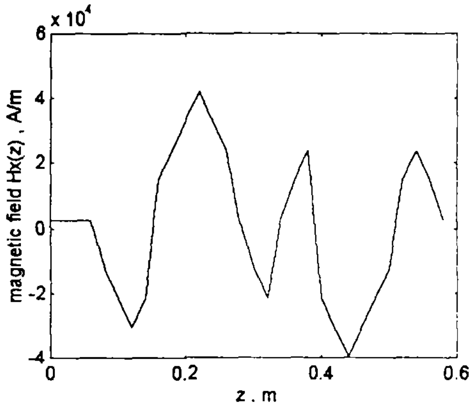


**Figure 5: Distribution of the magnetic field strength in the air-gap of LIS made by the FEM**

## 3. Conclusions

With the method represented in this paper one can determine the electromagnetic field vectors in the multi-polar linear induction system using four-dimensional potential. A priority of this method is the obtaining of analytical results for the electromagnetic field vectors. These results are also valid for linear media. The dependencies are valid also at high speeds of movement.

The results of the investigated linear induction system are comparable to those got by the finite elements method.

The investigations may be continued in the determination of other characteristics such as drag force, levitation force, etc.

The method proposed in this paper for an analysis of linear induction system can be used for optimization calculations.


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